# Analysis of field components of Infinitesimal Dipole

## Case I : Near Field Region ($kr << 1$) :

In this region $kr << 1$ or $r << \lambda/2\pi$. So, the values of E-field and H-field components are reduced to,

$$E_r \simeq \frac{-j\eta I_o l e^{-jkr}}{2\pi k r^3} \cdot \cos\theta \rightarrow (20)$$

$$E_\theta \simeq \frac{-j\eta I_o l e^{-jkr}}{4\pi k r^3} \sin\theta \rightarrow (21)$$

$$H\phi \simeq \frac{I_o l e^{-jkr}}{4\pi r^2} \sin\theta \rightarrow (22)$$

$$E\phi = H_r = H_\theta = 0 \rightarrow (23)$$

The average power density is given by

$$W_{av} = \frac{1}{2} \text{Re}\left[E \times H^*\right]$$

$$= \frac{1}{2} \text{Re}\left[\hat{a}_r E_\theta H_\phi^* - \hat{a}_\theta E_r H_\phi^*\right] \rightarrow (24)$$

$$E_\theta H_\phi^* = \frac{-j\eta I_o l e^{-jkr}}{4\pi k r^3} \cdot \sin\theta \times \frac{I_o l e^{jkr}}{4\pi r^2} \sin\theta$$

$$E_\theta H_\phi^* = \frac{-j\eta}{k}\left(\frac{I_o l}{4\pi}\right)^2 \frac{\sin^2\theta}{r^5} \rightarrow (25)$$

$$E_r H_\phi^* = \frac{-j\eta I_0 l e^{-jkr}}{2\pi k r^3} \cdot \cos\theta \times \frac{I_0 l e^{-jkr}}{4\pi r^2} \cdot \sin\theta$$

$$= \frac{-j\eta (I_0 l)^2}{k \cdot 8\pi^2} \cdot \frac{\sin\theta \cdot \cos\theta}{r^5} \rightarrow (26)$$

substitute (25) & (26) in (24)

$$W_{av} = \frac{1}{2} \text{Re}\left[-\hat{a}_r \, j \cdot \frac{\eta}{k}\left(\frac{I_0 l}{4\pi}\right)^2 \frac{\sin^2\theta}{r^5} + \hat{a}_\theta \cdot j\frac{\eta}{k} \frac{(I_0 l)^2}{8\pi^2} \cdot \frac{\sin\theta \cos\theta}{r^5}\right] = 0 \rightarrow (27)$$

Case ii :- Intermediate Field Region ($kr > 1$)

As the values of $kr$ becomes moderate ($kr > 1$), the field expressions can be approximated again but in a different form.

In contrast to the region where $kr \ll 1$, the first term within the brackets becomes more dominant and the second and third term can be neglected.

$$E_r \simeq \eta \frac{I_0 l e^{-jkr}}{2\pi r^2} \cdot \cos\theta \rightarrow (28)$$

$$E_\theta \simeq j\eta \frac{k I_0 l e^{-jkr}}{4\pi r} \sin\theta \rightarrow (29)$$

$$H_\phi \simeq \frac{jk I_0 l \cdot e^{-jkr}}{4\pi r} \sin\theta \rightarrow (30)$$

$$E_\phi = H_r = H_\theta = 0 \rightarrow (31)$$

The total Electric field is given by

$$E = \hat{a}_r E_r + \hat{a}_\theta E_\theta \rightarrow (32)$$

$$|E| = \sqrt{|E_r|^2 + |E_\theta|^2} \rightarrow (33)$$

## case iii : Far-Field Region $(kr \gg 1)$

$$E_\theta \simeq j\eta \frac{k I_0 l \, e^{-jkr}}{4\pi r} \cdot \sin\theta \rightarrow (34)$$

$$H_\phi \simeq \frac{j k . I_0 l . e^{-jkr}}{4\pi r} \cdot \sin\theta \rightarrow (35)$$

$$E_r \simeq E_\phi = H_r = H_\theta = 0 . \rightarrow (36)$$

The ratio of $E_\theta$ to $H_\phi$ is equal to

$$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta \rightarrow (37)$$

where,

$Z_w \rightarrow$ wave impedance

$\eta \rightarrow$ intrinsic Impedance ($377 \simeq 120\pi\ \Omega$ for free space)

