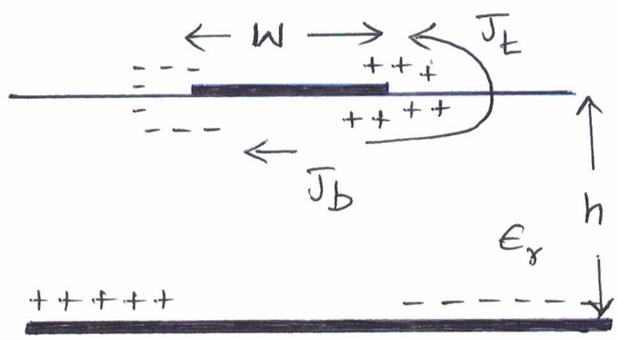
Cavity Model for the Rectangular Patch :

When the microstrip patch is energized, a charge distribution is established on the upper and lower surfaces of the patch as well as on the surface of the ground plane.



Charge distribution and current density creation on microstrip patch

The charge distribution is controlled by two mechanisms.

1. Attractive and
2. Repulsive mechanism

The attractive mechanism is between the corresponding opposite charges on the bottom side of the patch and the ground plane, which tends to maintain the charge concentration on the bottom of the patch.

The repulsive mechanism is between like charges on the bottom side of the patch which tends to push some charges from the bottom of the patch around its edges to its top surface. The movement of these charges creates corresponding current densities J_b and J_t at the bottom and top surfaces of the patch.

Since the height to width ratio is very small, the attractive mechanism dominates and most of the charge concentration and current flow remains underneath the patch.

A small amount of current flows around the edges of the patch to its top surface. This current flow decreases as the height to width ratio decreases.

The tangential magnetic fields at the edges would not be exactly zero due to finite height to width ratio. However, since they will be small, a good approximation to the cavity model is to treat the side walls as perfectly magnetic conducting. This model produces good normalized electric and magnetic field distributions beneath the Patch.

The cavity model represents the patch as a dielectric-loaded cavity with:

* electrical walls (above and below patch) and
* Magnetic walls (around the perimeter of the patch)

The magnetic wall is a wall at which

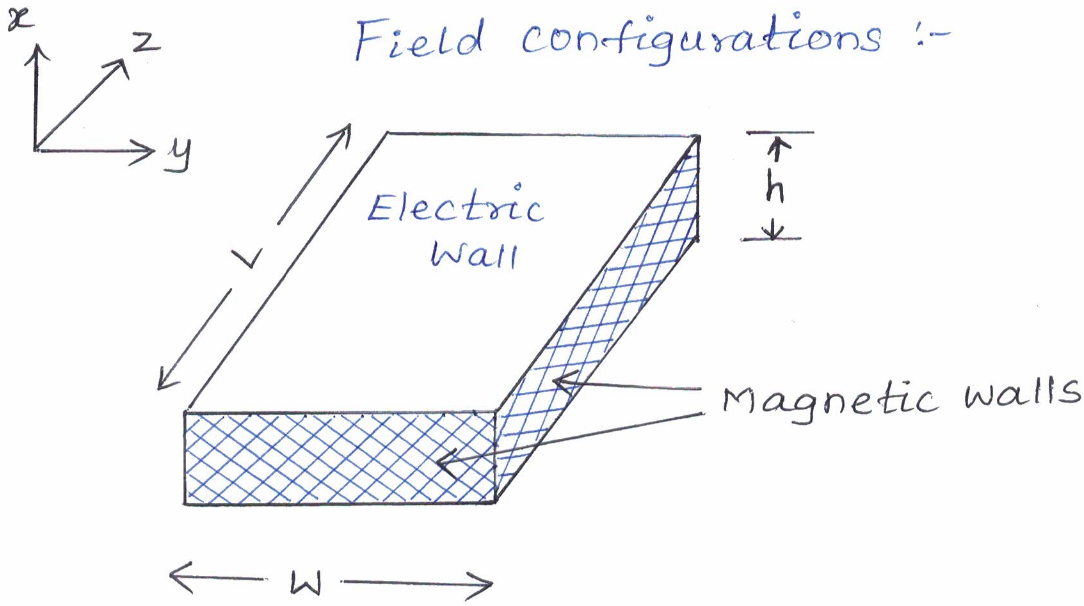
$\hat{n} \times H = 0$ (the H-field is purely normal)

$\hat{n} \cdot E = 0$ (the E-field is purely tangential)

Because the thickness of the microstrip is usually very small the waves generated within the dielectric substrate undergo considerable reflections when they arrive at the edge of the patch. Therefore only a small fraction of the incident energy is radiated. Thus the antenna is considered to be very inefficient.

