CHAPTER 2

Fundamental Parameters of Antennas

2.1 INTRODUCTION

To describe the performance of an antenna, definitions of various parameters are necessary. Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance. Parameter definitions will be given in this chapter. Many of those in quotation marks are from the *IEEE Standard Definitions of Terms for Antennas* (IEEE Std 145-1983).* This is a revision of the IEEE Std 145-1973.

2.2 RADIATION PATTERN

An antenna *radiation pattern* or *antenna pattern* is defined as "a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity, phase or polarization." The radiation property of most concern is the two- or three-dimensional spatial distribution of radiated energy as a function of the observer's position along a path or surface of constant radius. A convenient set of coordinates is shown in Figure 2.1. A trace of the received electric (magnetic) field at a constant radius is called the amplitude field *pattern*. On the other hand, a graph of the spatial variation of the power density along a constant radius is called an amplitude *power pattern*.

Often the *field* and *power* patterns are normalized with respect to their maximum value, yielding *normalized field* and *power patterns*. Also, the power pattern is usually plotted on a logarithmic scale or more commonly in decibels (dB). This scale is usually desirable because a logarithmic scale can accentuate in more details those parts of the

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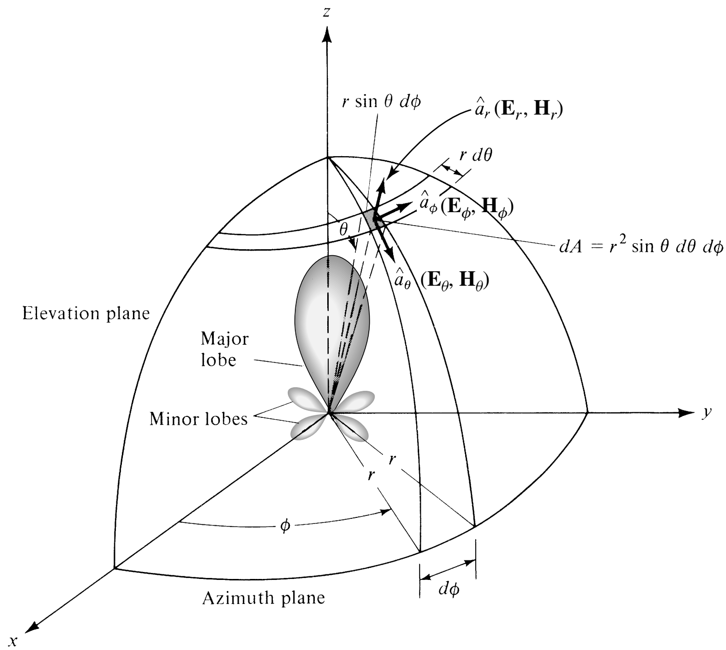


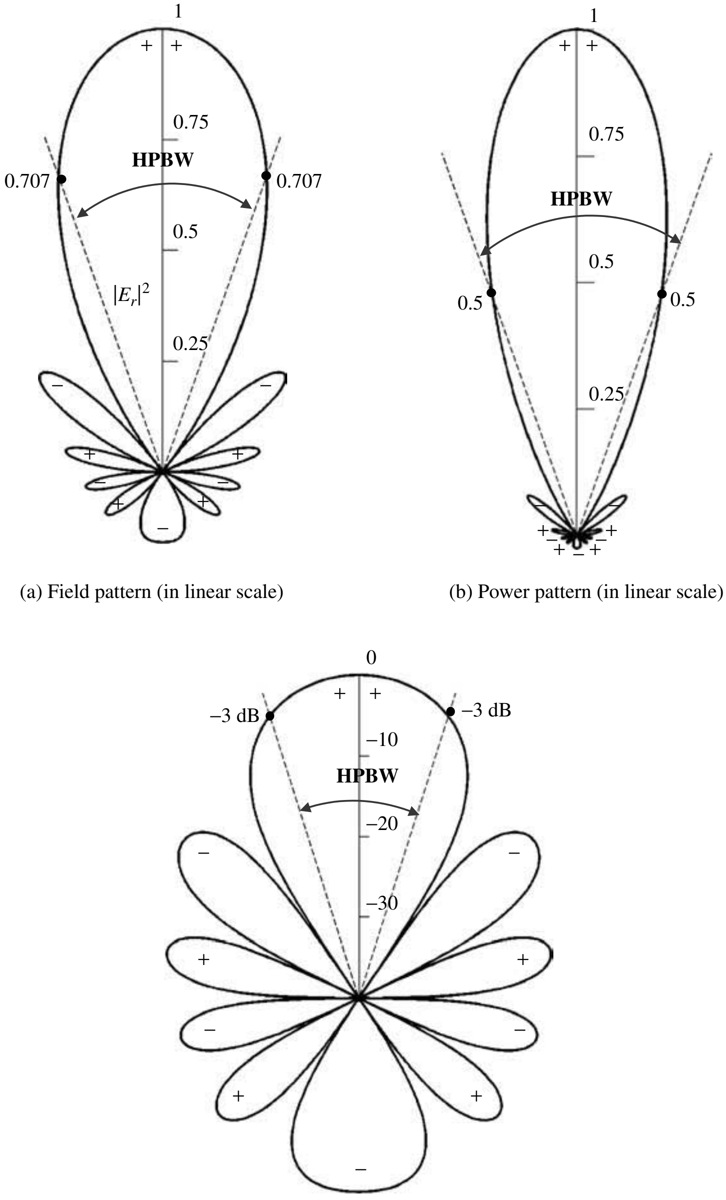
Figure 2.1 Coordinate system for antenna analysis.

pattern that have very low values, which later we will refer to as minor lobes. For an antenna, the

a. *field* pattern (*in linear scale*) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.
b. *power* pattern (*in linear scale*) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.
c. *power* pattern (*in dB*) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

To demonstrate this, the two-dimensional normalized field pattern (*plotted in linear scale*), power pattern (*plotted in linear scale*), and power pattern (*plotted on a logarithmic dB scale*) of a 10-element linear antenna array of isotropic sources, with a spacing of $d = 0.25\lambda$ between the elements, are shown in Figure 2.2. *In this and subsequent patterns, the plus* (+) *and minus* (−) *signs in the lobes indicate the relative polarization of the amplitude between the various lobes, which changes (alternates) as the nulls are crossed*. To find the points where the pattern achieves its half-power (−3 dB points), relative to the maximum value of the pattern, you set the value of the

a. field pattern at 0.707 value of its maximum, as shown in Figure 2.2(a)
b. power pattern (in a linear scale) at its 0.5 value of its maximum, as shown in Figure 2.2(b)
c. power pattern (in dB) at −3 *dB value of its maximum, as shown in Figure 2.2(c).*



(a) Field pattern (in linear scale)

(b) Power pattern (in linear scale)

(c) Power pattern (in dB)

Figure 2.2 Two-dimensional normalized *field* pattern (*linear scale*), *power* pattern (*linear scale*), and *power* pattern (*in dB*) of a 10-element linear array with a spacing of $d = 0.25\lambda$.

All three patterns yield the same angular separation between the two half-power points, 38.64°, on their respective patterns, *referred to as HPBW* and illustrated in Figure 2.2. This is discussed in detail in Section 2.5.

In practice, the three-dimensional pattern is measured and recorded in a series of two-dimensional patterns. However, for most practical applications, a few plots of the pattern as a function of θ for some particular values of ϕ, plus a few plots as a function of ϕ for some particular values of θ, give most of the useful and needed information.

2.2.1 Radiation Pattern Lobes

Various parts of a radiation pattern are referred to as *lobes*, which may be subclassified into *major* or *main, minor, side*, and *back* lobes.

A *radiation lobe* is a "portion of the radiation pattern bounded by regions of relatively weak radiation intensity." Figure 2.3(a) demonstrates a symmetrical three-dimensional polar pattern with a number of radiation lobes. Some are of greater radiation intensity than others, but all are classified as lobes. Figure 2.3(b) illustrates

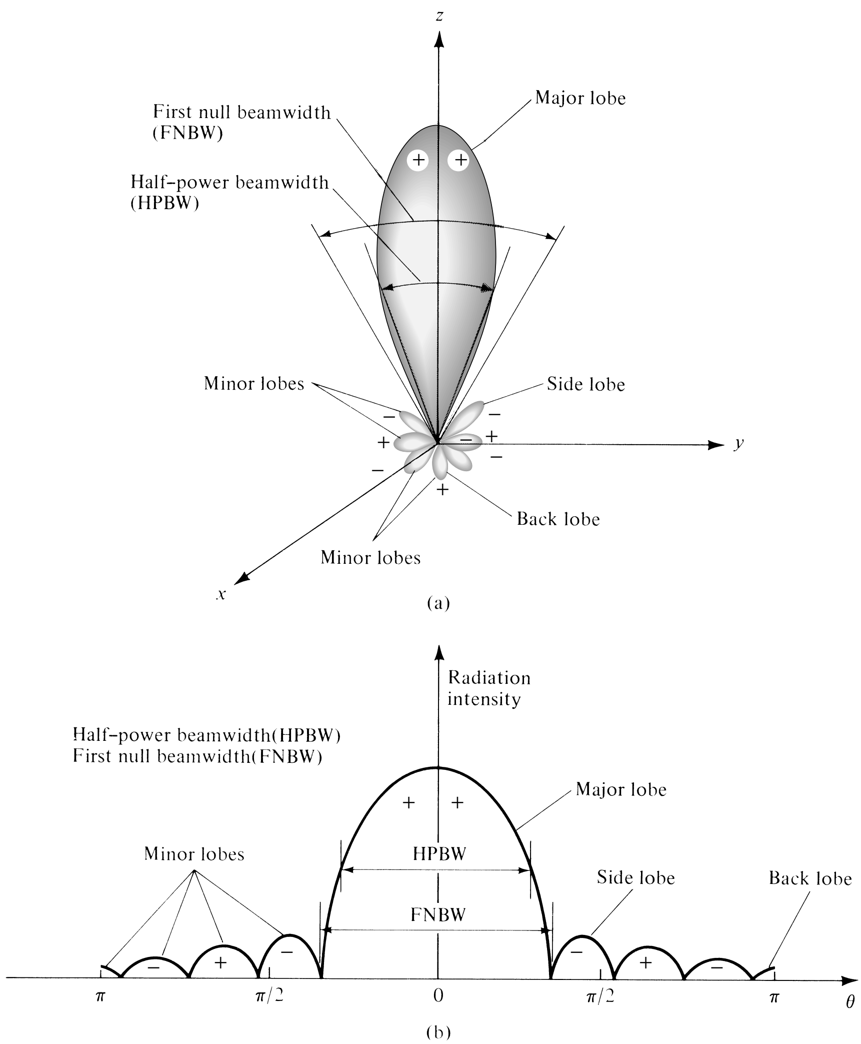


Figure 2.3 (a) Radiation lobes and beamwidths of an antenna pattern. (b) Linear plot of power pattern and its associated lobes and beamwidths.

a linear two-dimensional pattern [one plane of Figure 2.3(a)] where the same pattern characteristics are indicated.

MATLAB-based computer programs, designated as ***polar*** and ***spherical***, have been developed and are included in the CD of this book. These programs can be used to plot the two-dimensional patterns, both polar and semipolar (*in linear* and *dB scales*), in polar form and spherical three-dimensional patterns (*in linear* and *dB scales*). A description of these programs is found in the attached CD. Other programs that have been developed for plotting rectangular and polar plots are those of [1]–[3].

A *major lobe* (also called main beam) is defined as "the radiation lobe containing the direction of maximum radiation." In Figure 2.3 the major lobe is pointing in the $\theta = 0$ direction. In some antennas, such as split-beam antennas, there may exist more than one major lobe. A *minor lobe* is any lobe except a major lobe. In Figures 2.3(a) and (b) all the lobes with the exception of the major can be classified as minor lobes. A *side lobe* is "a radiation lobe in any direction other than the intended lobe." (Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam.) A *back lobe* is "a radiation lobe whose axis makes an angle of approximately 180° with respect to the beam of an antenna." Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.

Minor lobes usually represent radiation in undesired directions, and they should be minimized. Side lobes are normally the largest of the minor lobes. The level of minor lobes is usually expressed as a ratio of the power density in the lobe in question to that of the major lobe. This ratio is often termed the side lobe ratio or side lobe level. Side lobe levels of −20 dB or smaller are usually not desirable in most applications.

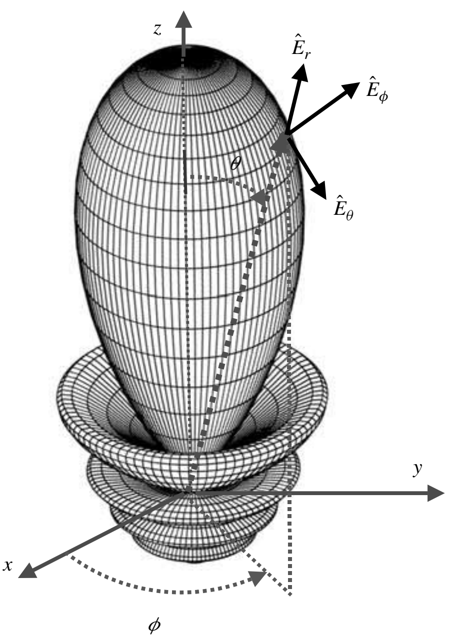


Figure 2.4 Normalized three-dimensional amplitude *field* pattern (*in linear scale*) of a 10-element linear array antenna with a uniform spacing of $d = 0.25\lambda$ and progressive phase shift $\beta = -0.6\pi$ between the elements.

Attainment of a side lobe level smaller than −30 dB usually requires very careful design and construction. In most radar systems, low side lobe ratios are very important to minimize false target indications through the side lobes.

A normalized three-dimensional far-field amplitude pattern, plotted on a linear scale, of a 10-element linear antenna array of isotropic sources with a spacing of $d = 0.25\lambda$ and progressive phase shift $\beta = -0.6\pi$, between the elements is shown in Figure 2.4. It is evident that this pattern has one major lobe, five minor lobes and one back lobe. The level of the side lobe is about −9 dB relative to the maximum. A detailed presentation of arrays is found in Chapter 6. For an amplitude pattern of an antenna, there would be, in general, three electric-field components (E_r, E_θ, E_ϕ) at each observation point on the surface of a sphere of constant radius $r = r_c$, as shown in Figure 2.1. In the far field, the radial E_r component for all antennas is zero or vanishingly small compared to either one, or both, of the other two components (see Section 3.6 of Chapter 3). Some antennas, depending on their geometry and also observation distance, may have only one, two, or all three components. In general, the magnitude of the total electric field would be $|\mathbf{E}| = \sqrt{|E_r|^2 + |E_\theta|^2 + |E_\phi|^2}$. The radial distance in Figure 2.4, and similar ones, represents the magnitude of $|\mathbf{E}|$.

