Solved Anna University Problems (NOV/DEC 2010 - NOV/DEC 2019)

EC8651-Transmission Lines and RF Systems

UNIT 1 - TRANSMISSION LINE THEORY

UNIT 1 - TRANSMISSION LINE THEORY PROBLEMS

1. Find the characteristic impedance of a line at 1600Hz if Z_{OC} = 750∠-30° Ω and Z_{SC} = 600∠-20° Ω. **[EC6503-NOV/DEC 2019](2 Marks), [EC6503-NOV/DEC 2016](2 Marks), [EC6503-APR/MAY 2019](2 Marks)**

Solution:

Given: f= 1600 Hz, Z_{OC} = 750∠-30° Ω, Z_{SC} = 600∠-20° Ω

$$Z_O = \sqrt{Z_{OC} \times Z_{SC}}$$

$$Z_O = \sqrt{750\angle -30^\circ \times 600\angle -20^\circ}$$

$$\mathbf{Z_O = 670.82\angle{-25}^\circ\ \Omega}$$

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2. The following measurement are made on a 25km line at a frequency of 796 Hz. Z_{SC} = 3220∠-79.29° Ω, Z_{OC} = 1301∠76.67° Ω. Determine the primary constants of the line. **[EC2305-NOV/DEC 2014](10 Marks)**

Solution:

Given: l=25Km, f=796Km, Z_{SC} = 3220∠-79.29° Ω, Z_{OC} = 1301∠76.67° Ω

$$Z_O = \sqrt{Z_{OC} \times Z_{SC}}$$

$$Z_O = \sqrt{1301\angle 76.67^\circ \times 3220\angle -79.29^\circ}$$

$$\mathbf{Z_O = 2046.76\angle{-1.31}^\circ}$$

$$\tanh\gamma = \sqrt{\frac{Z_{SC}}{Z_{OC}}}$$

$$\tanh\gamma = \sqrt{\frac{3220\angle -79.29^\circ}{1301\angle 76.67^\circ}}$$

$$\tanh\gamma = \frac{e^{2\gamma}-1}{e^{2\gamma}+1} = 1.5732\angle -77.98^\circ$$

$$\frac{e^{2\gamma}-1}{e^{2\gamma}+1} = 0.3276 - j1.5387$$

$$e^{2\gamma}-1 = (0.3276 - j1.5387)(e^{2\gamma}+1)$$

$$e^{2\gamma}-1 = 0.3276 \times e^{2\gamma} + 0.3276 - j1.5387 \times e^{2\gamma} - j1.5387$$

$$e^{2\gamma}[1 - 0.3276 + j1.5387] = 1 + 0.3276 - j1.5387$$

$$e^{2\gamma}[0.6724 + j1.5387] = 1.3276 - j1.5387$$

$$e^{2\gamma} = \frac{1.3276 - j1.5387}{0.6724 + j1.5387}$$

$$e^{2\gamma} = -0.5231 - j1.0914 = 1.2103\angle -115.61^\circ$$

$$2\gamma = \ln\left[1.2103\angle -115.61^\circ\right]$$

$$\gamma = \frac{1}{2}\ln\left[1.2103\angle -115.61^\circ\right]$$

Mathematically, $\ln[a\angle b] = \ln a + j\, b$

$$\gamma = \frac{1}{2}\left[\ln(1.2103) + j - 115.61^\circ\right]$$

$$\alpha + j\beta = \frac{1}{2}\ln 1.2103 + j\left(\frac{-115.61^\circ}{2}\right)$$

$$\alpha = \frac{1}{2}\ln 1.2103 = 0.0954 \text{ Nepers / Km}$$

α = 0.0954 Np/Km

$$\beta = \left(\frac{-115.61^\circ}{2}\right) = -57.81^\circ \text{ / Km} = 1.009 \text{ rad / Km}$$

β = 1.009 rad/Km

$\gamma=\alpha+j\beta$ = 0.0954+j1.009 = 1.0135$\angle$84.599°

To find Primary Constants:

$R+j\omega L = Z_oP = 2046.76\angle -1.31^\circ \times 1.0135\angle 84.599^\circ$

$= 242.42+j2060.18$

Comparing real and imaginary terms,

R = 242.42 ohms/Km

$\omega L = 2060.18$

$$L = \frac{2060.18}{4998.88} = 0.41213 \text{ H / Km}$$

L=0.41213 H/Km

Similarly,

$$G + j\omega C = \frac{P}{Z_o} = \frac{1.0135\angle 84.599^\circ}{2046.76\angle -1.31^\circ}$$

$$G + j\omega C = 3.533\times 10^{-5} + j4.939\times 10^{-4}$$

Comparing real and imaginary terms,

G = 3.533x10^{-5} mho/Km

$\omega C = 2.3365\text{x}10^{-5}$

$$C = \frac{4.939 \times 10^{-4}}{4998.88} = 98.8\,\text{nF / Km}$$

C = 98.8 nF/Km

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3. A transmission line has Z_o = 745∠-12° Ω and is terminated in Z_R=100Ω. Calculate reflection factor. **[EC6503-APR/MAY 2017](2 Marks)**

Solution:

Given: Z_o = 745∠-12° Ω, Z_R=100Ω

$$\text{Reflection Factor, } k = \left[\frac{2\sqrt{Z_R Z_O}}{Z_R + Z_O}\right]$$

$$\text{Reflection Factor, } k = \left[\frac{2\sqrt{100 \times 745\angle -12^\circ}}{100 + 745\angle -12^\circ}\right]$$

$$\text{Reflection Factor, } k = \left[\frac{545.894\angle -6^\circ}{843.07\angle -10.587^\circ}\right]$$

Reflection Factor = 0.6475∠4.587°

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4. A transmission line has Z_o = 745∠-12° Ω and is terminated in Z_R=100Ω. Calculate reflection loss in dB. **[EC2305-APR/MAY 2011](2 Marks)**

Solution:

Given: Z_o = 745∠-12° Ω, Z_R=100Ω

$$\text{Reflection loss in dB} = 20\log\left(\frac{1}{|k|}\right)\text{dB}$$

$$\text{Reflection Factor, } k = \left[\frac{2\sqrt{Z_R Z_O}}{Z_R + Z_O}\right]$$

$$\text{Reflection Factor, } k = \left[\frac{2\sqrt{100 \times 745\angle -12^\circ}}{100 + 745\angle -12^\circ}\right]$$

$$\text{Reflection Factor, } k = \left[\frac{545.894\angle -6^\circ}{843.07\angle -10.587^\circ}\right]$$

Reflection Factor = 0.6475∠4.587°

$$\text{Reflection loss in dB} = 20\log\left(\frac{1}{0.6475}\right)\text{dB}$$

Reflection loss in dB = 3.7752 dB

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5. A transmission line has a characteristic impedance of 400 Ω and is terminated by a load impedance of (650-j475) Ω. Determine the reflection coefficient. **[EC2305-NOV/DEC 2013](2 Marks)**

Solution:

Given: Z_O = 400 Ω, Z_R = (650-j475) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(650 - j475) - 400}{(650 - j475) + 400} = \frac{250 - j475}{1050 - j475}$$

$$K = 0.3675 - j0.2861 = 0.4658\angle -37.9^\circ$$

K = 0.4658∠-37.9°

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6. A transmission line has a characteristic impedance of 600 Ω. Determine magnitude of reflection coefficient if the receiving end impedance is (650-j475) Ω. **[EC2305-NOV/DEC 2013](2 Marks)**

Solution:

Given: Z_O = 600 Ω, Z_R = (650-j475) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(650 - j475) - 600}{(650 - j475) + 600} = \frac{50 - j475}{1250 - j475}$$

$$K = 0.1611 - j0.3188 = 0.3571\angle -63.18^\circ$$

K = 0.3571∠-63.18°

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7. A transmission line has a characteristic impedance of 300 Ω and is terminated in a load impedance of (150-j150) Ω. Determine the reflection coefficient. **[EC2305-APR/MAY 2014](2 Marks)**

Solution:

Given: Z_O = 300 Ω, Z_R = (150-j150) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(150 - j150) - 300}{(150 - j150) + 300} = \frac{-150 - j150}{450 - j150}$$

$$K = -0.2 - j0.4 = 0.4472\angle -116.57^\circ$$

K = 0.4472∠-116.57°

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8. Find the reflection coefficient of a 50 ohm line when it is terminated by a load impedance of 60+j 40 ohm. **[EC6503-NOV/DEC 2015](2 Marks), [EC2305-NOV/DEC 2015](2 Marks)[EC2305-APR/MAY 2013](2 Marks)**

Solution:

Given: Z_O = 50 Ω, Z_R = (60+j40) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(60 + j40) - 50}{(60 + j40) + 50} = \frac{10 + j40}{110 + j40}$$

$$K = 0.197 + j0.29197 = 0.3523\angle 55.98^\circ$$

K = 0.3523∠55.98°

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9. The constant of a transmission line are R=6Ω/Km, L=2.2mH/Km, C=0.005μF/Km and G=0.25x10^{-3} mho/Km, Calculate the characteristic impedance, attenuation constant and phase constant at 1000 Hz.

[EC6503-NOV/DEC 2019](6 Marks) [EC6503-APR/MAY 2019](6 Marks), [EC2305-APR/MAY 2014](8 Marks)

Solution:

Given: R=6Ω/Km, L=2.2mH/Km, C=0.005μF/Km and G=0.25x10^{-3} mho/Km, f=1000 Hz

ω = 2πf = 2x3.14x1000 = 6280

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{6 + j(6280)(2.2 \times 10^{-3})}{0.25 \times 10^{-3} + j(6280)(0.005 \times 10^{-6})}}$$

$$Z_o = \sqrt{\frac{6 + j13.816}{0.25 \times 10^{-3} + j3.14 \times 10^{-5}}} = \sqrt{\frac{15.063\angle 66.526^\circ}{2.52 \times 10^{-4}\angle 7.159^\circ}}$$

$$Z_o = \frac{3.881}{0.0159}\angle(66.526^\circ - 7.159^\circ)/2$$

Z_o = 244.088 ∠29.68°

The propagation constant is given by,

$$P = \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{\left(6 + j(6280)(2.2 \times 10^{-3})\right) \times \left(0.25 \times 10^{-3} + j(6280)(0.005 \times 10^{-6})\right)}$$

$$P = \gamma = \sqrt{(6 + j13.816) \times \left(0.25 \times 10^{-3} + j3.14 \times 10^{-5}\right)}$$

$$P = \gamma = \sqrt{15.063\angle 66.526^\circ \times 2.52 \times 10^{-4}\angle 7.159^\circ}$$

$$P = \gamma = \sqrt{15.063 \times 2.52 \times 10^{-4}\angle(7.159^\circ + 66.526^\circ)}$$

P=γ = α+jβ = 0.0616 ∠36.843° (Or) 0.0493+j0.0369

attenuation constant, α = 0.0493

and phase constant, β = 0.0369

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10. A transmission line has a characteristic impedance of (683-j138) ohm. the propagation constant is (0.0074+j0.0356) per Km. Determine the values of R and L of this line if the frequency is 1000Hz.

[EC2305-APR/MAY 2014](6 Marks)

Solution:

Given: Z_o = 683-j138Ω, γ=P= 0.0074+j0.0356, f=1000Hz

ω = 2πf = 2 x 3.14 x 1000 = 6280

R+jωL = Z_oP = (683-j138) x (0.0074+j0.0356)

= 9.967+j23.2936

Comparing real and imaginary terms,

R = 9.967 ohms/Km

ωL = 23.2936

$$L = \frac{23.2936}{6280} = 3.7092 \text{mH/Km}$$

L=3.7092 mH/Km

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11. A Communication line has L=3.67 mH/Km, G=0.08x10^{-6} mho/Km, C=0.0083 μF/Km and R=10.4 Ω/Km. Determine the characteristic impedance, propagation constant, phase constant, velocity of propagation, sending end current and receiving end current for a given frequency f=1000 Hz. sending end voltage is 1V and transmission line length is 100Kms.