## Power Density and Radiation Resistance :

For the infinitesimal dipole, the complex poynting vector can be written as,

$$W = \frac{1}{2}(E \times H^*)$$

$$= \frac{1}{2}(\hat{a}_r E_r + \hat{a}_\theta E_\theta) \times (\hat{a}_\phi H_\phi^*)$$

$$W = \frac{1}{2}(\hat{a}_r E_\theta H_\phi^* - \hat{a}_\theta E_r H_\phi^*) \rightarrow ①$$

$$W = (\hat{a}_r W_r + \hat{a}_\theta W_\theta) \rightarrow ②$$

The radial $W_r$ and transverse $W_\theta$ components are given as,

$$W_r = \frac{E_\theta H_\phi^*}{2} = \frac{j\eta k I_0 l \sin\theta}{2 \times 4\pi r}\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right]$$

$$\times \frac{-jkI_0 l \sin\theta}{4\pi r}\left[1 - \frac{1}{jkr}\right] e^{jkr}$$

$$= \frac{\eta (k l I_0)^2}{2 \times (4\pi r)^2} \cdot \sin^2\theta \left[1 - \cancel{\frac{1}{jkr}} + \cancel{\frac{1}{jkr}} - \cancel{\frac{1}{(jkr)^2}} + \cancel{\frac{1}{(jkr)^2}} - \frac{1}{(jkr)^3}\right]$$

$$W_r = \frac{\eta}{\cancel{2}} \cdot \frac{\cancel{4\pi^2}}{\lambda^2} (l I_0)^2 \cdot \frac{\sin^2\theta}{\overset{}{8}\,\cancel{16}\,\cancel{\pi^2} r^2}\left[1 - \frac{j}{(kr)^3}\right]$$

$$W_r = \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right] \rightarrow ③$$

$$W_\theta = -\frac{E_r H_\phi^*}{2} = -\eta \frac{I_0 l \cos\theta}{2 \times 2\pi r^2}\left[1+\frac{1}{jkr}\right] e^{-jkr} \times$$

$$\frac{-jkI_0 l \sin\theta}{4\pi r}\left[1-\frac{1}{jkr}\right] e^{jkr}$$

$$W_\theta = \frac{j\eta (I_0 l)^2 \cdot k \cdot \cos\theta \cdot \sin\theta}{2\times 8\pi^2 r^3}\left[1-\frac{1}{jkr}+\frac{1}{jkr}-\frac{1}{(jkr)^2}\right]$$

$$W_\theta = \frac{j\eta k.(I_0 l)^2 \cos\theta \sin\theta}{16\pi^2 r^3}\left[1+\frac{1}{(kr)^2}\right] \rightarrow (4)$$

The complex power moving in the radial direction is obtained by integrating (1) over a closed sphere of radius $r$.

$$P = \oiint_S W.ds = \int_0^{2\pi}\int_0^{\pi} (\hat{a}_r W_r + \hat{a}_\theta W_\theta).\hat{a}_r r^2 \sin\theta \, d\theta \, d\phi \rightarrow (5)$$

which reduces to

$$P = \int_0^{2\pi}\int_0^{\pi} W_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2 \frac{\sin^2\theta}{\cancel{r^2}}\left[1-\frac{j}{(kr)^3}\right] \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$P = \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2 \sin^3\theta \left[1 - \frac{j}{(kr)^3}\right] d\theta$$