2.2.2 Isotropic, Directional, and Omnidirectional Patterns

An *isotropic* radiator is defined as "a hypothetical lossless antenna having equal radiation in all directions." Although it is ideal and not physically realizable, it is often

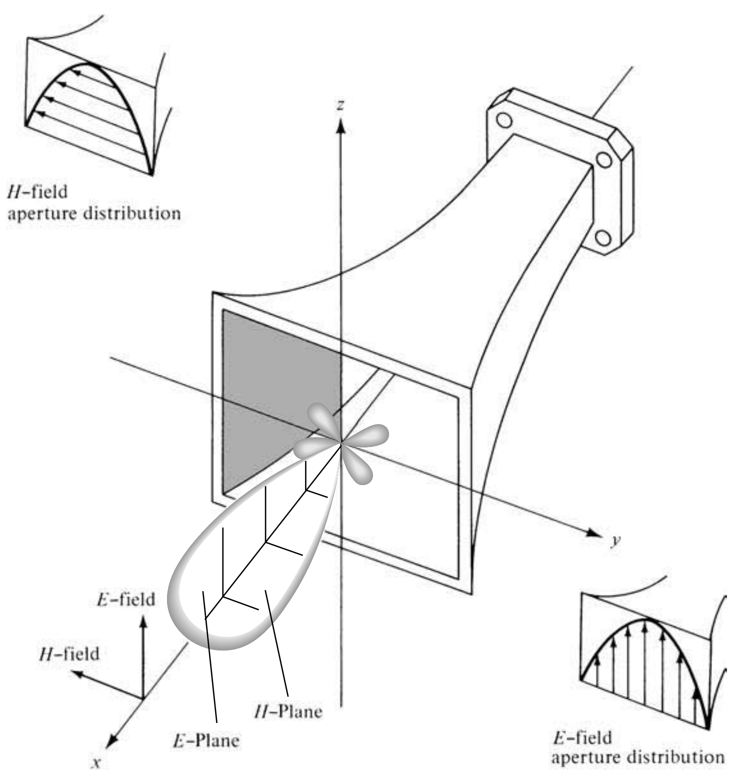


Figure 2.5 Principal E- and H-plane patterns for a pyramidal horn antenna.

taken as a reference for expressing the directive properties of actual antennas. A *directional* antenna is one "having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others. This term is usually applied to an antenna whose maximum directivity is significantly greater than that of a half-wave dipole." Examples of antennas with directional radiation patterns are shown in Figures 2.5 and 2.6. It is seen that the pattern in Figure 2.6 is nondirectional in the azimuth plane $[f(\phi), \theta = \pi/2]$ and directional in the elevation plane $[g(\theta), \phi = \text{constant}]$. This type of a pattern is designated as *omnidirectional*, and it is defined as one "having an essentially nondirectional pattern in a given plane (in this case in azimuth) and a directional pattern in any orthogonal plane (in this case in elevation)." An *omnidirectional* pattern is then a special type of a *directional* pattern.

2.2.3 Principal Patterns

For a linearly polarized antenna, performance is often described in terms of its principal E- and H-plane patterns. The *E-plane* is defined as "the plane containing the electric-field vector and the direction of maximum radiation," and the *H-plane* as "the plane containing the magnetic-field vector and the direction of maximum radiation." Although it is very difficult to illustrate the principal patterns without considering a specific example, it is the usual practice to orient most antennas so that at least one of the principal plane patterns coincide with one of the geometrical principal planes. An illustration is shown in Figure 2.5. For this example, the x-z plane (elevation plane; $\phi = 0$) is the principal E-plane and the x-y plane (azimuthal plane; $\theta = \pi/2$) is the principal H-plane. Other coordinate orientations can be selected.

The omnidirectional pattern of Figure 2.6 has an infinite number of principal E-planes (elevation planes; $\phi = \phi_c$) and one principal H-plane (azimuthal plane; $\theta = 90°$).

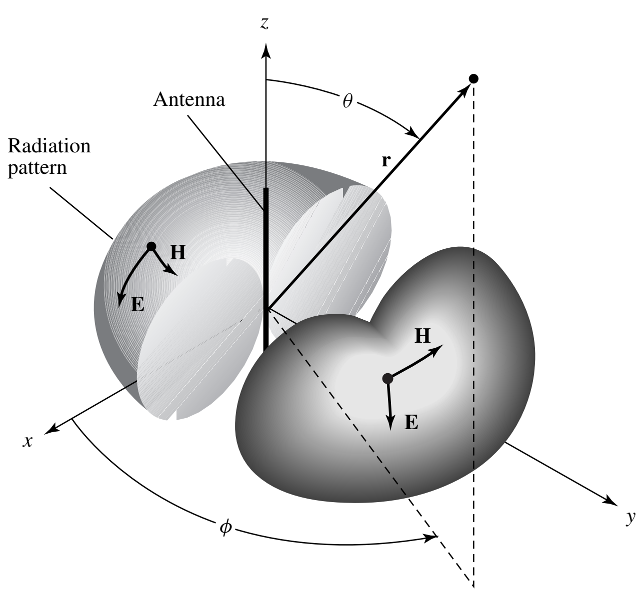


Figure 2.6 Omnidirectional antenna pattern.

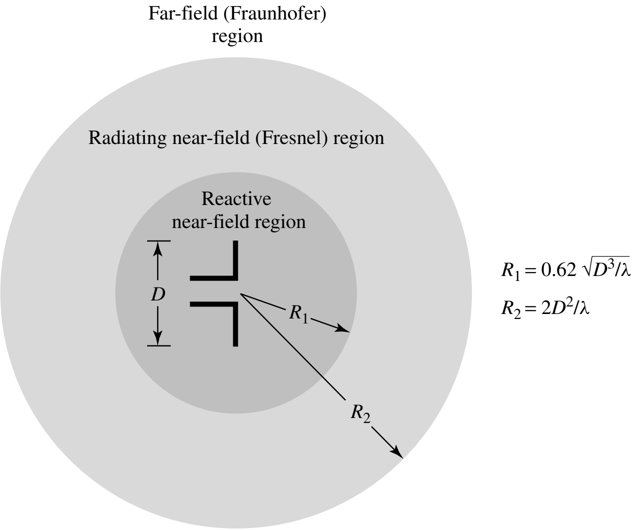


Figure 2.7 Field regions of an antenna.

2.2.4 Field Regions

The space surrounding an antenna is usually subdivided into three regions: (a) reactive near-field, (b) radiating near-field (Fresnel) and (c) far-field (Fraunhofer) regions as shown in Figure 2.7. These regions are so designated to identify the field structure in each. Although no abrupt changes in the field configurations are noted as the boundaries are crossed, there are distinct differences among them. The boundaries separating these regions are not unique, although various criteria have been established and are commonly used to identify the regions.

Reactive near-field region is defined as "that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates." For most antennas, the outer boundary of this region is commonly taken to exist at a distance $R < 0.62\sqrt{D^3/\lambda}$ from the antenna surface, where λ is the wavelength and D is the largest dimension of the antenna. "For a very short dipole, or equivalent radiator, the outer boundary is commonly taken to exist at a distance $\lambda/2\pi$ from the antenna surface."

Radiating near-field (Fresnel) region is defined as "that region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna. If the antenna has a maximum dimension that is not large compared to the wavelength, this region may not exist. For an antenna focused at infinity, the radiating near-field region is sometimes referred to as the Fresnel region on the basis of analogy to optical terminology. If the antenna has a maximum overall dimension which is very small compared to the wavelength, this field region may not exist." The inner boundary is taken to be the distance $R \geq 0.62\sqrt{D^3/\lambda}$ and the outer boundary the distance $R < 2D^2/\lambda$ where D is the largest* dimension of the antenna. This criterion is based on a maximum phase error of $\pi/8$. In this region the field pattern is, in general, a function of the radial distance and the radial field component may be appreciable.

*To be valid, D must also be large compared to the wavelength ($D > \lambda$).

Far-field (Fraunhofer) region is defined as "that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna. If the antenna has a maximum* overall dimension D, the far-field region is commonly taken to exist at distances greater than $2D^2/\lambda$ from the antenna, λ being the wavelength. The far-field patterns of certain antennas, such as multibeam reflector antennas, are sensitive to variations in phase over their apertures. For these antennas $2D^2/\lambda$ may be inadequate. In physical media, if the antenna has a maximum overall dimension, D, which is large compared to $\pi/|\gamma|$, the far-field region can be taken to begin approximately at a distance equal to $|\gamma|D^2/\pi$ from the antenna, γ being the propagation constant in the medium. For an antenna focused at infinity, the far-field region is sometimes referred to as the Fraunhofer region on the basis of analogy to optical terminology." In this region, the field components are essentially transverse and the angular distribution is independent of the radial distance where the measurements are made. The inner boundary is taken to be the radial distance $R = 2D^2/\lambda$ and the outer one at infinity.

The amplitude pattern of an antenna, as the observation distance is varied from the reactive near field to the far field, changes in shape because of variations of the fields, both magnitude and phase. A typical progression of the shape of an antenna, with the largest dimension D, is shown in Figure 2.8. It is apparent that in the reactive near-field region the pattern is more spread out and nearly uniform, with slight variations. As the observation is moved to the radiating near-field region (Fresnel), the pattern begins to smooth and form lobes. In the far-field region (Fraunhofer), the pattern is well formed, usually consisting of few minor lobes and one, or more, major lobes.

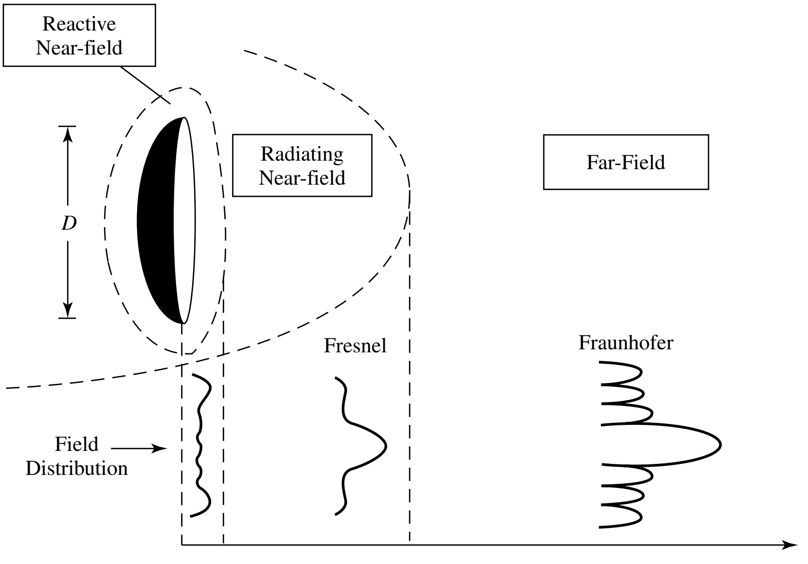


Figure 2.8 Typical changes of antenna amplitude pattern shape from reactive near field toward the far field. (SOURCE: Y. Rahmat-Samii, L. I. Williams, and R. G. Yoccarino, The UCLA Bi-polar Planar-Near-Field Antenna Measurement and Diagnostics Range," *IEEE Antennas & Propagation Magazine*, Vol. 37, No. 6, December 1995 © 1995 IEEE).

*To be valid, D must also be large compared to the wavelength ($D > \lambda$).

To illustrate the pattern variation as a function of radial distance beyond the minimum $2D^2/\lambda$ far-field distance, in Figure 2.9 we have included three patterns of a parabolic reflector calculated at distances of $R = 2D^2/\lambda, 4D^2/\lambda$, and infinity [4]. It is observed that the patterns are almost identical, except for some differences in the pattern structure around the first null and at a level below 25 dB. Because infinite distances are not realizable in practice, the most commonly used criterion for minimum distance of far-field observations is $2D^2/\lambda$.

2.2.5 Radian and Steradian

The measure of a plane angle is a radian. One *radian* is defined as the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r. A graphical illustration is shown in Figure 2.10(a). Since the circumference of a circle of radius r is $C = 2\pi r$, there are 2π rad $(2\pi r/r)$ in a full circle.

The measure of a solid angle is a steradian. One *steradian* is defined as the solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area equal to that of a square with each side of length r. A graphical illustration is shown in Figure 2.10(b). Since the area of a sphere of radius r is $A = 4\pi r^2$, there are 4π sr $(4\pi r^2/r^2)$ in a closed sphere.

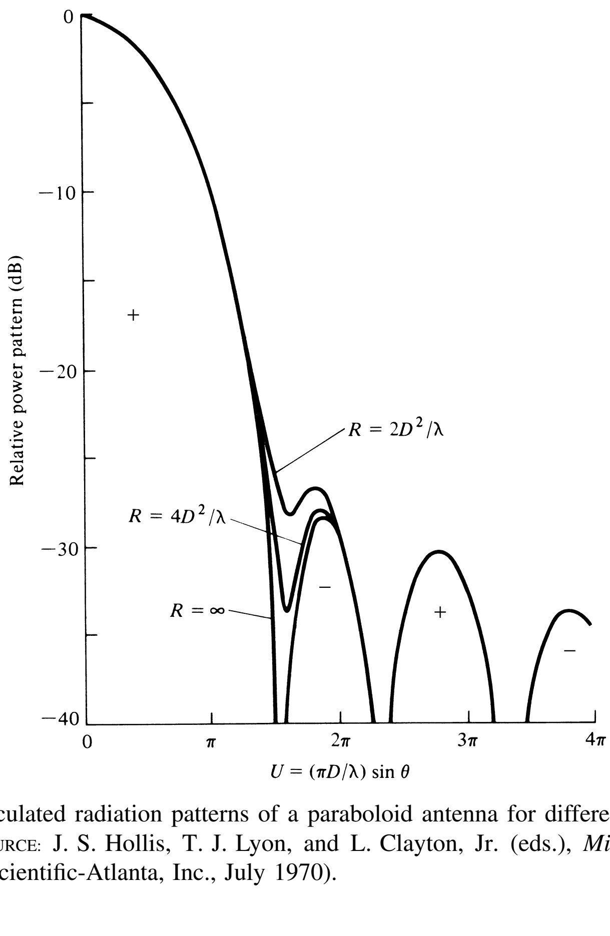
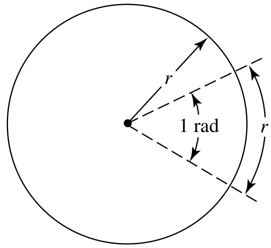
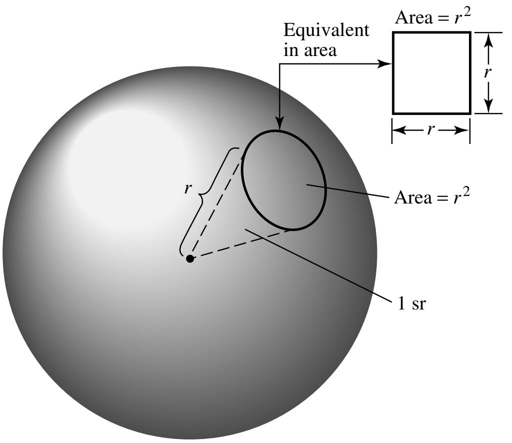


Figure 2.9 Calculated radiation patterns of a paraboloid antenna for different distances from the antenna. (SOURCE: J. S. Hollis, T. J. Lyon, and L. Clayton, Jr. (eds.), *Microwave Antenna Measurements*, Scientific-Atlanta, Inc., July 1970).