The cavity model assumes that the E-field is purely tangential to the slots formed between the ground plane and the patch edges (magnetic walls). Moreover, it considers only TM^x modes, i.e., modes with no H_x component.

Field configurations :-



The field configurations within the cavity can be found using the vector potential. The volume beneath the patch can be treated as rectangular cavity loaded with a dielectric material with dielectric constant ϵ_r.

The dielectric material of the substrate is assumed to be truncated and not extended beyond the edges of the patch.

The TM^x modes are fully described by a single function A_x, the x-component of the magnetic vector potential

$$A = A_x \hat{x} \longrightarrow (1)$$

In a homogeneous source free medium, A_x satisfies the wave equation :

$$\nabla^2 A_x + k^2 A_x = 0 \quad \rightarrow (2)$$

For regular shapes (like rectangular cavity), it is advantageous to use the separation of variables.

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} + k^2 A_x = 0 \quad \rightarrow (3)$$

$$A_x = X(x)\, Y(y)\, Z(z) \quad \rightarrow (4)$$

(or) $A_x = XYZ$

$\therefore$ equ (3) becomes

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \cdot \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} + k^2 XYZ = 0$$

$\div$ by XYZ

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k^2 \quad \rightarrow (5)$$

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0 \quad \rightarrow (6)$$

$$\frac{\partial^2 Y}{\partial y^2} + k_y^2 Y = 0 \quad \rightarrow (7)$$

$$\frac{\partial^2 Z}{\partial z^2} + k_z^2 Z = 0 \quad \rightarrow (8)$$

The eigen value equation is

$$k_x^2 + k_y^2 + k_z^2 = k^2 \quad \rightarrow (9)$$

The solution of equ. (6) is

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0.$$

$$m^2 X + k_x^2 X = 0.$$

$$(m^2 + k_x^2) X = 0$$

$$m^2 + k_x^2 = 0.$$

$$m^2 = -k_x^2$$

$$\therefore m = \pm j k_x \quad \rightarrow (10)$$

since the roots are imaginary, the solu. to the differential equ is

$$X(x) = A_1 \cos(k_x x) + B_1 \sin(k_x x) \quad \rightarrow (11)$$

similarly,

The solution of equs. (7) and (8) is

$$Y(y) = A_2 \cos(k_y y) + B_2 \sin(k_y y) \quad \rightarrow (12)$$

$$Z(z) = A_3 \cos(k_z z) + B_3 \sin(k_z z) \quad \rightarrow (13)$$

sub equs. (11), (12), (13) in (4)

$$\therefore A_x = \left[A_1 \cos(k_x x) + B_1 \sin(k_x x)\right]\left[A_2 \cos(k_y y) + B_2 \sin(k_y y)\right]$$

$$\left[A_3 \cos(k_z z) + B_3 \sin(k_z z)\right] \quad \rightarrow (14)$$

where k_x, k_y and k_z are the wave numbers along x, y and z directions

The electric and magnetic fields within the cavity are related to the vector potential A_x.

In our case, there are electric walls at $x=0$ and $x=h$. There, the tangential E-field components must vanish i.e., $E_y = E_z = 0 \big|_{x=0,h}$

$$E_x = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x^2} + k^2 A_x\right) \rightarrow (15)$$

$$E_y = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x \partial y}\right) \rightarrow (16)$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x \partial z}\right) \rightarrow (17)$$

we set A_x at the top and bottom walls as,

$$\left.\frac{\partial A_x}{\partial x}\right|_{x=0,h} = 0 \rightarrow (18)$$

At side walls, we set a vanishing normal derivative for A_x

$$\left.\frac{\partial A_x}{\partial z}\right|_{z=0,L} = 0 \quad ; \quad \left.\frac{\partial A_x}{\partial y}\right|_{y=0,W} = 0 \rightarrow (19)$$

This ensures vanishing H_x and H_y at $z=0$ and $z=L$, as well as vanishing H_x and H_z at $y=0$ and $y=W$ (magnetic walls), as follows from the relation between the H-field and A_x.

$$H_x = 0 \quad \rightarrow (20)$$

$$H_y = \frac{1}{\mu}\frac{\partial A_x}{\partial z} \quad \rightarrow (21)$$

$$H_z = -\frac{1}{\mu}\frac{\partial A_x}{\partial y} \quad \rightarrow (22)$$

The boundary conditions are,

$E_y = 0$ at $x=0$ and $x=h$ → TOP & Bottom plate

$H_y = 0$ at $z=0$ and $z=W$

$H_z = 0$ at $y=0$ and $y=L$

$\Big\}$ side wall.