$$= \cancel{2}\pi \cdot \frac{\eta}{4\,\cancel{8}}\left(\frac{I_0 l}{\lambda}\right)^2 \left[1 - \frac{j}{(kr)^3}\right] \int_0^{\pi} \sin^3\theta \cdot d\theta$$

$$= \frac{\pi\eta}{4} \frac{(I_0 l)^2}{\lambda^2} \left[1 - \frac{j}{(kr)^3}\right] \left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]_0^{\pi}$$

$$= \frac{\pi\eta}{4}\left(\frac{I_0 l}{\lambda}\right)^2 \left[1 - \frac{j}{(kr)^3}\right] \left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right]$$

$$P = \frac{\pi\eta}{\cancel{4}}\left(\frac{I_0 l}{\lambda}\right)^2 \left[1 - \frac{j}{(kr)^3}\right] \times \frac{\cancel{4}}{3}$$

$$\therefore P = \eta \cdot \frac{\pi}{3}\left(\frac{I_0 l}{\lambda}\right)^2 \left[1 - \frac{j}{(kr)^3}\right] \rightarrow (6)$$

Equation (5) gives the real and Imaginary power that is moving outwardly, can also be written as,

$$P = \frac{1}{2} \iint_S E \times H^* \cdot ds$$

$$P = \eta \cdot \frac{\pi}{3} \cdot \left(\frac{I_0 l}{\lambda}\right)^2 \left[1 - \frac{j}{(kr)^3}\right]$$

$$P = P_{rad} + j2\omega\left(\tilde{W}_m - \tilde{W}_e\right) \rightarrow (7)$$

where,

$P \rightarrow$ Power ( in radial direction )

$P_{rad} \rightarrow$ time - average power radiated

$\tilde{W}_m \rightarrow$ time-average magnetic energy density

$\tilde{W}_e \rightarrow$ time- average Electric energy density

$2\omega(\tilde{W}_m - \tilde{W}_e) \rightarrow$ time - average imaginary (reactive) power

From (15)

$$P_{rad} = \eta \cdot \frac{\pi}{3} \left(\frac{I_0 l}{\lambda}\right)^2 \rightarrow (16)$$

and

$$2\omega(\tilde{W}_m - \tilde{W}_e) = -\eta\left(\frac{\pi}{3}\right)\left(\frac{I_0 l}{\lambda}\right)^2 \frac{1}{(kr)^3} \rightarrow (17)$$

For large values of $kr$ $(kr \gg 1)$, the reactive power diminishes and vanishes.

since the antenna radiates its real power through the radiation resistance, for the infinitesimal dipole it is found by equating (16) to

$$\boxed{P_{rad} = \eta \cdot \frac{\pi}{3} \cdot \left(\frac{I_0 l}{\lambda}\right)^2 = \frac{1}{2} I_0^2 \cdot R_r \rightarrow (18)}$$

where, $R_r \rightarrow$ Radiation Resistance.

$$\boxed{R_r = \eta \cdot \frac{2\pi}{3}\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \rightarrow (19)}$$

Directivity :-

$$D = \frac{4\pi U}{P_{rad}} \rightarrow (1)$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_{rad} \cdot ds$$

$$= \int_0^{2\pi}\int_0^{\pi} W_{rad} \cdot r^2 \sin\theta \, d\theta \, d\phi \rightarrow (2)$$

$$W_{rad} = W_{av} = \frac{1}{2} Re\left[E \times H^*\right]$$

$$= \frac{1}{2} \hat{a}_r \left[\frac{|E_\theta|^2 + |E_\phi|^2}{\eta}\right] \rightarrow (3)$$