(a) Radian



(b) Steradian

Figure 2.10 Geometrical arrangements for defining a radian and a steradian.

The infinitesimal area dA on the surface of a sphere of radius r, shown in Figure 2.1, is given by

$$dA = r^2 \sin\theta \, d\theta \, d\phi \quad (\mathrm{m}^2) \tag{2-1}$$

Therefore, the element of solid angle $d\Omega$ of a sphere can be written as

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi \quad (\mathrm{sr}) \tag{2-2}$$

Example 2.1

For a sphere of radius r, find the solid angle Ω_A (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of $0 \le \theta \le 30^\circ$, $0 \le \phi \le 180^\circ$. Refer to Figures 2.1 and 2.10. Do this

a. exactly.
b. using $\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2$, where $\Delta\Theta_1$ and $\Delta\Theta_2$ are two perpendicular angular separations of the spherical cap passing through the north pole.

Compare the two.

Solution:

a. Using (2-2), we can write that

$$\Omega_A = \int_0^{360^\circ}\int_0^{30^\circ} d\Omega = \int_0^{2\pi}\int_0^{\pi/6} \sin\theta\, d\theta\, d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta\, d\theta$$

$$= 2\pi[-\cos\theta]|_0^{\pi/6} = 2\pi[-0.867 + 1] = 2\pi(0.133) = 0.83566$$

b. $\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2 \overbrace{=}^{\Delta\Theta_1=\Delta\Theta_2} (\Delta\Theta_1)^2 = \dfrac{\pi}{3}\left(\dfrac{\pi}{3}\right) = \dfrac{\pi^2}{9} = 1.09662$

It is apparent that the approximate beam solid angle is about 31.23% in error.

2.3 RADIATION POWER DENSITY

Electromagnetic waves are used to transport information through a wireless medium or a guiding structure, from one point to the other. It is then natural to assume that power and energy are associated with electromagnetic fields. The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as

$$\mathcal{W} = \mathcal{E} \times \mathcal{H} \tag{2-3}$$

$\mathcal{W}$ = instantaneous Poynting vector (W/m^2)
$\mathcal{E}$ = instantaneous electric-field intensity (V/m)
$\mathcal{H}$ = instantaneous magnetic-field intensity (A/m)

Note that script letters are used to denote instantaneous fields and quantities, while roman letters are used to represent their complex counterparts.

Since the Poynting vector is a power density, the total power crossing a closed surface can be obtained by integrating the normal component of the Poynting vector over the entire surface. In equation form

$$\mathcal{P} = \oiint_S \mathcal{W} \cdot d\mathbf{s} = \oiint_S \mathcal{W} \cdot \hat{\mathbf{n}}\, da \tag{2-4}$$

$\mathcal{P}$ = instantaneous total power (W)
$\hat{\mathbf{n}}$ = unit vector normal to the surface
da = infinitesimal area of the closed surface (m^2)

For applications of time-varying fields, it is often more desirable to find the average power density which is obtained by integrating the instantaneous Poynting vector over one period and dividing by the period. For time-harmonic variations of the form

$e^{j\omega t}$, we define the complex fields **E** and **H** which are related to their instantaneous counterparts $\mathscr{E}$ and $\mathscr{H}$ by

$$\mathscr{E}(x, y, z; t) = \text{Re}[\mathbf{E}(x, y, z)e^{j\omega t}] \tag{2-5}$$

$$\mathscr{H}(x, y, z; t) = \text{Re}[\mathbf{H}(x, y, z)e^{j\omega t}] \tag{2-6}$$

Using the definitions of (2-5) and (2-6) and the identity $\text{Re}[\mathbf{E}e^{j\omega t}] = \frac{1}{2}[\mathbf{E}e^{j\omega t} + \mathbf{E}^* e^{-j\omega t}]$, (2-3) can be written as

$$\mathscr{W} = \mathscr{E} \times \mathscr{H} = \tfrac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*] + \tfrac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}e^{j2\omega t}] \tag{2-7}$$

The first term of (2-7) is not a function of time, and the time variations of the second are twice the given frequency. The time average Poynting vector (average power density) can be written as

$$\boxed{\mathbf{W}_{\text{av}}(x, y, z) = [\mathscr{W}(x, y, z; t)]_{\text{av}} = \tfrac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*]} \quad (\text{W/m}^2) \tag{2-8}$$

The $\frac{1}{2}$ factor appears in (2-7) and (2-8) because the **E** and **H** fields represent peak values, and it should be omitted for RMS values.

A close observation of (2-8) may raise a question. If the real part of $(\mathbf{E} \times \mathbf{H}^*)/2$ represents the average (real) power density, what does the imaginary part of the same quantity represent? At this point it will be very natural to assume that the imaginary part must represent the reactive (stored) power density associated with the electromagnetic fields. In later chapters, it will be shown that the power density associated with the electromagnetic fields of an antenna in its far-field region is predominately real and will be referred to as *radiation density*.

Based upon the definition of (2-8), the average power radiated by an antenna (radiated power) can be written as

$$\boxed{\begin{aligned} P_{\text{rad}} = P_{\text{av}} &= \oiint_S \mathbf{W}_{\text{rad}} \cdot d\mathbf{s} = \oiint_S \mathbf{W}_{\text{av}} \cdot \hat{\mathbf{n}}\, da \\ &= \frac{1}{2} \oiint_S \text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} \end{aligned}} \tag{2-9}$$

The power pattern of the antenna, whose definition was discussed in Section 2.2, is just a measure, as a function of direction, of the average power density radiated by the antenna. The observations are usually made on a large sphere of constant radius extending into the far field. In practice, absolute power patterns are usually not desired. However, the performance of the antenna is measured in terms of the gain (to be discussed in a subsequent section) and in terms of relative power patterns. Three-dimensional patterns cannot be measured, but they can be constructed with a number of two-dimensional cuts.

Example 2.2

The radial component of the radiated power density of an antenna is given by

$$\mathbf{W}_{\text{rad}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the total radiated power.

Solution: For a closed surface, a sphere of radius r is chosen. To find the total radiated power, the radial component of the power density is integrated over its surface. Thus

$$P_{\text{rad}} = \oiint_S \mathbf{W}_{\text{rad}} \cdot \hat{\mathbf{n}}\, da$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2}\right) \cdot (\hat{\mathbf{a}}_r r^2 \sin\theta\, d\theta\, d\phi) = \pi^2 A_0 \quad (\text{W})$$

A three-dimensional normalized plot of the average power density at a distance of $r = 1$ m is shown in Figure 2.6.

An isotropic radiator is an ideal source that radiates equally in all directions. Although it does not exist in practice, it provides a convenient isotropic reference with which to compare other antennas. Because of its symmetric radiation, its Poynting vector will not be a function of the spherical coordinate angles θ and ϕ. In addition, it will have only a radial component. Thus the total power radiated by it is given by

$$P_{\text{rad}} = \oiint_S \mathbf{W}_0 \cdot d\mathbf{s} = \int_0^{2\pi}\int_0^{\pi} [\hat{\mathbf{a}}_r W_0(r)] \cdot [\hat{\mathbf{a}}_r r^2 \sin\theta\, d\theta\, d\phi] = 4\pi r^2 W_0 \qquad (2\text{-}10)$$

and the power density by

$$\mathbf{W}_0 = \hat{\mathbf{a}}_r W_0 = \hat{\mathbf{a}}_r \left(\frac{P_{\text{rad}}}{4\pi r^2}\right) \quad (\text{W/m}^2) \qquad (2\text{-}11)$$

which is uniformly distributed over the surface of a sphere of radius r.

2.4 RADIATION INTENSITY

Radiation intensity in a given direction is defined as “the power radiated from an antenna per unit solid angle.” The radiation intensity is a far-field parameter, and it can be obtained by simply multiplying the radiation density by the square of the distance. In mathematical form it is expressed as

$$\boxed{U = r^2 W_{\text{rad}}} \qquad (2\text{-}12)$$

where

$$U = \text{radiation intensity} \quad \text{(W/unit solid angle)}$$
$$W_{\text{rad}} = \text{radiation density} \quad (\text{W/m}^2)$$

The radiation intensity is also related to the far-zone electric field of an antenna, referring to Figure 2.4, by

$$\begin{aligned} U(\theta, \phi) = \frac{r^2}{2\eta}|\mathbf{E}(r, \theta, \phi)|^2 &\simeq \frac{r^2}{2\eta}\left[|E_\theta(r, \theta, \phi)|^2 + |E_\phi(r, \theta, \phi)|^2\right] \\ &\simeq \frac{1}{2\eta}\left[|E_\theta^\circ(\theta, \phi)|^2 + |E_\phi^\circ(\theta, \phi)|^2\right] \end{aligned} \tag{2-12a}$$

where

$$\mathbf{E}(r, \theta, \phi) = \text{far-zone electric-field intensity of the antenna} = \mathbf{E}^\circ(\theta, \phi)\frac{e^{-jkr}}{r}$$
$$E_\theta, E_\phi = \text{far-zone electric-field components of the antenna}$$
$$\eta = \text{intrinsic impedance of the medium}$$

The radial electric-field component (E_r) is assumed, if present, to be small in the far zone. Thus the power pattern is also a measure of the radiation intensity.

The total power is obtained by integrating the radiation intensity, as given by (2-12), over the entire solid angle of 4π. Thus

$$P_{\text{rad}} = \oint\!\!\oint_\Omega U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \tag{2-13}$$

where $d\Omega = \text{element of solid angle} = \sin\theta\, d\theta\, d\phi$.

Example 2.3

For the problem of Example 2.2, find the total radiated power using (2-13).

Solution: Using (2-12)

$$U = r^2 W_{\text{rad}} = A_0 \sin\theta$$

and by (2-13)

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi = A_0\int_0^{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\, d\phi = \pi^2 A_0$$

which is the same as that obtained in Example 2.2. A three-dimensional plot of the relative radiation intensity is also represented by Figure 2.6.

For an isotropic source U will be independent of the angles θ and ϕ, as was the case for W_{rad}. Thus (2-13) can be written as

$$P_{\text{rad}} = \oint\!\!\oint_\Omega U_0\, d\Omega = U_0 \oint\!\!\oint_\Omega d\Omega = 4\pi U_0 \tag{2-14}$$

or the radiation intensity of an isotropic source as

$$U_0 = \frac{P_{\text{rad}}}{4\pi} \tag{2-15}$$

2.5 BEAMWIDTH

Associated with the pattern of an antenna is a parameter designated as *beamwidth*. The *beamwidth* of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum. In an antenna pattern, there are a number of beamwidths. One of the most widely used beamwidths is the *Half-Power Beamwidth* (*HPBW*), which is defined by IEEE as: "In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam." This is demonstrated in Figure 2.2. Another important beamwidth is the angular separation between the first nulls of the pattern, and it is referred to as the *First-Null Beamwidth* (*FNBW*). Both the *HPBW* and *FNBW* are demonstrated for the pattern in Figure 2.11 for the pattern of Example 2.4. Other beamwidths are those where the pattern is −10 dB from the maximum, or any other value. However, in practice, the term *beamwidth*, with no other identification, usually refers to *HPBW*.

The beamwidth of an antenna is a very important figure of merit and often is used as a trade-off between it and the side lobe level; that is, as the beamwidth decreases, the side lobe increases and vice versa. In addition, the beamwidth of the antenna is also

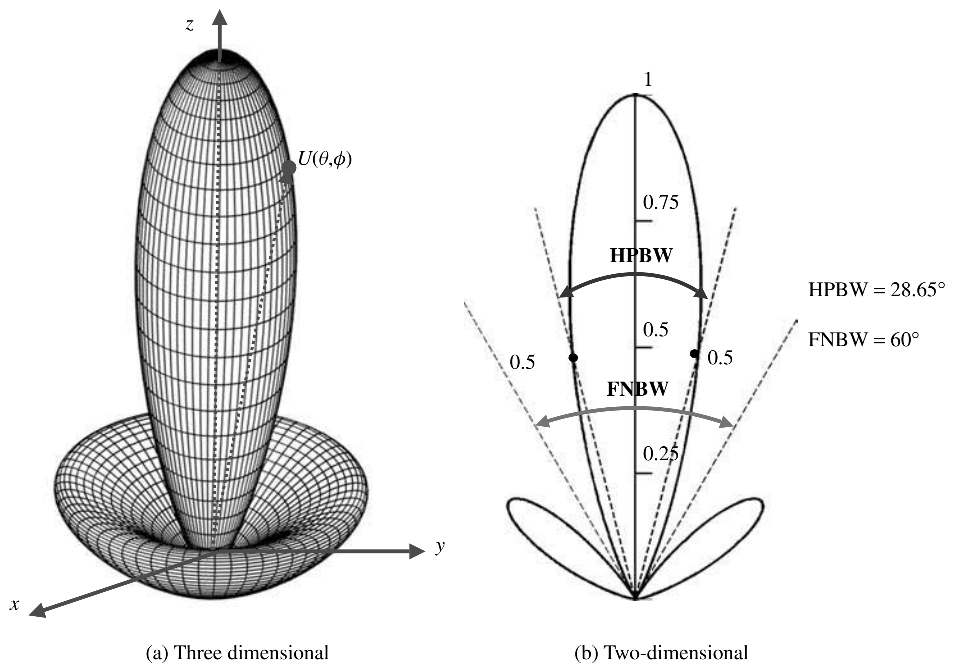


Figure 2.11 Three- and two-dimensional power patterns (in linear scale) of $U(\theta) = \cos^2(\theta)\cos^2(3\theta)$.

used to describe the resolution capabilities of the antenna to distinguish between two adjacent radiating sources or radar targets. The most common resolution criterion states that *the resolution capability of an antenna to distinguish between two sources is equal to half the first-null beamwidth (FNBW/2), which is usually used to approximate the half-power beamwidth (HPBW)* [5], [6]. That is, two sources separated by angular distances equal or greater than FNBW/2 $\approx$ HPBW of an antenna with a uniform distribution can be resolved. If the separation is smaller, then the antenna will tend to smooth the angular separation distance.