[EC6503-NOV/DEC 2016] (16 Marks) [EC6503-APR/MAY 2018](13 Marks)

Solution:

Given: E_s = 1V, f=1000 Hz, l=100 Km, R=10.4 Ω/Km, L = 0.00367H/Km, G =0.08x10^{-6} mho/Km, C = 0.0083 μF/Km

ω = 2πf = 2x3.14x1000 = 6280

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Z = R+jωL = 10.4 + j(6280)(.00367) = 25.285∠65.71°

Y = G+jωC = 0.08x10^{-6} +j(6280)(0.0083x10^{-6}) = 5.212x10^{-5}∠89.91°

$$Z_o = \sqrt{\frac{25.285\angle 65.71^\circ}{5.212 \times 10^{-5}\angle 89.91^\circ}}$$

$$Z_o = \frac{5.028}{7.2194 \times 10^{-3}} \angle(65.71^\circ - 89.91^\circ)/2$$

Z_o = 696.46 ∠-12.1° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.285\angle 65.71^\circ \times 5.212 \times 10^{-5} \angle 89.91^\circ}$$

$$P = \gamma = \sqrt{1.319 \times 10^{-3} \angle(65.71^\circ + 89.91^\circ)}$$

$P=\gamma = \alpha+j\beta = 0.0363 \angle 77.81^\circ$ (Or) $7.6649\times10^{-3}+j0.0355$

attenuation constant, α = 7.6649×10^{-3} Np/Km

and phase constant, β = 0.0355 rad/Km

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{6280}{0.0355} = 1.769 \times 10^5 \text{ Km/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0355} = 176.99 \text{ m}$$

NOTE: Z_R value is not given in the question. so, assume the line is perfectly matched line to proceed further. i.e., $Z_R=Z_O$, $Z_S=Z_O$

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{1}{0 + 696.46\angle -12.1^\circ} = 1.4358 \times 10^{-3} \angle 12.1^\circ \text{ A}$$

I_S = $1.4358\times10^{-3} \angle 12.1^\circ$

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)} e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, and put K=0, because $Z_R = Z_O$.

$$I_S = I_R\, e^{\sqrt{ZY}l}$$

$$I_R = I_S\, e^{-\sqrt{ZY}l}$$

$$e^{-\sqrt{ZY}l} = e^{-\gamma l} = e^{-(7.6649\times10^{-3} + j0.0355)\times 100} = e^{-0.76649} \angle 203.4^\circ$$

$$e^{-\sqrt{ZY}l} = 0.4646 \angle 203.4^\circ$$

$$\therefore I_R = 1.4358 \times 10^{-3} \angle 12.1^\circ \times 0.4646 \angle 203.4^\circ$$

I_R = $6.671\times10^{-4}\angle -144.5^\circ$ A

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12. A Parallel wire transmission line is having the following line parameters at 5 KHz. Series resistance (R = 2.59×10^{-3} Ω /m), Series inductance (L = 2μH/m), shunt conductance (G = 0 Ω /m) and capacitance between conductors (C = 5.56 pF/m). Find the characteristic impedance, attenuation constant (α Np/m), Phase shift constant (β rad/m), velocity of propagation and wavelength.

[EC6503-NOV/DEC 2015](10 Marks), [EC2305-NOV/DEC 2015](10 Marks)

Solution:

Given: f=5KHz, R = 2.59×10^{-3} Ω/m, L = 2μH/m, G = 0 mho/m, C = 5.56 pF/m

$\omega = 2\pi f = 2\times3.14\times5000 = 31400$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 2.59\times10^{-3} + j(31400)(2\times10^{-6}) = 0.0629\angle 87.64^\circ$

$Y = G+j\omega C = j(31400)(5.56\times10^{-12}) = 1.746\times10^{-7}\angle 90^\circ$

$$Z_o = \sqrt{\frac{0.0629\angle 87.64^\circ}{1.746\times10^{-7}\angle 90^\circ}}$$

$$Z_o = \frac{0.2508}{4.1785\times10^{-4}}\angle(87.64^\circ - 90^\circ)/2$$

Z_o = 600.21 $\angle$-1.18° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{0.0629\angle 87.64^\circ \times 1.746\times10^{-7}\angle 90^\circ}$$

$$P = \gamma = \sqrt{1.09823\times10^{-8}\angle(87.64^\circ + 90^\circ)}$$

$P=\gamma = \alpha+j\beta = 1.048\times10^{-4}\angle 88.82^\circ$ (Or) $2.1582\times10^{-6}+j1.04778\times10^{-4}$

attenuation constant, α = 2.158×10^{-6} Np/m

and phase constant, β = 104.778×10^{-6} rad/m

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{31400}{104.778\times10^{-6}} = 2.997\times10^{8}\ \text{m/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{104.778\times10^{-6}} = 59.94\times10^{3}\,\text{m}$$

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13. A 2 meter long transmission line with characteristics impedance of 60+j 40 ohm is operating at $\omega = 10^6$ rad / sec has attenuation constant of 0.921 Np/m and phase shift constant of 0 rad /m. If the line is terminated by a load of 20+j50 ohm, determine the input impedance of this line.

[EC6503-NOV/DEC 2015](6 Marks), [EC6503-APR/MAY 2017](8 Marks), [EC2305-NOV/DEC 2015](6 Marks), [EC2305-APR/MAY 2019](8 Marks)

Solution:

Given: l = 2m, Z_O = 60+j 40Ω, $\omega = 10^6$ rad/sec, α= 0.921 Np/m, β= 0 rad /m, Z_R = 20+j50 Ω

Reflection Coefficient, K

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{(20 + j50) - (60 + j40)}{(20 + j50) + (60 + j40)} = -0.1586 + j0.30345 = 0.3424\angle 117.6^\circ$$