W.K.T for far field Radiation,

$$E_\theta = j\eta \frac{K I_0 l e^{-jkr}}{4\pi r} \cdot \sin\theta \rightarrow (4)$$

$$E_\phi = 0 \rightarrow (5)$$

substitute (4) and (5) in (3)

$$W_{rad} = \hat{a}_r \cdot \frac{|E_\theta|^2}{2\eta}$$

$$= \hat{a}_r \cdot \frac{\cancel{\eta^2}}{2\cancel{\eta}} \left(\frac{K I_0 l}{4\pi}\right)^2 \cdot \frac{\sin^2\theta}{r^2}$$

$$W_{rad} = \hat{a}_r \cdot \frac{\eta}{2} \left(\frac{K I_0 l}{4\pi}\right)^2 \frac{\sin^2\theta}{r^2} \rightarrow (6)$$

substitute 6 in 2

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2}\left(\frac{K I_0 l}{4\pi}\right)^2 \cdot \frac{\sin^2\theta}{\cancel{r^2}} \cdot \cancel{r^2}\sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \cdot \frac{\eta}{2}\left(\frac{K I_0 l}{4\pi}\right)^2 \int_0^{\pi} \sin^3\theta \, d\theta$$

$$= \cancel{2}\pi \cdot \frac{\eta}{\cancel{2}}\left(\frac{K I_0 l}{4\pi}\right)^2 \left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\pi}$$

$$= \pi \cdot \eta \cdot \frac{K^2 \cdot I_0^2 \cdot l^2}{16\pi^2}\left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right]$$

$$P_{rad} = \cancel{\pi} \cdot \eta \cdot \frac{K^2 \cdot I_0^2 \cdot l^2}{4\,\cancel{16}\pi^{\cancel{2}}} \times \frac{4}{3} \Rightarrow \eta \cdot \frac{\cancel{4}\pi^{\cancel{2}}}{\lambda^2} \cdot \frac{I_0^2 \cdot l^2}{\cancel{4}\cancel{\pi}} \times \frac{1}{3}$$

$$\therefore P_{rad} = \eta \cdot \frac{\pi}{3}\left(\frac{I_0 l}{\lambda}\right)^2 \longrightarrow 7$$

$\because K = 2\pi/\lambda$

$K^2 = 4\pi^2/\lambda^2$

W.K.T $U = r^2 \cdot W_{rad}$

$$= \cancel{r^2} \cdot \frac{\eta}{2} \cdot \left(\frac{K I_0 \cdot l}{4\pi}\right)^2 \frac{\sin^2\theta}{\cancel{r^2}}$$

$$\therefore U = \frac{\eta}{2}\left(\frac{K I_0 l}{4\pi}\right)^2 \sin^2\theta \longrightarrow 8$$

$$U_{max} = \frac{\eta}{2}\left(\frac{K I_0 l}{4\pi}\right)^2 \rightarrow 9$$

; max. value occurs at $\theta = \pi/2$

W.K.T $$D_0 = \frac{4\pi U_{max}}{P_{rad}} \rightarrow 10$$

substitute 7 and 9 in 10

$$D_0 = \cancel{4\pi} \times \frac{\eta}{2} \frac{k^2 \cancel{I_0^2} \cancel{l^2}}{\cancel{16}\pi \cancel{^2}} \times \frac{3\lambda^2}{\eta \pi \cancel{I_0^2} \cancel{l^2}}$$

$4$

$$= \frac{\cancel{\eta}}{2} \cdot \frac{\cancel{4\pi^2}}{\cancel{\lambda^2}} \cdot \frac{1}{\cancel{4\pi^2}} \cdot \frac{3\cancel{\lambda^2}}{\cancel{\eta}}$$