Example 2.4

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$$

The three- and two-dimensional plots of this, plotted in a linear scale, are shown in Figure 2.11. Find the

a. half-power beamwidth HPBW (*in radians and degrees*)
b. first-null beamwidth FNBW (*in radians and degrees*)

Solution:

a. Since the $U(\theta)$ represents the *power* pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos\theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25 \; radians = 14.325^\circ$$

Since the function $U(\theta)$ is symmetrical about the maximum at $\theta = 0$, then the HPBW is

$$\text{HPBW} = 2\theta_h \approx 0.50 \; radians = 28.65^\circ$$

b. To find the first-null beamwidth (FNBW), you set the $U(\theta)$ equal to zero, or

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This leads to two solutions for θ_n.

$$\cos\theta_n = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2} \ radians = 90^\circ$$

$$\cos 3\theta_n = 0 \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = \frac{\pi}{6} \ radians = 30^\circ$$

The one with the smallest value leads to the FNBW. Again, because of the symmetry of the pattern, the FNBW is

$$\text{FNBW} = 2\theta_n = \frac{\pi}{3} \ radians = 60^\circ$$

2.6 DIRECTIVITY

In the 1983 version of the *IEEE Standard Definitions of Terms for Antennas*, there has been a substantive change in the definition of *directivity*, compared to the definition of the 1973 version. Basically the term *directivity* in the new 1983 version has been used to replace the term *directive gain* of the old 1973 version. In the new 1983 version the term *directive gain* has been deprecated. According to the authors of the new 1983 standards, "this change brings this standard in line with common usage among antenna engineers and with other international standards, notably those of the International Electrotechnical Commission (IEC)." Therefore *directivity of an antenna* defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied." Stated more simply, the directivity of a nonisotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source. In mathematical form, using (2-15), it can be written as

$$\boxed{D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}} \tag{2-16}$$

If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as

$$\boxed{D_{\max} = D_0 = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{\text{rad}}}} \tag{2-16a}$$

D = directivity (dimensionless)
D_0 = maximum directivity (dimensionless)

U = radiation intensity (W/unit solid angle)
U_{max} = maximum radiation intensity (W/unit solid angle)
U_0 = radiation intensity of isotropic source (W/unit solid angle)
P_{rad} = total radiated power (W)

For an isotropic source, it is very obvious from (2-16) or (2-16a) that the directivity is unity since U, U_{max}, and U_0 are all equal to each other.

For antennas with orthogonal polarization components, we define the *partial directivity of an antenna for a given polarization in a given direction* as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions." With this definition for the partial directivity, then in a given direction "the total directivity is the sum of the partial directivities for any two orthogonal polarizations." For a spherical coordinate system, the total maximum directivity D_0 for the orthogonal θ and ϕ components of an antenna can be written as

$$D_0 = D_\theta + D_\phi \tag{2-17}$$

while the partial directivities D_θ and D_ϕ are expressed as

$$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi} \tag{2-17a}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi} \tag{2-17b}$$

where

U_θ = radiation intensity in a given direction contained in θ field component
U_ϕ = radiation intensity in a given direction contained in ϕ field component
$(P_{rad})_\theta$ = radiated power in all directions contained in θ field component
$(P_{rad})_\phi$ = radiated power in all directions contained in ϕ field component

Example 2.5

As an illustration, find the maximum directivity of the antenna whose radiation intensity is that of Example 2.2. Write an expression for the directivity as a function of the directional angles θ and ϕ.

Solution: The radiation intensity is given by

$$U = r^2 W_{rad} = A_0 \sin\theta$$

The maximum radiation is directed along $\theta = \pi/2$. Thus

$$U_{max} = A_0$$

In Example 2.2 it was found that

$$P_{rad} = \pi^2 A_0$$

Using (2-16a), we find that the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4}{\pi} = 1.27$$

Since the radiation intensity is only a function of θ, the directivity as a function of the directional angles is represented by

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Before proceeding with a more general discussion of directivity, it may be proper at this time to consider another example, compute its directivity, compare it with that of the previous example, and comment on what it actually represents. This may give the reader a better understanding and appreciation of the directivity.

Example 2.6

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

Solution: The radiation intensity is given by

$$U = r^2 W_r = A_0 \sin^2\theta$$

The maximum radiation is directed along $\theta = \pi/2$. Thus

$$U_{max} = A_0$$

The total radiated power is given by

$$P_{rad} = \oiint_{\Omega} U\, d\Omega = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \sin\theta\, d\theta\, d\phi = A_0\left(\frac{8\pi}{3}\right)$$

Using (2-16a), we find that the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{\frac{8\pi}{3}(A_0)} = \frac{3}{2}$$

which is greater than 1.27 found in Example 2.5. Thus the directivity is represented by

$$D = D_0 \sin^2\theta = 1.5 \sin^2\theta$$

At this time it will be proper to comment on the results of Examples 2.5 and 2.6. To better understand the discussion, we have plotted in Figure 2.12 the relative radiation intensities of Example 2.5 ($U = A_0 \sin\theta$) and Example 2.6 ($U = A_0 \sin^2\theta$) where A_0 was set equal to unity. We see that both patterns are omnidirectional but that of Example 2.6 has more directional characteristics (is narrower) in the elevation plane. Since the directivity is a "figure of merit" describing how well the radiator directs energy in a certain direction, it should be convincing from Figure 2.12 that the directivity of Example 2.6 should be higher than that of Example 2.5.

To demonstrate the significance of directivity, let us consider another example; in particular let us examine the directivity of a half-wavelength dipole ($l = \lambda/2$), which is derived in Section 4.6 of Chapter 4 and can be approximated by

$$D = D_0 \sin^3\theta = 1.67 \sin^3\theta \tag{2-18}$$

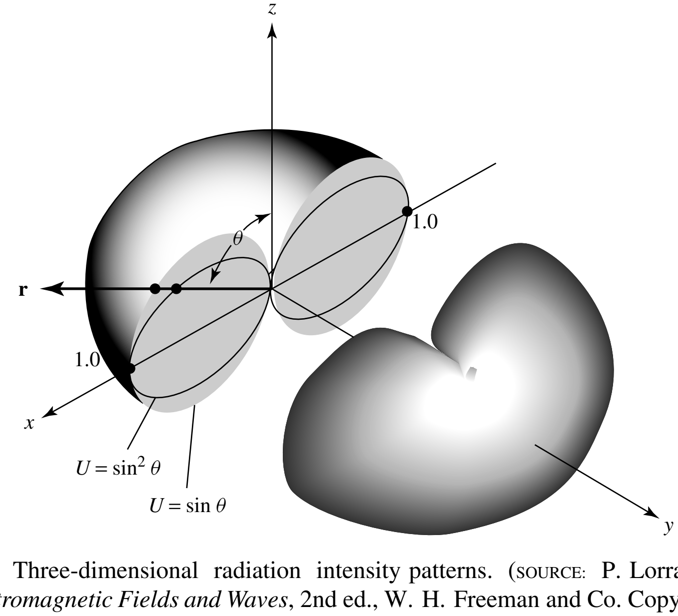


Figure 2.12 Three-dimensional radiation intensity patterns. (SOURCE: P. Lorrain and D. R. Corson, *Electromagnetic Fields and Waves*, 2nd ed., W. H. Freeman and Co. Copyright © 1970).

since it can be shown that

$$\sin^3\theta \simeq \left[\frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \tag{2-18a}$$

where θ is measured from the axis along the length of the dipole. The values represented by (2-18) and those of an isotropic source ($D = 1$) are plotted two- and three-dimensionally in Figure 2.13(a,b). For the three-dimensional graphical representation of Figure 2.13(b), at each observation point only the largest value of the two directivities is plotted. It is apparent that when $\sin^{-1}(1/1.67)^{1/3} = 57.44^\circ < \theta < 122.56^\circ$, the dipole radiator has greater directivity (greater intensity concentration) in those directions than that of an isotropic source. Outside this range of angles, the isotropic radiator has higher directivity (more intense radiation). The maximum directivity of the dipole (relative to the isotropic radiator) occurs when $\theta = \pi/2$, and it is 1.67 (or 2.23 dB) more intense than that of the isotropic radiator (with the same radiated power).

The three-dimensional pattern of Figure 2.13(b), and similar ones, are included throughout the book to represent the three-dimensional radiation characteristics of antennas. These patterns are plotted using software developed in [2] and [3], and can be used to visualize the three-dimensional radiation pattern of the antenna. These three-dimensional programs, along with the others, can be used effectively toward the design and synthesis of antennas, especially arrays, as demonstrated in [7] and [8]. A MATLAB-based program, designated as 3-D ***Spherical***, is also included in the attached CD to produce similar plots.

The directivity of an isotropic source is unity since its power is radiated equally well in all directions. *For all other sources, the maximum directivity will always be greater than unity, and it is a relative "figure of merit" which gives an indication of the directional properties of the antenna as compared with those of an isotropic source.* In equation form, this is indicated in (2-16a). The directivity can be smaller than unity; in fact it can be equal to zero. For Examples 2.5 and 2.6, the directivity is equal to zero in the $\theta = 0$ direction. *The values of directivity will be equal to or greater than zero and equal to or less than the maximum directivity* ($0 \leq D \leq D_0$).

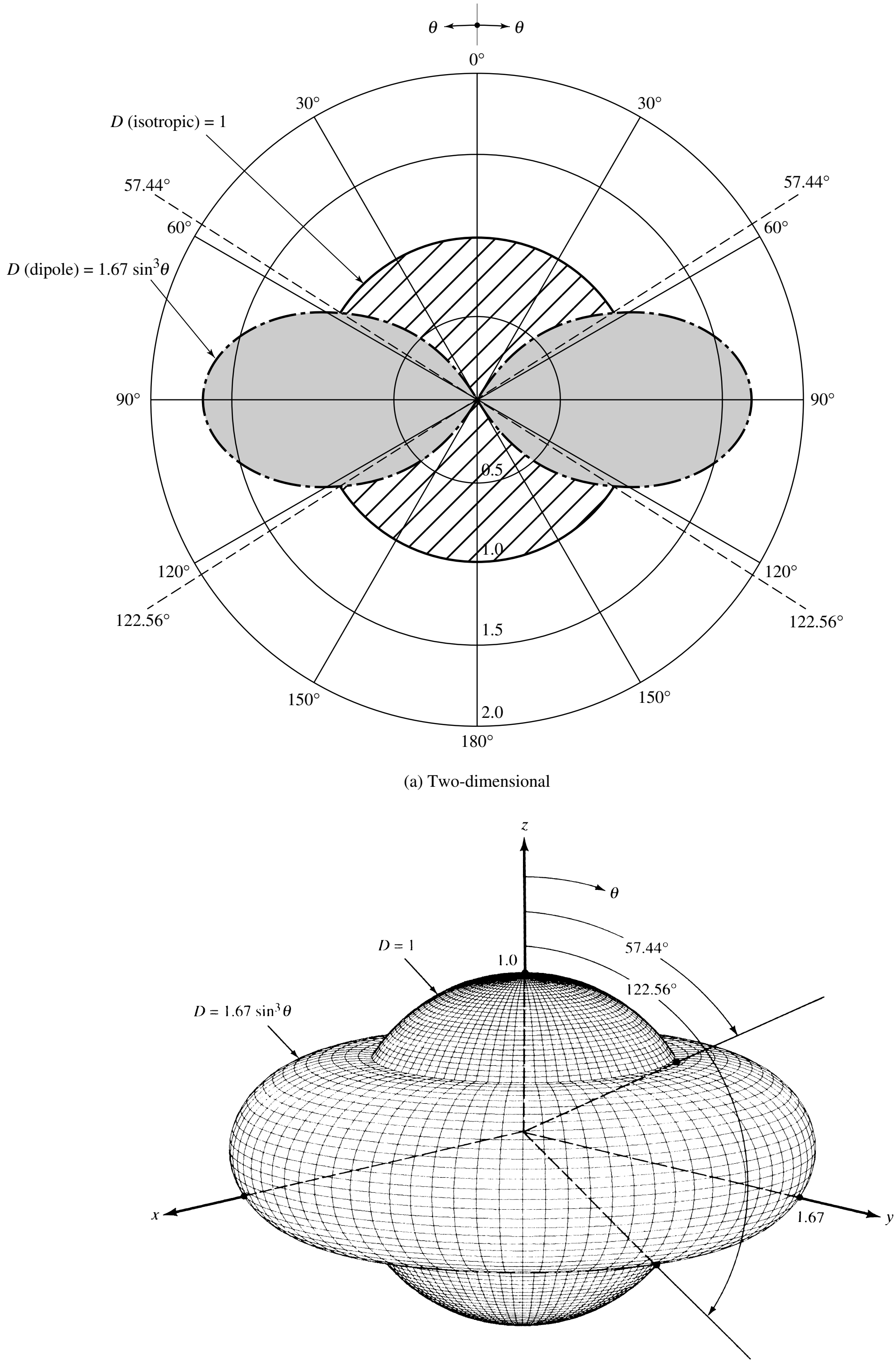
A more general expression for the directivity can be developed to include sources with radiation patterns that may be functions of both spherical coordinate angles θ and ϕ. In the previous examples we considered intensities that were represented by only one coordinate angle θ, in order not to obscure the fundamental concepts by the mathematical details. So it may now be proper, since the basic definitions have been illustrated by simple examples, to formulate the more general expressions.

Let the radiation intensity of an antenna be of the form

$$U = B_0 F(\theta,\phi) \simeq \frac{1}{2\eta}\left[|E_\theta^0(\theta,\phi)|^2 + |E_\phi^0(\theta,\phi)|^2\right] \tag{2-19}$$

where B_0 is a constant, and E_θ^0 and E_ϕ^0 are the antenna's far-zone electric-field components. The maximum value of (2-19) is given by

$$U_{\max} = B_0 F(\theta,\phi)|_{\max} = B_0 F_{\max}(\theta,\phi) \tag{2-19a}$$



(b) Three-dimensional

Figure 2.13 Two- and three-dimensional directivity patterns of a $\lambda/2$ dipole. (SOURCE: C. A. Balanis, "Antenna Theory: A Review." *Proc. IEEE*, Vol. 80, No. 1. January 1992. © 1992 IEEE).

The total radiated power is found using

$$P_{rad} = \oiint_{\Omega} U(\theta,\phi)\, d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi \tag{2-20}$$

We now write the general expression for the directivity and maximum directivity using (2-16) and (2-16a), respectively, as

$$\boxed{D(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\displaystyle\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}} \tag{2-21}$$

$$\boxed{D_0 = 4\pi \frac{F(\theta,\phi)|_{max}}{\displaystyle\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}} \tag{2-22}$$

Equation (2-22) can also be written as

$$\boxed{D_0 = \frac{4\pi}{\left[\displaystyle\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi\right] \Big/ F(\theta,\phi)|_{max}} = \frac{4\pi}{\Omega_A}} \tag{2-23}$$

where Ω_A is the beam solid angle, and it is given by

$$\Omega_A = \frac{1}{F(\theta,\phi)|_{max}}\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\, d\theta\, d\phi \tag{2-24}$$

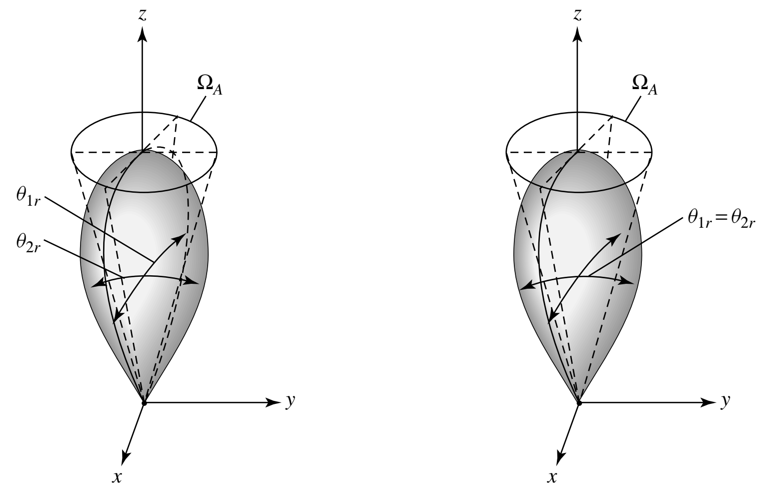
$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)|_{max}} \tag{2-25}$$

Dividing by $F(\theta,\phi)|_{max}$ merely normalizes the radiation intensity $F(\theta,\phi)$, and it makes its maximum value unity.