K=0.3424∠117.6°

$$e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l}\angle \beta l = e^{(0.921)(2)}\angle (0)(2)$$

$$e^{\gamma l} = e^{1.842}$$

$$\mathbf{e^{\gamma l} = 6.3091\ \angle 0^\circ}$$

$$e^{-\gamma l} = e^{-(\alpha + j\beta)l} = e^{-\alpha l}\angle -\beta l = e^{-(0.921)(2)}\angle -(0)(2)$$

$$e^{-\gamma l} = e^{-1.842}$$

$$\mathbf{e^{-\gamma l} = 0.1585\ \angle 0^\circ}$$

$$Z_s = Z_o\left\{\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}}\right\}$$

$$Z_S = 72.11\angle 33.69^\circ\left\{\frac{6.3091 + (0.3424\angle 117.6^\circ)(0.1585)}{6.3091 - (0.3424\angle 117.6^\circ)(0.1585)}\right\}$$

$$Z_S = 72.11\angle 33.69^\circ\left\{\frac{6.3091 + 0.0543\angle 117.6^\circ}{6.3091 - 0.0543\angle 117.6^\circ}\right\}$$

$$Z_S = 72.11\angle 33.69^\circ\left\{\frac{6.284\angle 0.4387^\circ}{6.3344\angle -0.4353^\circ}\right\}$$

$$Z_S = 72.11\angle 33.69^\circ\left\{0.9920\angle 0.8740^\circ\right\}$$

$$\mathbf{Z_S = 71.54\angle 34.56^\circ\ \Omega}$$

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14. A generator of 1V, 1000 Hz, supplies power to a 100 Km open wire line terminated in Z_O and having following parameters R = 10.4 ohm per Km, L = 0.00367 Henry per Km, G = 0.8×10^{-6} mho per Km, C = 0.00835 μF per Km. Calculate Z_o, α, β, λ, v. also find the received power.

[EC6503-MAY/JUN 2016](16 Marks), [EC6503-APR/MAY 2016](16 Marks), [EC6503-APR/MAY 2015](16 Marks)

Solution:

Given: E_g = 1V, f=1000 Hz, l=100 Km, when $Z_R = Z_O$ then $Z_S = Z_O$, R=10.4 Ω/Km, L = 0.00367H/Km, G =0.8×10^{-6} mho/Km, C = 0.00835 μF/Km

$\omega = 2\pi f = 2 \times 3.14 \times 1000 = 6280$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R + j\omega L = 10.4 + j(6280)(.00367) = 25.285\angle 65.71°$

$Y = G + j\omega C = 0.8 \times 10^{-6} + j(6280)(0.00835 \times 10^{-6}) = 5.244 \times 10^{-5}\angle 89.13°$

$$Z_o = \sqrt{\frac{25.285\angle 65.71°}{5.244 \times 10^{-5}\angle 89.13°}}$$

$$Z_o = \frac{5.028}{7.24 \times 10^{-3}}\angle(65.71° - 89.13°)/2$$

Z_o = 694.48 ∠-11.71° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.285\angle 65.71° \times 5.244 \times 10^{-5}\angle 89.13°}$$

$$P = \gamma = \sqrt{1.326 \times 10^{-3}\angle(65.71° + 89.13°)}$$

$P = \gamma = \alpha + j\beta = 0.0364\angle 77.42°$ (Or) $7.928 \times 10^{-3} + j0.0355$

attenuation constant, α = 7.928×10^{-3} Np/Km

and phase constant, β = 0.0355 rad/Km

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{6280}{0.0355} = 1.769 \times 10^5 \text{ Km/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0355} = 176.99 \text{ m}$$

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{1}{0 + 694.48\angle -11.71^\circ} = 1.4399 \times 10^{-3} \angle 11.71^\circ \text{ A}$$

$\mathbf{I_S = 1.4399x10^{-3} \angle 11.71^\circ}$

$E_S = I_S \text{ x } Z_S$

$E_S = 1.4399x10^{-3} \angle 11.71^\circ \text{ x } (694.48\angle -11.71^\circ)$

$\mathbf{E_S = 1 \text{ V}}$

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)} e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, and put K=0, because $Z_R = Z_O$.

$$I_S = I_R \, e^{\sqrt{ZY}l}$$

$$I_R = I_S \, e^{-\sqrt{ZY}l}$$

$$e^{-\sqrt{ZY}l} = e^{-\gamma l} = e^{-(7.928\times 10^{-3} + j0.0355)\times 100} = e^{-0.7928} \angle 203.4^\circ$$

$$e^{-\sqrt{ZY}l} = 0.4526 \angle 203.4^\circ$$

$$\therefore I_R = 1.4399 \times 10^{-3} \angle 11.71^\circ \times 0.4526 \angle 203.4^\circ$$

$\mathbf{I_R = 6.517x10^{-4} \angle -144.89^\circ}$

$E_R = I_R \text{ x } Z_R = 6.517x10^{-4} \angle -144.89^\circ \text{ x } (694.48\angle -11.71^\circ)$

$\mathbf{E_R = 0.4526\angle -156.6^\circ \text{ V}}$

$P_S = |E_S| . \ |I_S| .\cos (E_S \wedge I_S)$

$P_S = 1 \text{ x } 1.4399x \ 10^{-3} \text{ x } \cos (11.71)$

$\mathbf{P_S = 1.4099x10^{-3} \text{ W}}$

and $P_R = |E_R| . \ |I_R| .\cos (E_R \wedge I_R)$

$P_R = 0.4526 \text{ x } 6.517x10^{-4} \cos(-11.71)$

$\mathbf{P_R = 0.28882x10^{-3} \text{ W}}$

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15. A generator of 1V, 1000 cycles, supplies power to a 100 mile open wire line terminated in Z_O and having following parameters: Series resistance R = 10.4 ohm/ mile, Series inductance L = 0.00367 H/mile, Shunt conductance G = 0.8 x 10^{-6} mho/ mile, and capacitance between conductors C = 0.00835 μF/mile. Find the characteristic impedance, Propagation constant, attenuation constant, phase shift constant, velocity of propagation and wavelength. **[EC6503-APR/MAY 2017](6 Marks)**

Solution:

Given: E_g = 1V, f=1000 cycles = 1000 Hz, l=100 mile, when $Z_R = Z_O$ then $Z_S = Z_O$, R=10.4 Ω/ mile, L = 0.00367H/ mile, G =0.8x10^{-6} mho/ mile, C = 0.00835 μF/ mile

$\omega = 2\pi f = 2x3.14x1000 = 6280$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 10.4 + j(6280)(0.00367) = 25.285\angle 65.71°$