Now consider equ (16)

sub equ (14) in (16)

$$E_y = \frac{1}{j\omega\mu\varepsilon}\cdot\frac{\partial^2 A_x}{\partial x\,\partial y}$$

$$= \frac{1}{j\omega\mu\varepsilon}\cdot\frac{\partial^2}{\partial x\,\partial y}\Big[(A_1\cos k_x x + B_1 \sin k_x x)\times (A_2 \cos k_y y + B_2 \sin k_y y)\times (A_3\cos k_z z + B_3 \sin k_z z)\Big]$$

$$= \frac{1}{j\omega\mu\varepsilon}\cdot\frac{\partial}{\partial x}\Big[(A_1\cos k_x x + B_1 \sin k_x x)\times (-A_2 \sin k_y y\cdot k_y + B_2 \cos k_y y\cdot k_y)\times (A_3\cos k_z z + B_3 \sin k_z z)\Big]$$

$$E_y = \frac{1}{j\omega\mu\varepsilon}\Big[\left(-A_1 \sin k_x x \cdot k_x + B_1 \cos k_x x\, k_x\right) \times$$
$$\left(-A_2 \sin k_y \cdot y \cdot k_y + B_2 \cos k_y y \cdot k_y\right) \times$$
$$\left(A_3 \cos k_z z + B_3 \sin k_z z\right)\Big] \rightarrow (23)$$

sub $x = 0$ in (23)

$$E_y = \frac{1}{j\omega\mu\varepsilon}\Big[B_1 k_x\left(-A_2 \sin k_y y \cdot k_y + B_2 \cos k_y y \cdot k_y\right) \times$$
$$\left(A_3 \cos k_z z + B_3 \sin k_z z\right)\Big]$$

Assume $B_1 = 0$ to make $E_y = 0$ at $x = 0$.

$$\boxed{B_1 = 0}$$

sub $x = h$ and $B_1 = 0$ in equ (23)

$$E_y = \frac{1}{j\omega\mu\varepsilon}\left[-A_1 \sin k_x \cdot h \cdot k_x\right]$$

$$k_x h = m\pi$$

$$k_x = \frac{m\pi}{h} \rightarrow (24)$$

similarly,

$$k_z = \frac{p\pi}{w} \rightarrow (25)$$

$$k_y = \frac{n\pi}{L} \rightarrow (26)$$

Applying the second boundary condition in equ (21)

$H_y = 0$ at $z = 0$ and $z = w$

$$H_y = \frac{1}{\mu} \frac{\partial A_x}{\partial z}$$

$$= \frac{1}{\mu} \frac{\partial}{\partial z} \Big[\big(A_1 \cos(k_x x) + B_1 \sin(k_x x)\big) \times \big(A_2 \cos(k_y y) + B_2 \sin(k_y y)\big) \times \big(A_3 \cos(k_z z) + B_3 \sin(k_z z)\big) \Big]$$

$$H_y = \frac{1}{\mu} \Big[\big(A_1 \cos(k_x x) + B_1 \sin(k_x x)\big) \times \big(A_2 \cos(k_y y) + B_2 \sin(k_y y)\big) \times \big(-A_3 \sin(k_z z) . k_z + B_3 \cos(k_z . z) . k_z\big) \Big] \rightarrow (27)$$

sub $z = 0$ in (27)

$$H_y = \frac{1}{\mu} \Big[\big(A_1 \cos(k_x x) + B_1 \sin(k_x x)\big) \times \big(A_2 \cos(k_y y) + B_2 \sin(k_y y)\big) \times \big(B_3 k_z\big) \Big]$$

Assume $B_3 = 0$ to make $H_y = 0$ at $z = 0$

$$\boxed{B_3 = 0}$$

sub $B_3 = 0$ and $z = w$ in (27)

$$H_y = \frac{1}{\mu} \Big[\big(A_1 \cos(k_x x) + B_1 \sin(k_x x)\big) \times \big(A_2 \cos(k_y y) + B_2 \sin(k_y y)\big) \times \big(-A_3 \sin(k_z w) . k_z\big) \Big]$$

$$-A_3 \sin k_z W \cdot k_z = 0$$

$$\sin k_z W = 0$$

$$k_z W = p\pi$$

$$\boxed{k_z = \frac{p\pi}{W}} \rightarrow (28)$$

$p = 0, 1, 2, 3, \ldots$

sub $B_1 = B_2 = B_3 = 0$ in equ (14)

Thus the final form for the vector potential A_x within the cavity is

$$A_x = A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z') \rightarrow (29)$$

where A_{mnp} represents the amplitude coefficients of each mnp mode. The wavenumbers k_x, k_y, k_z are equal to

$$\left.\begin{aligned} k_x &= \left(\frac{m\pi}{h}\right), \quad m = 0, 1, 2, \ldots \\ k_y &= \left(\frac{n\pi}{L}\right), \quad n = 0, 1, 2, \ldots \\ k_z &= \left(\frac{p\pi}{W}\right), \quad p = 0, 1, 2, \ldots \end{aligned}\right\} \rightarrow (30)$$

$m = n = p \neq 0$

where m, n, p represents the number of half cycle field variations along the x, y, z directions.