The beam solid angle Ω_A *is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within* Ω_A.

2.6.1 Directional Patterns

Instead of using the exact expression of (2-23) to compute the directivity, it is often convenient to derive simpler expressions, even if they are approximate, to compute the directivity. These can also be used for design purposes. For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beamwidths in two perpendicular planes [5] shown



(a) Nonsymmetrical pattern (b) Symmetrical pattern

Figure 2.14 Beam solid angles for nonsymmetrical and symmetrical radiation patterns.

in Figure 2.14(a). For a rotationally symmetric pattern, the half-power beamwidths in any two perpendicular planes are the same, as illustrated in Figure 2.14(b).

With this approximation, (2-23) can be approximated by

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}} \tag{2-26}$$

The beam solid angle Ω_A has been approximated by

$$\boxed{\Omega_A \simeq \Theta_{1r}\Theta_{2r}} \tag{2-26a}$$

where

Θ_{1r} = half-power beamwidth in one plane (rad)
Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

If the beamwidths are known in degrees, (2-26) can be written as

$$D_0 \simeq \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}} \tag{2-27}$$

where

Θ_{1d} = half-power beamwidth in one plane (degrees)
Θ_{2d} = half-power beamwidth in a plane at a right angle to the other (degrees)

For planar arrays, a better approximation to (2-27) is [9]

$$\boxed{D_0 \simeq \frac{32{,}400}{\Omega_A(\text{degrees})^2} = \frac{32{,}400}{\Theta_{1d}\Theta_{2d}}} \tag{2-27a}$$

The validity of (2-26) and (2-27) is based on a pattern that has only one major lobe and any minor lobes, if present, should be of very low intensity. For a pattern with two identical major lobes, the value of the maximum directivity using (2-26) or (2-27) will be twice its actual value. For patterns with significant minor lobes, the values of maximum directivity obtained using (2-26) or (2-27), which neglect any minor lobes, will usually be too high.

Example 2.7

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos\theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.
b. maximum directivity; exact using (2-23) and approximate using (2-26).

Solution: The half-power point of the pattern occurs at $\theta = 60^\circ$. Thus the beamwidth in the θ direction is 120° or

$$\Theta_{1r} = \frac{2\pi}{3}$$

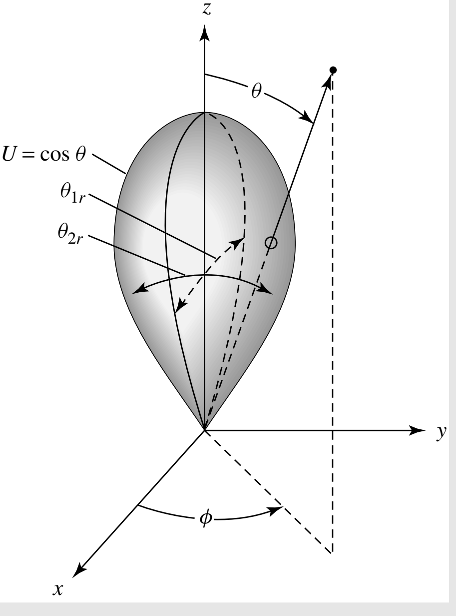


Figure 2.15 Radiation intensity pattern of the form $U = \cos\theta$ in the upper hemisphere.

Since the pattern is independent of the ϕ coordinate, the beamwidth in the other plane is also equal to

$$\Theta_{2r} = \frac{2\pi}{3}$$

a. *Beam solid angle* Ω_A:
Exact: Using (2-24), (2-25)

$$\Omega_A = \int_0^{360^\circ} \int_0^{90^\circ} \cos\theta \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

$$= 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \pi \int_0^{\pi/2} \sin(2\theta) \, d\theta = \pi \text{ steradians}$$

Approximate: Using (2-26a)

$$\Omega_A \approx \Theta_{1r}\Theta_{2r} = \frac{2\pi}{3}\left(\frac{2\pi}{3}\right) = \left(\frac{2\pi}{3}\right)^2 = 4.386 \text{ steradians}$$

b. *Directivity* D_0:
Exact: $D_0 = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{\pi} = 4$ (dimensionless) $= 6.02$ dB
The same exact answer is obtained using (2-16a).
Approximate: $D_0 \approx \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{4.386} = 2.865$ (dimensionless) $= 4.57$ dB

The exact maximum directivity is 4 and its approximate value, using (2-26), is 2.865. Better approximations can be obtained if the patterns have much narrower beamwidths, which will be demonstrated later in this section.

Many times it is desirable to express the directivity in decibels (dB) instead of dimensionless quantities. The expressions for converting the dimensionless quantities of directivity and maximum directivity to decibels (dB) are

$$D(\text{dB}) = 10\log_{10}[D(\text{dimensionless})] \tag{2-28a}$$

$$D_0(\text{dB}) = 10\log_{10}[D_0(\text{dimensionless})] \tag{2-28b}$$

It has also been proposed [10] that the maximum directivity of an antenna can also be obtained approximately by using the formula

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1} + \frac{1}{D_2}\right) \tag{2-29}$$

where

$$D_1 \simeq \frac{1}{\left[\dfrac{1}{2\ln 2}\displaystyle\int_0^{\Theta_{1r}/2} \sin\theta \, d\theta\right]} \simeq \frac{16\ln 2}{\Theta_{1r}^2} \tag{2-29a}$$

$$D_2 \simeq \frac{1}{\left[\dfrac{1}{2\ln 2}\displaystyle\int_0^{\Theta_{2r}/2} \sin\theta \, d\theta\right]} \simeq \frac{16\ln 2}{\Theta_{2r}^2} \tag{2-29b}$$

Θ_{1r} and Θ_{2r} are the half-power beamwidths (in radians) of the E- and H-planes, respectively. The formula of (2-29) will be referred to as the arithmetic mean of the maximum directivity. Using (2-29a) and (2-29b) we can write (2-29) as

$$\frac{1}{D_0} \simeq \frac{1}{2\ln 2}\left(\frac{\Theta_{1r}^2}{16} + \frac{\Theta_{2r}^2}{16}\right) = \frac{\Theta_{1r}^2 + \Theta_{2r}^2}{32\ln 2} \tag{2-30}$$

or

$$\boxed{D_0 \simeq \frac{32\ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2}} \tag{2-30a}$$

$$D_0 \simeq \frac{22.181(180/\pi)^2}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2} \tag{2-30b}$$

where Θ_{1d} and Θ_{2d} are the half-power beamwidths in degrees. Equation (2-30a) is to be contrasted with (2-26) while (2-30b) should be compared with (2-27).

In order to make an evaluation and comparison of the accuracies of (2-26) and (2-30a), examples whose radiation intensities (power patterns) can be represented by

$$U(\theta, \phi) = \begin{cases} B_0 \cos^n(\theta) & 0 \le \theta \le \pi/2, \quad 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases} \tag{2-31}$$

where $n = 1 - 10$, 11.28, 15, and 20 are considered. The maximum directivities were computed using (2-26) and (2-30a) and compared with the exact values as obtained using (2-23). The results are shown in Table 2.1. From the comparisons it is evident that the error due to Tai & Pereira's formula is always negative (i.e., it predicts lower values of maximum directivity than the exact ones) and monotonically decreases as n increases (the pattern becomes more narrow). However, the error due to Kraus' formula is negative for small values of n and positive for large values of n. For small values of n the error due to Kraus' formula is negative and positive for large values of n; the error is zero when $n = 5.497 \simeq 5.5$ (half-power beamwidth of 56.35°). In addition, for symmetrically rotational patterns the absolute error due to the two approximate formulas is identical when $n = 11.28$, which corresponds to a half-power beamwidth of 39.77°. From these observations we conclude that, Kraus' formula is more accurate for small values of n (broader patterns) while Tai & Pereira's is more accurate for large values of n (narrower patterns). Based on absolute error and symmetrically rotational

TABLE 2.1 Comparison of Exact and Approximate Values of Maximum Directivity for $U = \cos^n \theta$ Power Patterns

n	Exact Equation (2-22)	Kraus Equation (2-26)	Kraus % Error	Tai and Pereira Equation (2-30a)	Tai and Pereira % Error
1	4	2.86	−28.50	2.53	−36.75
2	6	5.09	−15.27	4.49	−25.17
3	8	7.35	−8.12	6.48	−19.00
4	10	9.61	−3.90	8.48	−15.20
5	12	11.87	−1.08	10.47	−12.75
6	14	14.13	+0.93	12.46	−11.00
7	16	16.39	+2.48	14.47	−9.56
8	18	18.66	+3.68	16.47	−8.50
9	20	20.93	+4.64	18.47	−7.65
10	22	23.19	+5.41	20.47	−6.96
11.28	24.56	26.08	+6.24	23.02	−6.24
15	32	34.52	+7.88	30.46	−4.81
20	42	45.89	+9.26	40.46	−3.67

patterns, Kraus' formula leads to smaller error for $n < 11.28$ (half-power beamwidth greater than 39.77°) while Tai & Pereira's leads to smaller error for $n > 11.28$ (half-power beamwidth smaller than 39.77°). The results are shown plotted in Figure 2.16 for $0 < n \leq 450$.

2.6.2 Omnidirectional Patterns

Some antennas (such as dipoles, loops, broadside arrays) exhibit omnidirectional patterns, as illustrated by the three-dimensional patterns in Figure 2.17 (a,b). As single-lobe directional patterns can be approximated by (2-31), omnidirectional patterns can often be approximated by

$$U = |\sin^n(\theta)| \qquad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi \tag{2-32}$$

where n represents both integer and noninteger values. The directivity of antennas with patterns represented by (2-32) can be determined in closed from using the definition of (2-16a). However, as was done for the single-lobe patterns of Figure 2.14, approximate directivity formulas have been derived [11], [12] for antennas with omnidirectional patterns similar to the ones shown in Figure 2.17 whose main lobe is approximated by (2-32). The approximate directivity formula for an omnidirectional pattern as a function of the pattern half-power beamwidth (in degrees), which is reported by McDonald in [11], was derived based on the array factor of a broadside collinear array [see Section 6.4.1 and (6-38a)] and is given by

$$D_0 \simeq \frac{101}{\text{HPBW (degrees)} - 0.0027\,[\text{HPBW (degrees)}]^2} \tag{2-33a}$$

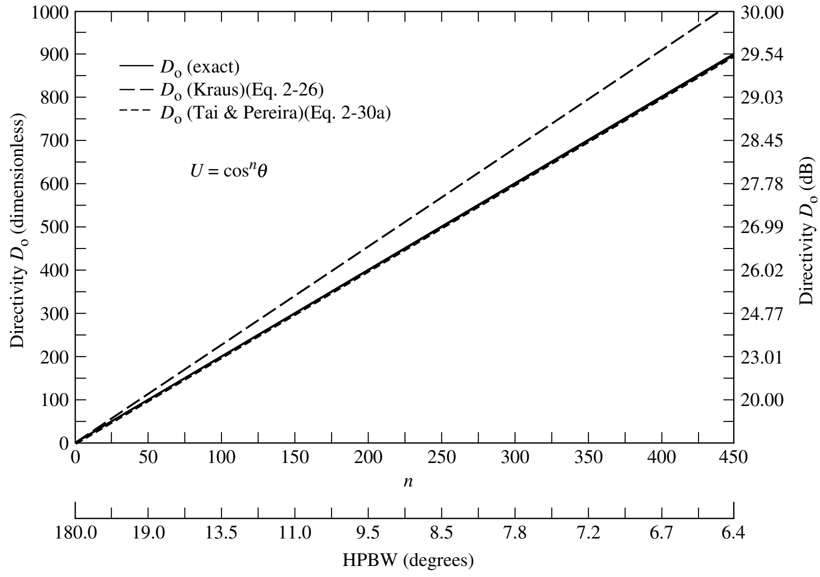


Figure 2.16 Comparison of exact and approximate values of directivity for directional $U = \cos^n \theta$ power patterns.

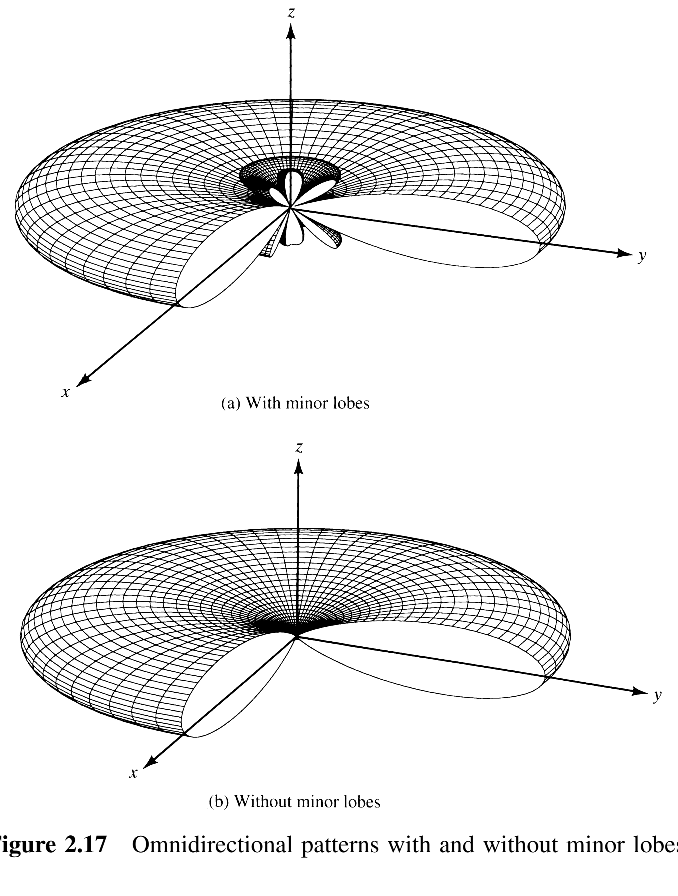


Figure 2.17 Omnidirectional patterns with and without minor lobes.

However, that reported by Pozar in [12] is derived based on the exact values obtained using (2-32) and then representing the data in closed-form using curve-fitting, and it is given by

$$D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/\text{HPBW (degrees)}} \tag{2-33b}$$

The approximate formula of (2-33a) should, in general, be more accurate for omnidirectional patterns with minor lobes, as shown in Figure 2.17(a), while (2-33b) should be more accurate for omnidirectional patterns with minor lobes of very low intensity (ideally no minor lobes), as shown in Figure 2.17(b).