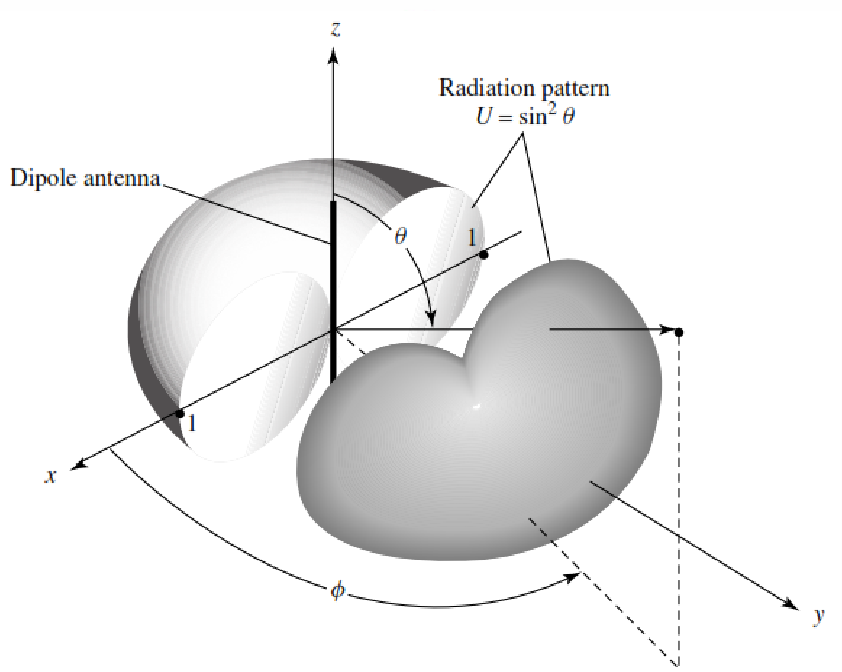
$$\boxed{D_0 = \frac{3}{2}} \rightarrow 11$$

Maximum Effective aperture

$$A_{em} = \frac{\lambda^2}{4\pi} \cdot D_0$$

$$= \frac{\lambda^2}{4\pi} \times \frac{3}{2}$$

$$\boxed{\therefore A_{em} = \frac{3\lambda^2}{8\pi}} \rightarrow 12$$




Three-dimensional radiation pattern of infinitesimal dipole.

**Summary of Procedure to Determine the Far-Field Radiation Characteristics of an Antenna**

1. Specify electric and/or magnetic current densities **J, M**

2. Determine vector potential components $A_\theta,\ A_\phi$ and/or $F_\theta,\ F_\phi$ in far field

3. Find far-zone **E** and **H** radiated fields $(E_\theta,\ E_\phi;\ H_\theta,\ H_\phi)$
4. Form either

a. $$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \mathbf{W}_{\text{av}}(r,\theta,\phi) = \frac{1}{2}\text{Re}[\mathbf{E}\times\mathbf{H}^*]$$

$$\simeq \frac{1}{2}\text{Re}\ [(\hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi)\times(\hat{a}_\theta H_\theta^* + \hat{a}_\phi H_\phi^*)]$$

$$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \hat{a}_r\frac{1}{2}\left[\frac{|E_\theta|^2+|E_\phi|^2}{\eta}\right] = \hat{a}_r\frac{1}{r^2}|f(\theta,\phi)|^2$$

or

b. $U(\theta,\phi) = r^2\ W_{\text{rad}}(r,\theta,\phi) = |f(\theta,\phi)|^2$

5. Determine either

a. $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} W_{\text{rad}}(r,\theta,\phi)r^2\sin\theta\,d\theta\,d\phi$

or

b. $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\,d\theta\,d\phi$

6. Find directivity using

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}}$$

$$D_0 = D_{\max} = D(\theta,\phi)|_{\max} = \frac{U(\theta,\phi)|_{\max}}{U_0} = \frac{4\pi U(\theta,\phi)|_{\max}}{P_{\text{rad}}}$$

7. Form *normalized* power amplitude pattern:

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U_{\max}}$$

8. Determine radiation and input resistance:

$$R_r = \frac{2P_{\text{rad}}}{|I_0|^2};\qquad R_{in} = \frac{R_r}{\sin^2\left(\frac{kl}{2}\right)}$$

9. Determine maximum effective area

$$A_{em} = \frac{\lambda^2}{4\pi}D_0$$