$Y = G+j\omega C = 0.8x10^{-6} +j(6280)(0.00835x10^{-6}) = 5.244x10^{-5}\angle 89.13°$

$$Z_o = \sqrt{\frac{25.285\angle 65.71°}{5.244\times 10^{-5}\angle 89.13°}}$$

$$Z_o = \frac{5.028}{7.24\times 10^{-3}}\angle(65.71° - 89.13°)/2$$

Z_o = 694.48 $\angle$-11.71° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.285\angle 65.71° \times 5.244\times 10^{-5}\angle 89.13°}$$

$$P = \gamma = \sqrt{1.326\times 10^{-3}\angle(65.71° + 89.13°)}$$

$P=\gamma = \alpha+j\beta = 0.0364 \angle 77.42°$ (Or) $7.928x10^{-3}+j0.0355$

attenuation constant, α = $7.928x10^{-3}$ Np/Km

and phase constant, β = 0.0355 rad/Km

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{6280}{0.0355} = 1.769\times 10^5 \text{ Km/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0355} = 176.99\,\text{m}$$

16. A generator of 1V, 1KHz, supplies power to a 100 Km open wire line terminated in 200 Ω resistance. The line parameters are R = 10 Ω/Km, L = 3.8mH/Km, G =1×10^{-6} mho/Km, C = 0.0085 μF/Km. Calculate Z_o, α, β, λ, v. also find the received power. **[EC2305-NOV/DEC 2018](16 Marks)**

Solution:

Given: E_g = 1V, f=1000 Hz, l=100 Km, Z_R =200 Ω, R=10 Ω/Km, L = 3.8mH/Km, G =1×10^{-6} mho/Km, C = 0.0085 μF/Km

$\omega = 2\pi f = 2\times3.14\times1000 = 6280$

The characteristic impedance is given by,

$$Z_O = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 10 + j(6280)(3.8\times10^{-3}) = 25.87\angle67.26°$

$Y = G+j\omega C = 1\times10^{-6} + j(6280)(0.0085\times10^{-6}) = 5.339\times10^{-5}\angle88.93°$

$$Z_O = \sqrt{\frac{25.87\angle67.26°}{5.339\times10^{-5}\angle88.93°}}$$

$$Z_O = \frac{5.086}{7.307\times10^{-3}}\angle(67.26° - 88.93°)/2$$

$\mathbf{Z_o = 696.06 \angle{-10.84}°\ \Omega}$

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.87\angle67.26° \times 5.339\times10^{-5}\angle88.93°}$$

$$P = \gamma = \sqrt{1.3812\times10^{-3}\angle(67.26° + 88.93°)}$$

$P=\gamma = \alpha+j\beta = 0.03716\ \angle78.095°$ (Or) $7.6657\times10^{-3}+j0.03636$

attenuation constant, α = $\mathbf{7.6657\times10^{-3}}$ Np/Km

and phase constant, β = 0.03636 rad/Km

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{6280}{0.03636} = 1.727\times10^5\ \text{Km/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.03636} = 172.72\,\text{m}$$

Reflection Coefficient, K

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{200 - 696.06\angle -10.84^\circ}{200 + 696.06\angle -10.84^\circ} = -0.557 + j0.0656 = 0.5609\angle 173.28^\circ$$

$K = 0.5609\angle 173.28^\circ$

$$e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l}\angle \beta l = e^{(7.6657\times 10^{-3})(100)}\angle (0.03636)(100)$$

$$e^{\gamma l} = e^{(7.6657\times 10^{-3})(100)}\angle (0.03636)(100)\,\text{rad}$$

$e^{\gamma l} = e^{0.76657}\ \angle 208.3^\circ$

$e^{\gamma l} = 2.1524\ \angle 208.3^\circ$

$$e^{-\gamma l} = e^{-(\alpha + j\beta)l} = e^{-\alpha l}\angle -\beta l = e^{-(7.6657\times 10^{-3})(100)}\angle -(0.03636)(100)$$

$$e^{-\gamma l} = e^{-(7.6657\times 10^{-3})(100)}\angle -(0.03636)(100)\,\text{rad}$$

$e^{-\gamma l} = e^{-0.76657}\ \angle -208.3^\circ$

$e^{-\gamma l} = 0.4646\ \angle -208.3^\circ$

$$Z_s = Z_o \left\{ \frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}} \right\}$$

$$Z_S = 696.06\angle -10.84^\circ \left\{ \frac{2.1524\angle 208.3^\circ + (0.5609\angle 173.28^\circ)(0.4646\angle -208.3^\circ)}{2.1524\angle 208.3^\circ - (0.5609\angle 173.28^\circ)(0.4646\angle -208.3^\circ)} \right\}$$

$$Z_S = 696.06\angle -10.84^\circ \left\{ \frac{2.1524\angle 208.3^\circ + 0.2606\angle -35.02^\circ}{2.1524\angle 208.3^\circ - 0.2606\angle -35.02^\circ} \right\}$$

$$Z_S = 696.06\angle -10.84^\circ \left\{ \frac{2.0487\angle -145.17^\circ}{2.2813\angle -157.56^\circ} \right\}$$

$Z_S = 696.06\ \angle -10.84^\circ \times 0.8980\angle 12.39^\circ$

$Z_S = 625.08\angle 1.548^\circ$

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{1}{0 + 625.08\angle 1.548^\circ} = 1.5992\times 10^{-3}\angle -1.548^\circ\ \text{A}$$

$I_S = 1.5992\times 10^{-3}\ \angle -1.548^\circ$

$E_S = I_S \times Z_S$

$E_S = 1.5992\times 10^{-3}\ \angle -1.548^\circ \times 625.08\ \angle 1.548^\circ$

$E_S = 1\angle -3.4484\times 10^{-4}$ V

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)}e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, as s is measured from the receiving end.

$$I_S = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}l} - K\,e^{-\sqrt{ZY}l}\right]$$