The approximate formulas of (2-33a) and (2-33b) can be used to design omnidirectional antennas with specified radiation pattern characteristics. To facilitate this procedure, the directivity of antennas with omnidirectional patterns approximated by (2-32) is plotted in Figure 2.18 versus n and the half-power beamwidth (in degrees). Three curves are plotted in Figure 2.18; one using (2-16a) and referred as *exact*, one using (2-33a) and denoted as *McDonald*, and the third using (2-33b) and denoted as *Pozar*. Thus, the curves of Figure 2.18 can be used for design purposes, as follows:

a. Specify the desired directivity and determine the value of n and half-power beamwidth of the omnidirectional antenna pattern, or
b. Specify the desired value of n or half-power beamwidth and determine the directivity of the omnidirectional antenna pattern.

To demonstrate the procedure, an example is taken.

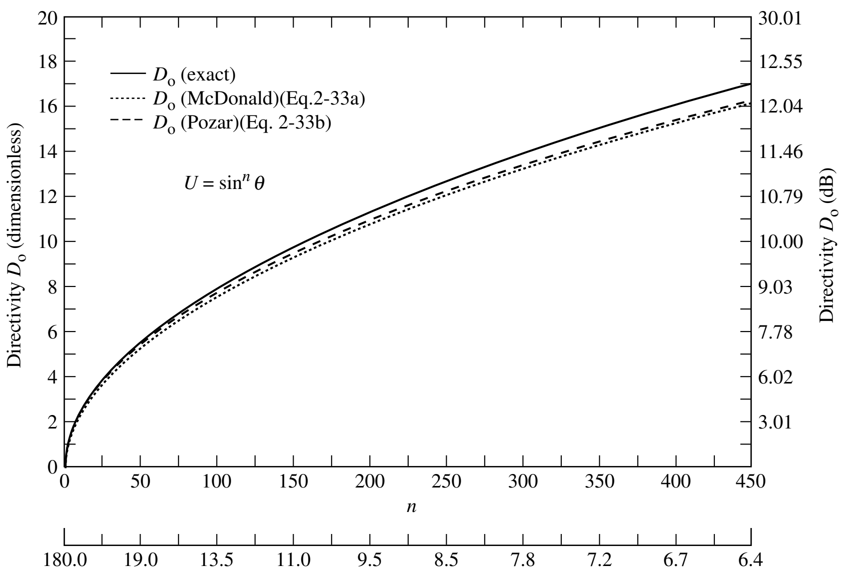


Figure 2.18 Comparison of exact and approximate values of directivity for omnidirectional $U = \sin^n \theta$ power patterns.

Example 2.8

Design an antenna with omnidirectional amplitude pattern with a half-power beamwidth of 90°. Express its radiation intensity by $U = \sin^n \theta$. Determine the value of n and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna using (2-16a), (2-33a), and (2-33b).

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45^\circ) = 0.5 = \sin^n(45^\circ) = (0.707)^n$$

or

$$n = 2$$

Therefore, the radiation intensity of the omnidirectional antenna is represented by $U = \sin^2 \theta$. An infinitesimal dipole (see Chapter 4) or a small circular loop (see Chapter 5) are two antennas which possess such a pattern.

Using the definition of (2-16a), the exact directivity is

$$U_{max} = 1$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \, \sin \theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

Since the half-power beamwidth is equal to 90°, then the directivity based on (2-33a) is equal to

$$D_0 = \frac{101}{90 - 0.0027(90)^2} = 1.4825 = 1.71 \text{ dB}$$

while that based on (2-33b) is equal to

$$D_0 = -172.4 + 191\sqrt{0.818 + 1/90} = 1.516 = 1.807 \text{ dB}$$

The value of n and the three values of the directivity can also be obtained using Figure 2.18, although they may not be as accurate as those given above because they have to be taken off the graph. However, the curves can be used for other problems.

2.7 NUMERICAL TECHNIQUES

For most practical antennas, their radiation patterns are so complex that closed-form mathematical expressions are not available. Even in those cases where expressions are available, their form is so complex that integration to find the radiated power, required to compute the maximum directivity, cannot be performed. Instead of using the approximate expressions of Kraus, Tai and Pereira, McDonald, or Pozar alternate and more accurate techniques may be desirable. With the high-speed computer systems now available, the answer may be to apply numerical methods.

Let us assume that the radiation intensity of a given antenna is separable, and it is given by

$$U = B_0 f(\theta) g(\phi) \tag{2-34}$$

where B_0 is a constant. The directivity for such a system is given by

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} \tag{2-35}$$

where

$$P_{\text{rad}} = B_0 \int_0^{2\pi} \left\{ \int_0^{\pi} f(\theta) g(\phi) \sin\theta \, d\theta \right\} d\phi \tag{2-36}$$

which can also be written as

$$P_{\text{rad}} = B_0 \int_0^{2\pi} g(\phi) \left\{ \int_0^{\pi} f(\theta) \sin\theta \, d\theta \right\} d\phi \tag{2-37}$$

If the integrations in (2-37) cannot be performed analytically, then from integral calculus we can write a series approximation

$$\int_0^{\pi} f(\theta) \sin\theta \, d\theta = \sum_{i=1}^{N} [f(\theta_i) \sin\theta_i] \Delta\theta_i \tag{2-38}$$

For N uniform divisions over the π interval,

$$\Delta\theta_i = \frac{\pi}{N} \tag{2-38a}$$

Referring to Figure 2.19, θ_i can take many different forms. Two schemes are shown in Figure 2.19 such that

$$\theta_i = i\left(\frac{\pi}{N}\right), \quad i = 1, 2, 3, \ldots, N \tag{2-38b}$$

or

$$\theta_i = \frac{\pi}{2N} + (i-1)\frac{\pi}{N}, \quad i = 1, 2, 3, \ldots, N \tag{2-38c}$$

In the former case, θ_i is taken at the trailing edge of each division; in the latter case, θ_i is selected at the middle of each division. The scheme that is more desirable will depend upon the problem under investigation. Many other schemes are available.

In a similar manner, we can write for the ϕ variations that

$$\int_0^{2\pi} g(\phi) \, d\phi = \sum_{j=1}^{M} g(\phi_j) \Delta\phi_j \tag{2-39}$$

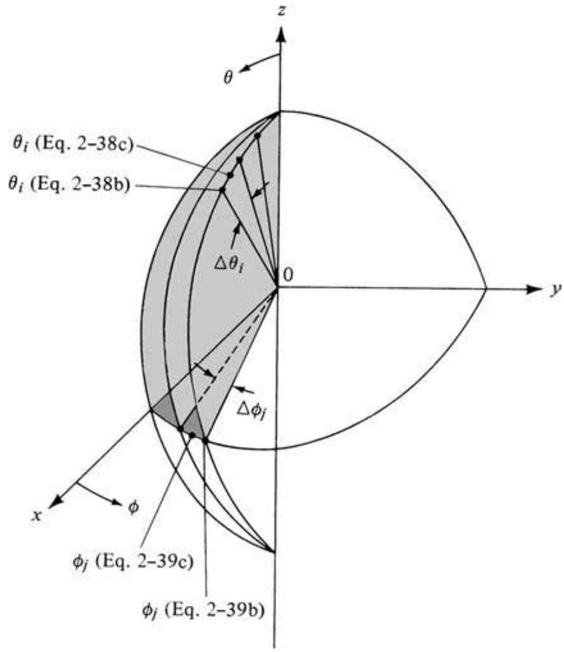


Figure 2.19 Digitization scheme of pattern in spherical coordinates.

where for M uniform divisions

$$\Delta\phi_j = \frac{2\pi}{M} \tag{2-39a}$$

Again referring to Figure 2.19

$$\phi_j = j\left(\frac{2\pi}{M}\right), \quad j = 1, 2, 3, \ldots, M \tag{2-39b}$$

or

$$\phi_j = \frac{2\pi}{2M} + (j-1)\frac{2\pi}{M}, \quad j = 1, 2, 3, \ldots, M \tag{2-39c}$$

Combining (2-38), (2-38a), (2-39), and (2-39a) we can write (2-37) as

$$P_{\text{rad}} = B_0\left(\frac{\pi}{N}\right)\left(\frac{2\pi}{M}\right)\sum_{j=1}^{M}\left\{g(\phi_j)\left[\sum_{i=1}^{N} f(\theta_i)\sin\theta_i\right]\right\} \tag{2-40}$$

The double summation of (2-40) is performed by adding for each value of $j\,(j = 1, 2, 3, \ldots, M)$ all values of $i\,(i = 1, 2, 3, \ldots, N)$. In a computer program flowchart, this can be performed by a loop within a loop. Physically, (2-40) can be interpreted by referring to Figure 2.19. It simply states that for each value of $g(\phi)$ at the azimuthal angle $\phi = \phi_j$, the values of $f(\theta)\sin\theta$ are added for all values of $\theta = \theta_i\,(i = 1, 2, 3, \ldots, N)$. The values of θ_i and ϕ_j can be determined by using either of the forms as given by (2-38b) or (2-38c) and (2-39b) or (2-39c).

Since the θ and ϕ variations are separable, (2-40) can also be written as

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{N}\right)\left(\frac{2\pi}{M}\right)\left[\sum_{j=1}^{M} g(\phi_j)\right]\left[\sum_{i=1}^{N} f(\theta_i)\sin\theta_i\right] \tag{2-41}$$

in which case each summation can be performed separately.

If the θ and ϕ variations are not separable, and the radiation intensity is given by

$$U = B_0 F(\theta, \phi) \tag{2-42}$$

the digital form of the radiated power can be written as

$$\boxed{P_{\text{rad}} = B_0 \left(\frac{\pi}{N}\right)\left(\frac{2\pi}{M}\right)\sum_{j=1}^{M}\left[\sum_{i=1}^{N} F(\theta_i, \phi_j)\sin\theta_i\right]} \tag{2-43}$$

θ_i and ϕ_j take different forms, two of which were introduced and are shown pictorially in Figure 2.19. The evaluation and physical interpretation of (2-43) is similar to that of (2-40).

To examine the accuracy of the technique, two examples will be considered.

Example 2.9(a)

The radiation intensity of an antenna is given by

$$U(\theta, \phi) = \begin{cases} B_0 \sin\theta \sin^2\phi, & 0 \le \theta \le \pi, \quad 0 \le \phi \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

The three-dimensional pattern of $U(\theta, \phi)$ is shown in Figure 2.20.

Determine the maximum directivity numerically by using (2-41) with θ_i and ϕ_j of (2-38b) and (2-39b), respectively. Compare it with the exact value.

Solution: Let us divide the θ and ϕ intervals each into 18 equals segments ($N = M = 18$). Since $0 \le \phi \le \pi$, then $\Delta\phi_j = \pi/M$ and (2-41) reduces to

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 \left[\sum_{j=1}^{18} \sin^2\phi_j\right]\left[\sum_{i=1}^{18} \sin^2\theta_i\right]$$

with

$$\theta_i = i\left(\frac{\pi}{18}\right) = i(10^\circ), \quad i = 1, 2, 3, \ldots, 18$$

$$\phi_j = j\left(\frac{\pi}{18}\right) = j(10^\circ), \quad j = 1, 2, 3, \ldots, 18$$

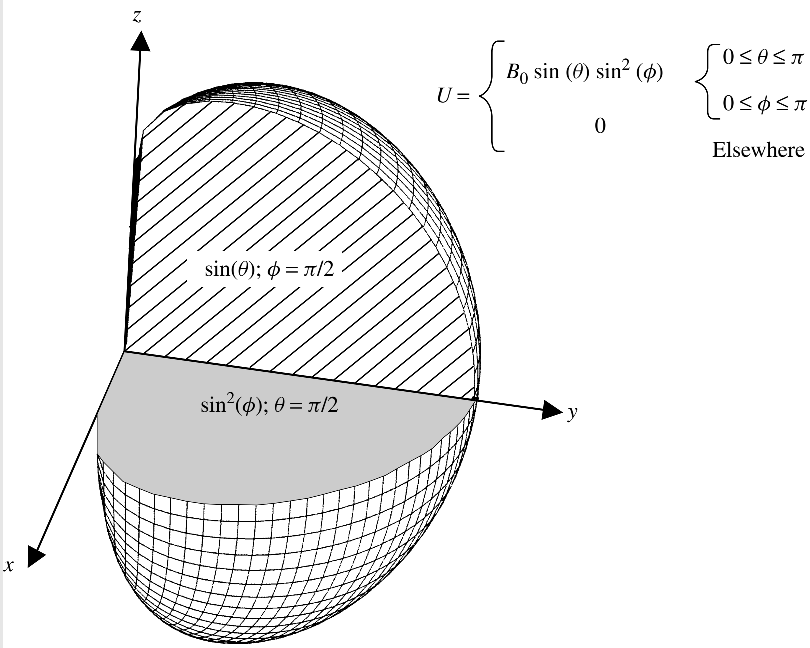


Figure 2.20 Three-dimensional pattern of the radiation of Examples 2.9(a,b).

Thus

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 [\sin^2(10^\circ) + \sin^2(20^\circ) + \cdots + \sin^2(180^\circ)]^2$$

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 (9)^2 = B_0 \left(\frac{\pi^2}{4}\right)$$

and

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.0929$$

The exact value is given by

$$P_{\text{rad}} = B_0 \int_0^{\pi} \sin^2\phi\, d\phi \int_0^{\pi} \sin^2\theta\, d\theta = \frac{\pi}{2}\left(\frac{\pi}{2}\right) B_0 = \frac{\pi^2}{4} B_0$$

and

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.0929$$

which is the same as the value obtained numerically!

Example 2.9(b)

Given the same radiation intensity as that in Example 2.9(a), determine the directivity using (2-41) with θ_i and ϕ_j of (2-38c) and (2-39c).

Solution: Again using 18 divisions in each interval, we can write (2-41) as

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 \left[\sum_{j=1}^{18} \sin^2 \phi_j\right] \left[\sum_{i=1}^{18} \sin^2 \theta_i\right]$$

with

$$\theta_i = \frac{\pi}{36} + (i-1)\frac{\pi}{18} = 5^\circ + (i-1)10^\circ, \quad i = 1, 2, 3, \ldots, 18$$

$$\phi_j = \frac{\pi}{36} + (j-1)\frac{\pi}{18} = 5^\circ + (j-1)10^\circ, \quad j = 1, 2, 3, \ldots, 18$$

Because of the symmetry of the divisions about the $\theta = \pi/2$ and $\phi = \pi/2$ angles, we can write

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 \left[2\sum_{j=1}^{9} \sin^2 \phi_j\right] \left[2\sum_{i=1}^{9} \sin^2 \theta_i\right]$$

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 4[\sin^2(5^\circ) + \sin^2(15^\circ) + \cdots + \sin^2(85^\circ)]^2$$

$$P_{\text{rad}} = B_0 \left(\frac{\pi}{18}\right)^2 4(4.5)^2 = B_0 \left(\frac{\pi}{18}\right)^2 (81) = B_0 \left(\frac{\pi^2}{4}\right)$$

which is identical to that of the previous example. Thus

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.0929$$

which again is equal to the exact value!