Since the wavenumbers k_x, k_y, and k_z are subject to the constraint equation

$$K_x^2 + K_y^2 + K_z^2 = \left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2 = K_r^2 = \omega^2 \mu \varepsilon_0 \rightarrow (31)$$

The resonant frequencies for the cavity are given by

$$(f_r)_{mnp} = \frac{1}{2\pi\sqrt{\mu\varepsilon_0}} \sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2} \rightarrow (32)$$

substitute equ (32) in equs. (15) → (17) and equs. (20) → (22)

$$E_x = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x^2} + K^2 A_x\right)$$

$$= \frac{1}{j\omega\mu\varepsilon} \cdot \frac{\partial^2}{\partial x^2}\left(A_{mnp} \cos(k_x x') \cdot \cos(k_y \cdot y') \cdot \cos(k_z \cdot z')\right)$$

$$+ \frac{K^2}{j\omega\mu\varepsilon}\left(A_{mnp} \cdot \cos(k_x x') \cos(k_y y') \cdot \cos(k_z \cdot z')\right)$$

$$E_x = \frac{1}{j\omega\mu\varepsilon} \cdot \frac{\partial}{\partial x}\left(-A_{mnp} \sin(k_x x') \cdot k_x \cdot \cos(k_y y') \cdot \cos(k_z \cdot z')\right)$$

$$+ \frac{K^2}{j\omega\mu\varepsilon}\left(A_{mnp} \cdot \cos(k_x x') \cdot \cos(k_y y') \cdot \cos(k_z z')\right)$$

$$E_x = \frac{1}{j\omega\mu\varepsilon}\left(-A_{mnp} \cos(k_x x') \cdot k_x^2 \cdot \cos(k_y y') \cdot \cos(k_z \cdot z')\right)$$

$$+ \frac{K^2}{j\omega\mu\varepsilon}\left(A_{mnp} \cdot \cos(k_x x') \cdot \cos(k_y y') \cos(k_z z')\right)$$

$$\therefore E_x = \frac{-j\left(k^2 - k_x^2\right)}{\omega\mu\epsilon} \cdot A_{mnp} \cdot \cos(k_x x') \cdot \cos(k_y y') \cdot \cos(k_z \cdot z') \rightarrow (33)$$

similarly,

$$E_y = \frac{-j k_x k_y}{\omega\mu\epsilon} A_{mnp} \cdot \sin(k_x x') \cdot \sin(k_y y') \cos(k_z z') \rightarrow (34)$$

$$E_z = \frac{-j k_x k_z}{\omega\mu\epsilon} A_{mnp} \cdot \sin(k_x x') \cdot \cos(k_y y') \sin(k_z z') \rightarrow (35)$$

$$H_x = 0 \rightarrow (36)$$

$$H_y = \frac{-k_z}{\mu} \cdot A_{mnp} \cdot \cos(k_x x') \cdot \cos(k_y y') \sin(k_z z') \rightarrow (37)$$

$$H_z = \frac{k_y}{\mu} A_{mnp} \cdot \cos(k_x x') \sin(k_y y') \cos(k_z z') \rightarrow (38)$$

To determine the dominant mode with the lowest resonance, the resonant frequencies are examined. The mode with the lower order frequency is called as dominant mode.

For all microstrip antennas $h \ll L$ and $h \ll W$. if $L > W > h$, the mode with the lowest frequency is TM^x_{010} whose resonant freq. is given by

$$(f_r)_{010} = \frac{1}{2L\sqrt{\mu\epsilon}} = \frac{v_0}{2L\sqrt{\epsilon_r}} \rightarrow (39)$$

If $L > W > \frac{L}{2} > h$, the next higher order (second) mode is TM^x_{001}, whose resonant frequency is

$$(f_r)_{001} = \frac{1}{2W\sqrt{\mu\varepsilon}} = \frac{v_0}{2W\sqrt{\varepsilon_r}} \rightarrow (40)$$