$Z_R + Z_O = 893.28\angle{-8.427}^\circ$

$Ke^{\sqrt{ZY}l} = 0.5609\angle 173.28^\circ \times 2.1524\angle 208.3^\circ$

$Ke^{\sqrt{ZY}l} = 1.2073\angle 21.58^\circ$

$$1.5992\text{x}10^{-3}\angle{-1.548}^\circ = \frac{I_R}{2}\left(\frac{893.28\angle{-8.427}^\circ}{696.06\angle{-10.84}^\circ}\right)\left[2.1524\angle 208.3^\circ - 1.2073\angle 21.58^\circ\right]$$

$$1.5992\text{x}10^{-3}\angle{-1.548}^\circ = \frac{I_R}{2}\left(1.2833\angle 2.413^\circ\right)\left[3.354\angle{-154.11}^\circ\right]$$

$$1.5992\text{x}10^{-3}\angle{-1.548}^\circ = I_R \times 2.1523\angle{-151.70}^\circ$$

$I_R = 7.43\text{x}10^{-4}\angle 150.15^\circ$

$E_R = I_R \text{ x } Z_R = 7.43\text{x}10^{-4}\angle 150.15^\circ \text{ x } 200 = 0.1486\angle 150.153^\circ$

$E_R = 0.1486\angle 150.15^\circ$ V

$P_S = |E_S|.\ |I_S|.\cos(E_S \wedge I_S)$

$P_S = 1 \text{ x } 1.5992 \text{ x } 10^{-3} \text{ x } \cos(-1.5477)$

$P_S = 1.5986 \text{ x } 10^{-3}$ W

and $P_R = |E_R|.\ |I_R|.\cos(E_R \wedge I_R)$

$P_R = 0.1486 \text{ x } 7.43\text{x}10^{-4}\cos(0)$

$P_R = 0.1486 \text{ x } 7.43\text{x}10^{-4}$ W

$P_R = 1.1041\text{x}10^{-4}$ W

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17. A generator of 1V, 1KHz, supplies power to a 100 Km open wire line terminated in 200 Ω resistance. The line parameters are R = 10 Ω/Km, L = 3.8mH/Km, G =1x10^{-6} mho/Km, C = 0.0085 μF/Km. Calculate the impedance, reflection coefficient, power and transmission efficiency.

[EC2305-APR/MAY 2011](16 Marks)

Solution:

Given: E_g = 1V, f=1000 Hz, l=100 Km, Z_R =200 Ω, R=10 Ω/Km, L = 3.8mH/Km, G =1x10^{-6} mho/Km, C = 0.0085 μF/Km

$\omega = 2\pi f = 2\text{x}3.14\text{x}1000 = 6280$

The characteristic impedance is given by,

$$Z_O = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 10 + j(6280)(3.8\text{x}10^{-3}) = 25.87\angle 67.26^\circ$

$Y = G+j\omega C = 1\text{x}10^{-6} + j(6280)(0.0085\text{x}10^{-6}) = 5.339\text{x}10^{-5}\angle 88.93^\circ$

$$Z_O = \sqrt{\frac{25.87\angle 67.26^\circ}{5.339\times 10^{-5}\angle 88.93^\circ}}$$

$$Z_O = \frac{5.086}{7.307\times 10^{-3}}\angle(67.26^\circ - 88.93^\circ)/2$$

Z_o = 696.06 $\angle$-10.84° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.87\angle 67.26^\circ \times 5.339\times 10^{-5}\angle 88.93^\circ}$$

$$P = \gamma = \sqrt{1.3812\times 10^{-3}\angle(67.26^\circ + 88.93^\circ)}$$

P=γ = α+jβ = 0.03716 ∠78.095° (Or) 7.6657×10^{-3}+j0.03636

attenuation constant, α = 7.6657×10^{-3} Np/Km

and phase constant, β = 0.03636 rad/Km

Reflection Coefficient, K

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{200 - 696.06\angle -10.84^\circ}{200 + 696.06\angle -10.84^\circ} = -0.557 + j0.0656 = 0.5609\angle 173.28^\circ$$

K=0.5609∠173.28°

$$e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l}\angle \beta l = e^{(7.6657\times 10^{-3})(100)}\angle(0.03636)(100)$$

$$e^{\gamma l} = e^{(7.6657\times 10^{-3})(100)}\angle(0.03636)(100)\,\text{rad}$$

$e^{\gamma l}$ = $e^{0.76657}$ ∠208.3°

$e^{\gamma l}$ = 2.1524 ∠208.3°

$$e^{-\gamma l} = e^{-(\alpha + j\beta)l} = e^{-\alpha l}\angle -\beta l = e^{-(7.6657\times 10^{-3})(100)}\angle -(0.03636)(100)$$

$$e^{-\gamma l} = e^{-(7.6657\times 10^{-3})(100)}\angle -(0.03636)(100)\,\text{rad}$$

$e^{-\gamma l}$ = $e^{-0.76657}$ ∠-208.3°

$e^{-\gamma l}$ = 0.4646 ∠-208.3°

$$Z_s = Z_o\left\{\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}}\right\}$$

$$Z_S = 696.06\angle -10.84^\circ\left\{\frac{2.1524\angle 208.3^\circ + (0.5609\angle 173.28^\circ)(0.4646\angle -208.3^\circ)}{2.1524\angle 208.3^\circ - (0.5609\angle 173.28^\circ)(0.4646\angle -208.3^\circ)}\right\}$$

$$Z_S = 696.06\angle -10.84^\circ\left\{\frac{2.1524\angle 208.3^\circ + 0.2606\angle -35.02^\circ}{2.1524\angle 208.3^\circ - 0.2606\angle -35.02^\circ}\right\}$$

$$Z_S = 696.06\angle -10.84^\circ \left\{\frac{2.0487\angle -145.17^\circ}{2.2813\angle -157.56^\circ}\right\}$$

Z_S = 696.06 $\angle$-10.84° x 0.8980$\angle$12.39°

Z_S = 625.08$\angle$1.548°

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{1}{0 + 625.08\angle 1.548^\circ} = 1.5992 \times 10^{-3}\angle -1.548^\circ \text{ A}$$

I_S = 1.5992x10^{-3} $\angle$-1.548°

E_S = I_S x Z_S

E_S = 1.5992x10^{-3} $\angle$-1.548° x 625.08 $\angle$1.548°

E_S = 1$\angle$-3.4484x10^{-4} V

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)}e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, as s is measured from the receiving end.