It is interesting to note that decreasing the number of divisions (M and/or N) to 9, 6, 4, and even 2 leads to the same answer, which also happens to be the exact value! To demonstrate as to why the number of divisions does not affect the answer for this pattern, let us refer to Figure 2.21 where we have plotted the $\sin^2 \phi$ function and divided the $0^\circ \leq \phi \leq 180^\circ$ interval into six divisions. The exact value of the directivity uses the area under the solid curve. Doing the problem numerically, we find the area under the rectangles, which is shown shaded. Because of the symmetrical nature of the function, it can be shown that the shaded area in section #1 (included in the numerical evaluation) is equal to the blank area in section #1′ (left out by the numerical method). The same is true for the areas in sections #2 and #2′, and #3 and #3′. Thus, there is a one-to-one compensation. Similar justification is applicable for the other number of divisions.

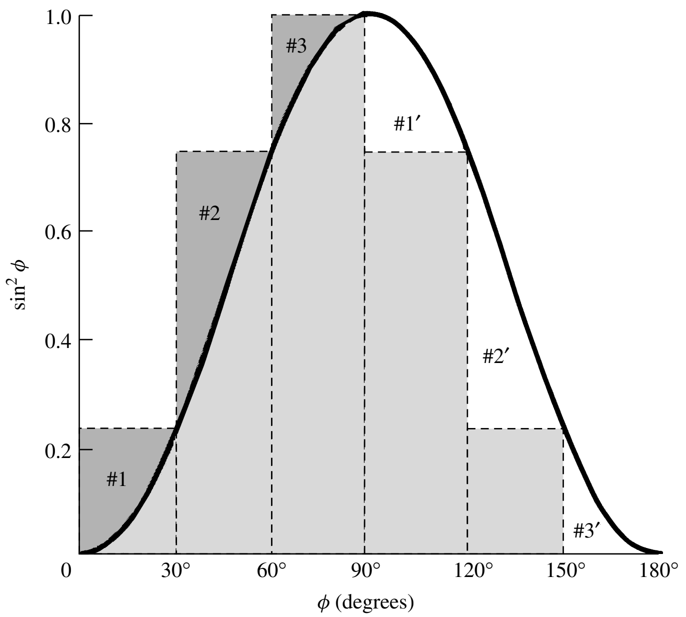


Figure 2.21 Digitized form of $\sin^2 \phi$ function.

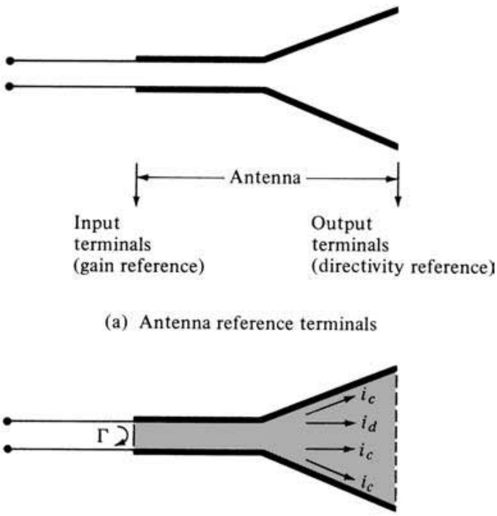
It should be emphasized that all functions, even though they may contain some symmetry, do not give the same answers independent of the number of divisions. As a matter of fact, in most cases the answer only approaches the exact value as the number of divisions is increased to a large number.

A MATLAB and FORTRAN computer program called ***Directivity*** has been developed to compute the maximum directivity of any antenna whose radiation intensity is $U = F(\theta, \phi)$ based on the formulation of (2-43). The intensity function F does not have to be a function of both θ and ϕ. The numerical evaluations are made at the trailing edge, as defined by (2-38b) and (2-39b). The program is included in the attached CD. It contains a *subroutine* for which the intensity factor $U = F(\theta, \phi)$ for the required application must be specified by the user. As an illustration, the antenna intensity $U = \sin\theta \sin^2 \phi$ has been inserted in the subroutine. In addition, the upper and lower limits of θ and ϕ must be specified for each application of the same pattern.

2.8 ANTENNA EFFICIENCY

Associated with an antenna are a number of efficiencies and can be defined using Figure 2.22. The total antenna efficiency e_0 is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due, referring to Figure 2.22(b), to

1. reflections because of the mismatch between the transmission line and the antenna
2. $I^2 R$ losses (conduction and dielectric)



(b) Reflection, conduction, and dielectric losses

Figure 2.22 Reference terminals and losses of an antenna.

In general, the overall efficiency can be written as

$$\boxed{e_0 = e_r e_c e_d} \tag{2-44}$$

where

e_0 = total efficiency (dimensionless)
e_r = reflection (mismatch) efficiency $= (1 - |\Gamma|^2)$ (dimensionless)
e_c = conduction efficiency (dimensionless)
e_d = dielectric efficiency (dimensionless)
Γ = voltage reflection coefficient at the input terminals of the antenna [$\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ where $Z_{in} =$ antenna input impedance, $Z_0 =$ characteristic impedance of the transmission line]
VSWR = voltage standing wave ratio $= \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

Usually e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write (2-44) as

$$e_0 = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2) \tag{2-45}$$

where $e_{cd} = e_c e_d$ = antenna radiation efficiency, which is used to relate the gain and directivity.

2.9 GAIN

Another useful measure describing the performance of an antenna is the *gain*. Although the gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities. Remember that directivity is a measure that describes only the directional properties of the antenna, and it is therefore controlled only by the pattern.

Gain of an antenna (in a given direction) is defined as "the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π." In equation form this can be expressed as

$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta,\phi)}{P_{in}} \quad \text{(dimensionless)} \tag{2-46}$$

In most cases we deal with *relative gain*, which is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction." The power input must be the same for both antennas. The reference antenna is usually a dipole, horn, or any other antenna whose gain can be calculated or it is known. In most cases, however, the reference antenna is a *lossless isotropic source*. Thus

$$G = \frac{4\pi U(\theta,\phi)}{P_{in}\,\text{(lossless isotropic source)}} \quad \text{(dimensionless)} \tag{2-46a}$$

When the direction is not stated, the power gain is usually taken in the direction of maximum radiation.

Referring to Figure 2.22(a), we can write that the total radiated power (P_{rad}) is related to the total input power (P_{in}) by

$$P_{\text{rad}} = e_{cd} P_{in} \tag{2-47}$$

where e_{cd} is the antenna radiation efficiency (dimensionless) which is defined in (2-44), (2-45) and Section 2.14 by (2-90). According to the IEEE Standards, "gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses)."

In this edition of the book we define two gains; one, referred to as *gain* (G), and the other, referred to as *absolute gain* (G_{abs}), that also takes into account the reflection/mismatch losses represented in both (2-44) and (2-45).

Using (2-47) reduces (2-46a) to

$$G(\theta,\phi) = e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{\text{rad}}}\right] \tag{2-48}$$

which is related to the directivity of (2-16) and (2-21) by

$$\boxed{G(\theta,\phi) = e_{cd} D(\theta,\phi)} \tag{2-49}$$

In a similar manner, the maximum value of the gain is related to the maximum directivity of (2-16a) and (2-23) by

$$\boxed{G_0 = G(\theta,\phi)|_{\max} = e_{cd} D(\theta,\phi)|_{\max} = e_{cd} D_0} \tag{2-49a}$$

While (2-47) does take into account the losses of the antenna element itself, *it does not take into account the losses when the antenna element is connected to a transmission line*, as shown in Figure 2.22. These connection losses are usually referred to as *reflections (mismatch) losses*, and they are taken into account by introducing a reflection (mismatch) efficiency e_r, which is related to the reflection coefficient as shown in (2-45) or $e_r = (1 - |\Gamma|^2)$. Thus, we can introduce an *absolute gain* G_{abs} that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line), and it can be written as

$$\boxed{\begin{aligned} G_{abs}(\theta,\phi) &= e_r G(\theta,\phi) = (1-|\Gamma|^2)G(\theta,\phi) \\ &= e_r e_{cd} D(\theta,\phi) = e_o D(\theta,\phi) \end{aligned}} \tag{2-49b}$$

where e_o is the overall efficiency as defined in (2-44), (2-45). Similarly, the *maximum absolute* gain G_{0abs} of (2-49a) is related to the maximum directivity D_0 by

$$\boxed{\begin{aligned} G_{0abs} &= G_{abs}(\theta,\phi)|_{\max} = e_r G(\theta,\phi)|_{\max} = (1-|\Gamma|^2)G(\theta,\phi)|_{\max} \\ &= e_r e_{cd} D(\theta,\phi)|_{\max} = e_o D(\theta,\phi)|_{\max} = e_o D_0 \end{aligned}} \tag{2-49c}$$

If the antenna is matched to the transmission line, that is, the antenna input impedance Z_{in} *is equal to the characteristic impedance* Z_c *of the line* ($|\Gamma| = 0$)*, then the two gains are equal* ($G_{abs} = G$).

As was done with the directivity, we can define the *partial gain of an antenna for a given polarization in a given direction* as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically." With this definition for the partial gain, then, in a given direction, "the total gain is the sum of the partial gains for any two orthogonal polarizations." For a spherical coordinate system, the total maximum gain G_0 for the orthogonal θ and ϕ components of an antenna can be written, in a similar form as was the maximum directivity in (2-17)–(2-17b), as

$$G_0 = G_\theta + G_\phi \tag{2-50}$$

while the partial gains G_θ and G_ϕ are expressed as

$$G_\theta = \frac{4\pi U_\theta}{P_{in}} \tag{2-50a}$$

$$G_\phi = \frac{4\pi U_\phi}{P_{in}} \tag{2-50b}$$

where

U_θ = radiation intensity in a given direction contained in E_θ field component
U_ϕ = radiation intensity in a given direction contained in E_ϕ field component
P_{in} = total input (accepted) power

For many practical antennas an approximate formula for the gain, corresponding to (2-27) or (2-27a) for the directivity, is

$$\boxed{G_0 \simeq \frac{30{,}000}{\Theta_{1d}\Theta_{2d}}} \tag{2-51}$$

In practice, whenever the term "gain" is used, it usually refers to the *maximum gain* as defined by (2-49a) or (2-49c).

Usually the gain is given in terms of decibels instead of the dimensionless quantity of (2-49a). The conversion formula is given by

$$G_0(\text{dB}) = 10\log_{10}[e_{cd}D_0 \text{ (dimensionless)}] \tag{2-52}$$

Example 2.10

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3\theta$$

find the maximum absolute gain of this antenna.

Solution: Let us first compute the maximum directivity of the antenna. For this

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi B_0 \int_0^{\pi}\sin^4\theta\, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi\frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

Thus, the total maximum gain is equal to

$$G_0 = e_{cd}D_0 = 1(1.697) = 1.697$$

$$G_0(\text{dB}) = 10\log_{10}(1.697) = 2.297$$

which is identical to the directivity because the antenna is lossless.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency of (2-44) or (2-45), and it is equal to

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73-50}{73+50}\right|^2\right) = 0.965$$

$$e_r(\text{dB}) = 10\log_{10}(0.965) = -0.155$$

Therefore the overall efficiency is

$$e_0 = e_r e_{cd} = 0.965$$
$$e_0(\text{dB}) = -0.155$$

Thus, the overall losses are equal to 0.155 dB. The absolute gain is equal to

$$G_{0abs} = e_0 D_0 = 0.965(1.697) = 1.6376$$
$$G_{0abs}(\text{dB}) = 10 \log_{10}(1.6376) = 2.142$$

The gain in dB can also be obtained by converting the directivity and radiation efficiency in dB and then adding them. Thus,

$$e_{cd}(\text{dB}) = 10 \log_{10}(1.0) = 0$$
$$D_0(\text{dB}) = 10 \log_{10}(1.697) = 2.297$$
$$G_0(\text{dB}) = e_{cd}(\text{dB}) + D_0(\text{dB}) = 2.297$$

which is the same as obtained previously. The same procedure can be used for the absolute gain.

2.10 BEAM EFFICIENCY

Another parameter that is frequently used to judge the quality of transmitting and receiving antennas is the *beam efficiency*. For an antenna with its major lobe directed along the z-axis ($\theta = 0$), as shown in Figure 2.1(a), the beam efficiency (BE) is defined by

$$\text{BE} = \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}} \text{(dimensionless)} \qquad \text{(2-53)}$$

where θ_1 is the half-angle of the cone within which the percentage of the total power is to be found. Equation (2-53) can be written as

$$\text{BE} = \frac{\displaystyle\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\displaystyle\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi} \qquad \text{(2-54)}$$

If θ_1 is chosen as the angle where the first null or minimum occurs (see Figure 2.1), then the beam efficiency will indicate the amount of power in the major lobe compared to the total power. A very high beam efficiency (between the nulls or minimums), usually in the high 90s, is necessary for antennas used in radiometry, astronomy, radar, and other applications where received signals through the minor lobes must be minimized. The beam efficiencies of some typical rectangular and circular aperture antennas will be discussed in Chapter 12.

2.11 BANDWIDTH

The *bandwidth* of an antenna is defined as "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard." The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency. For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower. For narrowband antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency difference of acceptable operation is 5% of the center frequency of the bandwidth.

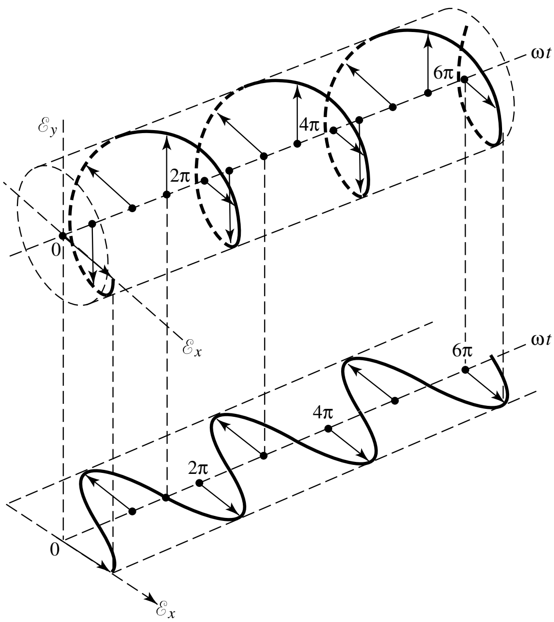
Because the characteristics (input impedance, pattern, gain, polarization, etc.) of an antenna do not necessarily vary in the same manner or are even critically affected by the frequency, there is no unique characterization of the bandwidth. The specifications are set in each case to meet the needs of the particular application. Usually there is a distinction made between pattern and input impedance variations. Accordingly *pattern bandwidth* and *impedance bandwidth* are used to emphasize this distinction. Associated with pattern bandwidth are gain, side lobe level, beamwidth, polarization, and beam direction while input impedance and radiation efficiency are related to impedance bandwidth. For example, the pattern of a linear dipole with overall length less than a half-wavelength ($l < \lambda/2$) is insensitive to frequency. The limiting factor for this antenna is its impedance, and its bandwidth can be formulated in terms of the Q. The Q of antennas or arrays with dimensions large compared to the wavelength, excluding superdirective designs, is near unity. Therefore the bandwidth is usually formulated in terms of beamwidth, side lobe level, and pattern characteristics. For intermediate length antennas, the bandwidth may be limited by either pattern or impedance variations, depending upon the particular application. For these antennas, a 2:1 bandwidth indicates a good design. For others, large bandwidths are needed. Antennas with very large bandwidths (like 40:1 or greater) have been designed in recent years. These are known as *frequency independent* antennas, and they are discussed in Chapter 11.