If $L > \frac{L}{2} > W > h$, the second order mode is TM^x_{020} instead of TM_{001}, whose resonant freq., is,

$$(f_r)_{020} = \frac{1}{L\sqrt{\mu\varepsilon}} = \frac{v_0}{L\sqrt{\varepsilon_r}} \rightarrow (41)$$

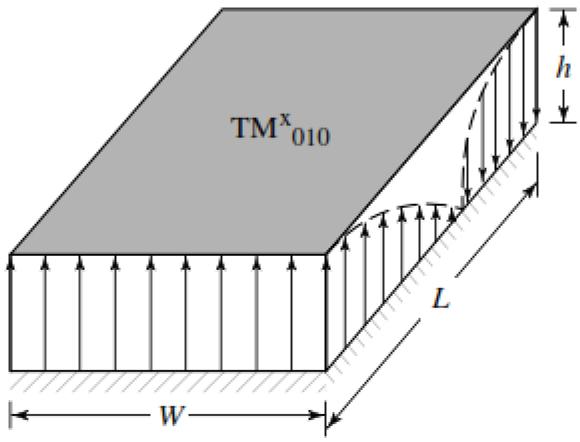
If $W > L > h$, the dominant mode is TM^x_{001}, whose resonant freq., is

$$(f_r)_{001} = \frac{1}{2W\sqrt{\mu\varepsilon}} = \frac{v_0}{2W\sqrt{\varepsilon_r}} \rightarrow (42)$$

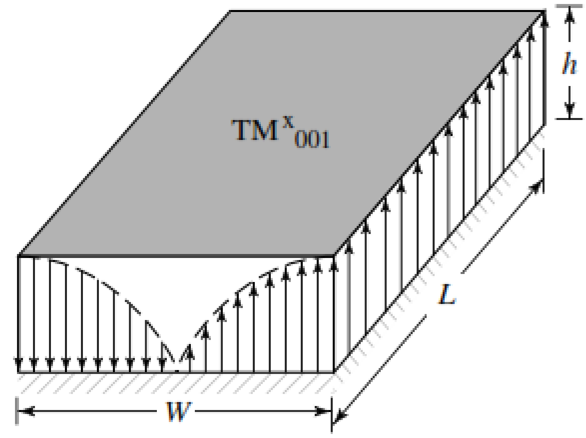
If $W > \frac{W}{2} > L > h$, the second order mode is TM^x_{002} whose resonant freq., is

$$(f_r)_{002} = \frac{1}{W\sqrt{\mu\varepsilon}} = \frac{v_0}{W\sqrt{\varepsilon_r}} \rightarrow (43)$$

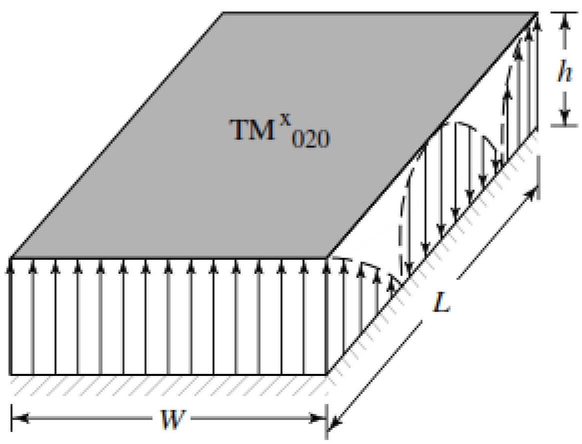
Based on z-electric component (E_z), the distribution of the tangential electric field along the side walls of the cavity for TM^x_{010}, TM^x_{001}, TM^x_{020}, TM^x_{002} is shown in figure below.



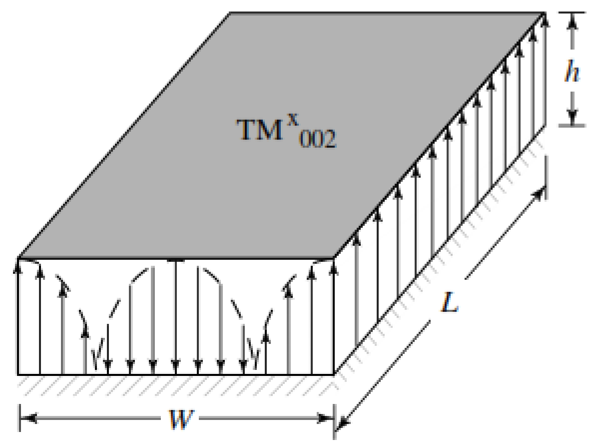
(a) $TM^x{}_{010}$



(b) $TM^x{}_{001}$



(c) $TM^x{}_{020}$



(d) $TM^x{}_{002}$

Figure 14.14 Field configurations (modes) for rectangular microstrip patch.