$$I_S = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}l} - K\,e^{-\sqrt{ZY}l}\right]$$

Z_R + Z_O = 893.28$\angle$-8.427°

Ke$^{\sqrt{ZY}l}$ = 0.5609$\angle$173.28° x 2.1524 $\angle$208.3°

Ke$^{\sqrt{ZY}l}$ = 1.2073$\angle$21.58°

$$1.5992\text{x}10^{-3}\ \angle -1.548^\circ = \frac{I_R}{2}\left(\frac{893.28\ \angle -8.427^\circ}{696.06\ \angle -10.84^\circ}\right)\left[2.1524\angle 208.3^\circ - 1.2073\angle 21.58^\circ\right]$$

$$1.5992\text{x}10^{-3}\ \angle -1.548^\circ = \frac{I_R}{2}\left(1.2833\angle 2.413^\circ\right)\left[3.354\angle -154.11^\circ\right]$$

$$1.5992\text{x}10^{-3}\ \angle -1.548^\circ = I_R \times 2.1523\angle -151.70^\circ$$

I_R = 7.43x10^{-4}$\angle$150.15°

E_R = I_R x Z_R = 7.43x10^{-4}$\angle$150.15° x 200 = 0.1486$\angle$150.153°

E_R = 0.1486$\angle$150.15° V

P_S = |E_S|. |I_S|.cos (E_S^ I_S)

P_S = 1 x 1.5992 x 10^{-3} x cos (-1.5477)

P_S = 1.5986 x 10^{-3} W

and P_R = |E_R|. |I_R|.cos (E_R^ I_R)

P_R = 0.1486 x 7.43x10^{-4} cos(0)

P_R = 0.1486 x 7.43x10^{-4} W

P_R = 1.1041x10^{-4} W

Transmission Efficiency, $\eta = \frac{P_R}{P_S} \times 100$

$$\eta = \frac{1.1041\text{x}10^{-4}}{1.5986 \times 10^{-3}} \times 100$$

η = 6.907 %

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18. A telephone cable 64 km long has a resistance of 13 Ω/km and a capacitance of 0.008 μf /km. Calculate the attenuation constant, velocity and wavelength of the line at 1000 Hz.

[EC2305-NOV/DEC 2016](6 Marks), [EC2305-MAY/JUN 2015](6 Marks), [EC2305-APR/MAY/JUN 2016](4 Marks)

Solution:

Given: l=64Km, R=13 Ω/km, C=0.008 μf /km, f=1000 Hz

ω = 2πf = 2x3.14x1000 = 6280

The characteristic impedance is given by,

$$Z_O = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_O = \sqrt{\frac{13}{j(6280)(0.008 \times 10^{-6})}}$$

$$Z_o = \sqrt{\frac{13}{j5.024 \times 10^{-5}}} = \sqrt{\frac{13\angle 0^\circ}{5.024 \times 10^{-5} \angle 90^\circ}}$$

$$Z_o = \frac{3.61}{7.088 \times 10^{-3}} \angle(0^\circ - 90^\circ)/2$$

Z_o = 509.31 ∠-45°

The propagation constant is given by,

$$P = \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{(13) \times \left(j(6280)(0.008 \times 10^{-6})\right)}$$

$$P = \gamma = \sqrt{13 \times j5.024 \times 10^{-5}}$$

$$P = \gamma = \sqrt{6.5312 \times 10^{-4} \angle 90^\circ}$$

P=γ = α+jβ = 25.56x10^{-3} ∠45° (Or) 0.01807+j0.01807

attenuation constant, α = 0.01807 Np/Km

and phase constant, β = 0.01807 rad/Km

The velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{6280}{0.01807} = 3.48 \times 10^5 \text{ Km/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{.01807} = 347.54 \text{ m}$$

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19. A transmission line has L = 10 mH/m, C = 10 $^{-7}$ F/m, R = 20 Ω/m and G =10 $^{-5}$ mho/m. Find the input impedance at a frequency of $\left(\frac{5000}{2\pi}\right)$Hz, if the line is very long. **[EC2305-NOV/DEC 2013] (6 Marks)**

Solution:

Given: L = 10 mH/m, C = 10 $^{-7}$ F/m, R = 20 Ω/m and G =10 $^{-5}$ mho/m,

$$f = \left(\frac{5000}{2\pi}\right)$$

$$\omega = 2\pi f = 2\pi \text{ x} \left(\frac{5000}{2\pi}\right) = 5000$$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{20 + j(5000)(10 \times 10^{-3})}{10^{-5} + j(5000)(10 \times 10^{-7})}}$$

$$Z_o = \sqrt{\frac{20 + j50}{10^{-5} + j5 \times 10^{-4}}} = \sqrt{\frac{53.85\angle 68.199^\circ}{5.0 \times 10^{-4}\angle 88.85^\circ}}$$

$$Z_o = \frac{7.338}{0.0224}\angle(68.199^\circ - 88.85^\circ)/2$$

$\mathbf{Z_o = 327.59 \angle{-10.33}^\circ\ \Omega}$

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20. The characteristic impedance of a uniform transmission line is 2309.6 ohms at a frequency of 800 Hz. At this frequency, the propagation constant is 0.054(0.0366+j0.99). Determine R and L.

[EC2305-NOV/DEC 2013] (6 Marks), [EC2305-NOV/DEC 2010] (6 Marks)

Solution:

Given: Z_o=2309.6 Ω , f=800Hz, P=γ=0.054(0.0366+j0.99) = 0.054∠87.9°

ω = 2πf = 2 x 3.14 x 800 = 5024

R=jωL = Z_oP = 2309.6 x 0.054∠87.9°

= 4.57+j124.63

Comparing real and imaginary terms,

R = 4.57 ohms/Km

$\omega L = 124.63$

$$L = \frac{124.63}{5024} = 0.0248\,H/Km\,(Or)\,24.8mH/Km$$

L=24.8 mH/Km

---

21. A transmission line has the following per unit length parameters: L=0.1μ H, R=5Ω, C=300pF and G= 0.01mhos. Calculate the propagation constant and characteristic impedance at 500 MHz. **[EC2305-NOV/DEC 2010] (8 Marks)**

Solution:

Given: f=500MHz, R = 5 Ω, L = 0.1μH, G = 0.01 mho, C = 300 pF

$\omega = 2\pi f = 2x3.14x500x10^6 = 3140x10^6$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 5 + j(3140x10^6)(0.1x10^{-6}) = 314.04\angle 89.09°$

$Y = G+j\omega C = 0.01+ j(3140x10^6)(300x10^{-12}) = 0.9421\angle 89.39°$

$$Z_o = \sqrt{\frac{314.04\angle 89.09°}{0.9421\angle 89.39°}}$$

$$Z_o = \frac{17.721}{0.9706}\angle(89.09° - 89.39°)/2$$

Z_o = 18.258 ∠-0.15° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{314.04\angle 89.09° \times 0.9421\angle 89.39°}$$

$$P = \gamma = \sqrt{295.86\angle(89.09° + 89.39°)}$$

$P = \gamma = \alpha + j\beta = 17.2\ \angle 89.24°$ (Or) $0.2281+j17.198$

---

22. A transmission line has the following constants R = 10.4 Ω/km, L = 3.666 mH/km, G = 0.08 x 10^{-6} mho/km and C = 0.00835 μF/km. Calculate its characteristic impedance, attenuation, phase constant and phase velocity. **[EC2305-MAY/JUN 2012](8 Marks)**

Solution:

Given: R = 10.4 Ω/km, L = 3.666 mH/km, G = 0.08 x 10^{-6} mho/km and C = 0.00835 μF/km

Let f =1000 Hz,

$\omega = 2\pi f = 2x3.14x1000 = 6280$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 10.4 + j(6280)(3.666x10^{-3}) = 25.263\angle 65.69°$

$Y = G+j\omega C = 0.08x10^{-6}+ j(6280)(0.00835x10^{-6}) = 5.244x10^{-5}\angle 89.91°$

$$Z_o = \sqrt{\frac{25.263\angle 65.69°}{5.244x10^{-5}\angle 89.91°}}$$

$$Z_o = \frac{5.0262}{7.242 \times 10^{-3}}\angle(65.69° - 89.91°)/2$$

Z_o = 694.08 $\angle$-12.11° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{25.263\angle 65.69° \times 5.244 \times 10^{-5}\angle 89.91°}$$

$$P = \gamma = \sqrt{1.3248 \times 10^{-3}\angle(65.69° + 89.91°)}$$

P=γ = α+jβ = 0.0364 $\angle$77.8° (Or) 7.6922x10^{-3}+j0.0356

attenuation constant, α = 7.6922x10^{-3} Np/Km

phase constant, β = 0.0356 rad/Km

$$\text{Phase Velocity}, v_p = \frac{\omega}{\beta} = \frac{6280}{0.0356} = 1.76 \times 10^5 \text{ Km/sec}$$

---

23. A telephone line has R=6Ω/Km, L=2.2mH/Km, C=0.005μF/Km and G=0.05x10^{-6} mho/Km. Determine the characteristic impedance and propagation constant at 1 KHz. **[EC2305-APR/MAY 2016] (6 Marks)**

Solution:

Given: R = 6 Ω/km, L = 2.2 mH/km, G = 0.05 x 10^{-6} mho/km and C = 0.005 μF/km, f =1000 Hz,

$\omega = 2\pi f = 2x3.14x1000 = 6280$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 6 + j(6280)(2.2x10^{-3}) = 15.063\angle 66.53°$

$Y = G+j\omega C = 0.05x10^{-6}+ j(6280)(0.005x10^{-6}) = 3.14x10^{-5}\angle 89.91^{\circ}$

$$Z_o = \sqrt{\frac{15.063\angle 66.53°}{3.14x10^{-5}\angle 89.91°}}$$

$$Z_o = \frac{3.881}{5.604 \times 10^{-3}}\angle(66.53° - 89.91°)/2$$

Z_o = 692.59 ∠-11.69° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{15.063\angle 66.53° \times 3.14 \times 10^{-5}\angle 89.91°}$$

$$P = \gamma = \sqrt{0.473 \times 10^{-3}\angle(66.53° + 89.91°)}$$

P=γ = α+jβ = 0.0217 ∠78.22° (Or) 4.43x10^{-3}+j0.0212

attenuation constant, α = 4.43x10^{-3} Np/Km

phase constant, β = 0.0212 rad/Km

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24. A cable has the following parameters: R=48.75 ohm/Km, L=1.09mH/Km, G=38.75μmho/Km, C=0.059 μF/Km. Determine the characteristic impedance, propagation constant and wavelength for a source of f=1600Hz and E_s=1V. **[EC2305-APR/MAY 2013](6 Marks)**

Solution:

Given: R=48.75 ohm/Km, L=1.09mH/Km, G=38.75μmho/Km, C=0.059 μF/Km

$\omega = 2\pi f = 2x3.14x1600 = 10048$

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$Z = R+j\omega L = 48.75 + j(10048)(1.09\text{x}10^{-3}) = 49.965\angle 12.66°$

$Y = G+j\omega C = 38.75\text{x}10^{-6} + j(10048)(0.059\text{x}10^{-6}) = 5.941\text{x}10^{-4}\angle 86.26°$

$$Z_o = \sqrt{\frac{49.965\angle 12.66°}{5.941 \times 10^{-4}\angle 86.26°}}$$

$$Z_o = \frac{7.069}{0.0244}\angle(12.66° - 86.26°)/2$$

Z_o = 290.02 $\angle$-36.8° Ω

The propagation constant is given by,

$$P = \gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{49.965\angle 12.66° \times 5.941 \times 10^{-4}\angle 86.26°}$$

$$P = \gamma = \sqrt{0.02968\angle(12.66° + 86.26°)}$$

$P=\gamma = \alpha+j\beta = 0.1723\ \angle 49.46°$ (Or) $0.112+j0.131$

attenuation constant, α = 0.112 Np/Km

and phase constant, β = 0.131 rad/Km

Wavelength is given by,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.131} = 47.96\,\text{m}$$