The above discussion presumes that the coupling networks (transformers, baluns, etc.) and/or the dimensions of the antenna are not altered in any manner as the frequency is changed. It is possible to increase the acceptable frequency range of a narrowband antenna if proper adjustments can be made on the critical dimensions of the antenna and/or on the coupling networks as the frequency is changed. Although not an easy or possible task in general, there are applications where this can be accomplished. The most common examples are the antenna of a car radio and the "rabbit ears" of a television. Both usually have adjustable lengths which can be used to tune the antenna for better reception.

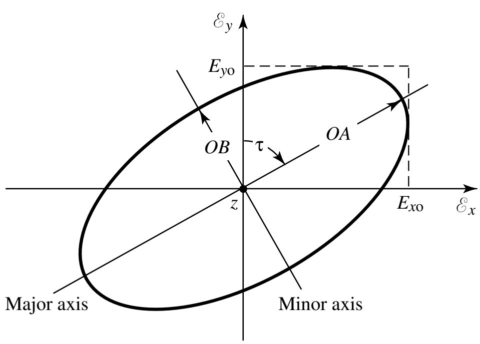
2.12 POLARIZATION

Polarization of an antenna in a given direction is defined as "the polarization of the wave transmitted (radiated) by the antenna. *Note:* When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain." In practice, polarization of the radiated energy varies with the direction from the center of the antenna, so that different parts of the pattern may have different polarizations.

Polarization of a radiated wave is defined as "that property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector; specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, *as observed along the direction of propagation*." Polarization then is the curve traced by the end point of the arrow (vector) representing the instantaneous electric field. The field must be observed along the direction of propagation. A typical trace as a function of time is shown in Figures 2.23(a) and (b).



(a) Rotation of wave



(b) Polarization ellipse

Figure 2.23 Rotation of a plane electromagnetic wave and its polarization ellipse at $z = 0$ as a function of time.

The polarization of a wave can be defined in terms of a wave *radiated (transmitted)* or *received* by an antenna in a given direction. The polarization of a wave *radiated* by an antenna in a specified direction at a point in the far field is defined as "the polarization of the (locally) plane wave which is used to represent the radiated wave at that point. At any point in the far field of an antenna the radiated wave can be represented by a plane wave whose electric-field strength is the same as that of the wave and whose direction of propagation is in the radial direction from the antenna. As the radial distance approaches infinity, the radius of curvature of the radiated wave's phase front also approaches infinity and thus in any specified direction the wave appears locally as a plane wave." This is a far-field characteristic of waves radiated by all practical antennas, and it is illustrated analytically in Section 3.6 of Chapter 3. The polarization of a wave *received* by an antenna is defined as the "polarization of a plane wave, incident from a given direction and having a given power flux density, which results in maximum available power at the antenna terminals."

Polarization may be classified as linear, circular, or elliptical. If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be *linearly* polarized. In general, however, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized. Linear and circular polarizations are special cases of elliptical, and they can be obtained when the ellipse becomes a straight line or a circle, respectively. The figure of the electric field is traced in a *clockwise* (CW) or *counterclockwise* (CCW) sense. *Clockwise* rotation of the electric-field vector is also designated as *right-hand polarization* and *counterclockwise* as *left-hand polarization*.

In general, the polarization characteristics of an antenna can be represented by its *polarization pattern* whose one definition is "the spatial distribution of the polarizations of a field vector excited (radiated) by an antenna taken over its radiation sphere. When describing the polarizations over the radiation sphere, or portion of it, reference lines shall be specified over the sphere, in order to measure the tilt angles (see tilt angle) of the polarization ellipses and the direction of polarization for linear polarizations. An obvious choice, though by no means the only one, is a family of lines tangent at each point on the sphere to either the θ or ϕ coordinate line associated with a spherical coordinate system of the radiation sphere. At each point on the radiation sphere the polarization is usually resolved into a pair of orthogonal polarizations, the *co-polarization* and *cross polarization*. To accomplish this, the co-polarization must be specified at each point on the radiation sphere." "*Co-polarization* represents the polarization the antenna is intended to radiate (receive) while *cross-polarization* represents the polarization orthogonal to a specified polarization, which is usually the co-polarization."

"For certain linearly polarized antennas, it is common practice to define the co-polarization in the following manner: First specify the orientation of the co-polar electric-field vector at a pole of the radiation sphere. Then, for all other directions of interest (points on the radiation sphere), require that the angle that the co-polar electric-field vector makes with each great circle line through the pole remain constant over that circle, the angle being that at the pole."

"In practice, the axis of the antenna's main beam should be directed along the polar axis of the radiation sphere. The antenna is then appropriately oriented about this axis to align the direction of its polarization with that of the defined co-polarization at the pole." "This manner of defining co-polarization can be extended to the case of elliptical polarization by defining the constant angles using the major axes of the polarization ellipses rather than the co-polar electric-field vector. The sense of polarization (rotation) must also be specified."

The polarization of the wave radiated by the antenna can also be represented on the Poincaré sphere [13]–[16]. Each point on the Poincaré sphere represents a unique polarization. The north pole represents left circular polarization, the south pole represents right circular, and points along the equator represent linear polarization of different tilt angles. All other points on the Poincaré sphere represent elliptical polarization. For details, see Figure 17.24 of Chapter 17.

The polarization of an antenna is measured using techniques described in Chapter 17.

2.12.1 Linear, Circular, and Elliptical Polarizations

The instantaneous field of a plane wave, traveling in the negative z direction, can be written as

$$\mathscr{E}(z;t) = \hat{\mathbf{a}}_x \mathscr{E}_x(z;t) + \hat{\mathbf{a}}_y \mathscr{E}_y(z;t) \tag{2-55}$$

According to (2-5), the instantaneous components are related to their complex counterparts by

$$\begin{aligned} \mathscr{E}_x(z;t) &= \text{Re}[E_x^- e^{j(\omega t+kz)}] = \text{Re}[E_{xo} e^{j(\omega t+kz+\phi_x)}] \\ &= E_{xo}\cos(\omega t + kz + \phi_x) \end{aligned} \tag{2-56}$$

$$\begin{aligned} \mathscr{E}_y(z;t) &= \text{Re}[E_y^- e^{j(\omega t+kz)}] = \text{Re}[E_{yo} e^{j(\omega t+kz+\phi_y)}] \\ &= E_{yo}\cos(\omega t + kz + \phi_y) \end{aligned} \tag{2-57}$$

where E_{xo} and E_{yo} are, respectively, the maximum magnitudes of the x and y components.

A. Linear Polarization

For the wave to have linear polarization, the time-phase difference between the two components must be

$$\Delta\phi = \phi_y - \phi_x = n\pi, \quad n = 0, 1, 2, 3, \ldots \tag{2-58}$$

B. Circular Polarization

Circular polarization can be achieved *only* when the magnitudes of the two components are the same *and* the time-phase difference between them is odd multiples of $\pi/2$. That is,

$$|\mathscr{E}_x| = |\mathscr{E}_y| \Rightarrow E_{xo} = E_{yo} \tag{2-59}$$

$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +\left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CW} \quad (2\text{-}60) \\ -\left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CCW} \quad (2\text{-}61) \end{cases}$$

If the direction of wave propagation is reversed (i.e., $+z$ direction), the phases in (2-60) and (2-61) for CW and CCW rotation must be interchanged.

C. Elliptical Polarization

Elliptical polarization can be attained *only* when the time-phase difference between the two components is odd multiples of $\pi/2$ *and* their magnitudes are not the same *or* when the time-phase difference between the two components is not equal to multiples of $\pi/2$ (irrespective of their magnitudes). That is,

$$|\mathscr{E}_x| \neq |\mathscr{E}_y| \Rightarrow E_{xo} \neq E_{yo}$$

$$\text{when } \Delta\phi = \phi_y - \phi_x = \begin{cases} +(\frac{1}{2} + 2n)\pi & \text{for CW} \quad (2\text{-}62a) \\ -(\frac{1}{2} + 2n)\pi & \text{for CCW} \quad (2\text{-}62b) \end{cases} \quad n = 0, 1, 2, \ldots$$

or

$$\Delta\phi = \phi_y - \phi_x \neq \pm\frac{n}{2}\pi = \begin{cases} > 0 & \text{for CW} \quad (2\text{-}63) \\ < 0 & \text{for CCW} \quad (2\text{-}64) \end{cases} \quad n = 0, 1, 2, 3, \ldots$$

For elliptical polarization, the curve traced at a given position as a function of time is, in general, a tilted ellipse, as shown in Figure 2.23(b). The ratio of the major axis to the minor axis is referred to as the axial ratio (AR), and it is equal to

$$\text{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, \quad 1 \leq \text{AR} \leq \infty \tag{2-65}$$

where

$$OA = \left[\tfrac{1}{2}\{E_{xo}^2 + E_{yo}^2 + [E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2 E_{yo}^2 \cos(2\Delta\phi)]^{1/2}\}\right]^{1/2} \tag{2-66}$$

$$OB = \left[\tfrac{1}{2}\{E_{xo}^2 + E_{yo}^2 - [E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2 E_{yo}^2 \cos(2\Delta\phi)]^{1/2}\}\right]^{1/2} \tag{2-67}$$

The tilt of the ellipse, *relative to the y axis*, is represented by the angle τ given by

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left[\frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2}\cos(\Delta\phi)\right] \tag{2-68}$$

When the ellipse is aligned with the principal axes [$\tau = n\pi/2, n = 0, 1, 2, \ldots$], the major (minor) axis is equal to $E_{xo}(E_{yo})$ or $E_{yo}(E_{xo})$ and the axial ratio is equal to E_{xo}/E_{yo} or E_{yo}/E_{xo}.

SUMMARY

We will summarize the preceding discussion on polarization by stating the general characteristics, and the *necessary and sufficient* conditions that the wave must have in order to possess *linear, circular* or *elliptical* polarization.

Linear Polarization A *time-harmonic wave is linearly polarized at a given point in space if the electric-field (or magnetic-field) vector at that point is always oriented along the same straight line at every instant of time*. This is accomplished if the field vector (electric or magnetic) possesses:

a. Only one component, or
b. Two orthogonal linear components that are in time phase or 180° (or multiples of 180°) out-of-phase.

Circular Polarization *A time-harmonic wave is circularly polarized at a given point in space if the electric (or magnetic) field vector at that point traces a circle as a function of time.*

The *necessary and sufficient* conditions to accomplish this are if the field vector (electric or magnetic) possesses *all* of the following:

a. The field must have two orthogonal linear components, and
b. The two components must have the same magnitude, and
c. The two components must have a time-phase difference of odd multiples of 90°.

The sense of rotation is always determined by rotating the phase-leading component toward the phase-lagging component and observing the field rotation as the wave is viewed as it travels away from the observer. If the rotation is clockwise, the wave is right-hand (or clockwise) circularly polarized; if the rotation is counterclockwise, the wave is left-hand (or counterclockwise) circularly polarized. The rotation of the phase-leading component toward the phase-lagging component should be done along the angular separation between the two components that is less than 180°. *Phases equal to or greater than* 0° *and less than* 180° *should be considered leading whereas those equal to or greater than* 180° *and less than* 360° *should be considered lagging.*

Elliptical Polarization *A time-harmonic wave is elliptically polarized if the tip of the field vector (electric or magnetic) traces an elliptical locus in space. At various instants of time the field vector changes continuously with time at such a manner as to describe an elliptical locus. It is right-hand (clockwise) elliptically polarized if the field vector rotates clockwise, and it is left-hand (counterclockwise) elliptically polarized if the field vector of the ellipse rotates counterclockwise* [13]. The sense of rotation is determined using the same rules as for the circular polarization. In addition to the sense of rotation, elliptically polarized waves are also specified by their axial ratio whose magnitude is the ratio of the major to the minor axis.

A wave is elliptically polarized if it is not linearly or circularly polarized. Although linear and circular polarizations are special cases of elliptical, usually in practice elliptical polarization refers to other than linear or circular. The *necessary and sufficient* conditions to accomplish this are if the field vector (electric or magnetic) possesses *all* of the following:

a. The field must have two orthogonal linear components, and
b. The two components can be of the same or different magnitude.
c. (1) If the two components are not of the same magnitude, the time-phase difference between the two components must not be 0° or multiples of 180° (because it will then be linear). (2) If the two components are of the same magnitude, the time-phase difference between the two components must not be odd multiples of 90° (because it will then be circular).

If the wave is elliptically polarized with two components not of the same magnitude but with odd multiples of 90° time-phase difference, the polarization ellipse will not be tilted but it will be aligned with the principal axes of the field components. The major axis of the ellipse will align with the axis of the field component which is larger of the two, while the minor axis of the ellipse will align with the axis of the field component which is smaller of the two.

2.12.2 Polarization Loss Factor and Efficiency

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as "polarization mismatch." The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss. Assuming that the electric field of the incoming wave can be written as

$$\mathbf{E}_i = \hat{\boldsymbol{\rho}}_w E_i \tag{2-69}$$

where $\hat{\boldsymbol{\rho}}_w$ is the unit vector of the wave, and the polarization of the electric field of the receiving antenna can be expressed as

$$\mathbf{E}_a = \hat{\boldsymbol{\rho}}_a E_a \tag{2-70}$$

where $\hat{\boldsymbol{\rho}}_a$ is its unit vector (polarization vector), the polarization loss can be taken into account by introducing a *polarization loss factor* (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = |\cos\psi_p|^2 \text{ (dimensionless)} \tag{2-71}$$

where ψ_p is the angle between the two unit vectors. The relative alignment of the polarization of the incoming wave and of the antenna is shown in Figure 2.24. If the antenna is polarization matched, its PLF will be unity and the antenna will extract maximum power from the incoming wave.

Another figure of merit that is used to describe the polarization characteristics of a wave and that of an antenna is the *polarization efficiency* (*polarization mismatch* or *loss factor*) which is defined as "the ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of

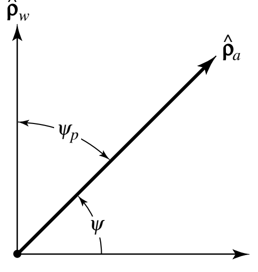


Figure 2.24 Polarization unit vectors of incident wave ($\hat{\boldsymbol{\rho}}_w$) and antenna ($\hat{\boldsymbol{\rho}}_a$), and polarization loss factor (PLF).

propagation, whose state of polarization has been adjusted for a maximum received power." This is similar to the PLF and it is expressed as

$$p_e = \frac{|\boldsymbol{\ell}_\mathbf{e} \cdot \mathbf{E}^{\text{inc}}|^2}{|\boldsymbol{\ell}_\mathbf{e}|^2 |\mathbf{E}^{\text{inc}}|^2} \tag{2-71a}$$

where

$\boldsymbol{\ell}_e$ = vector effective length of the antenna
$\mathbf{E}^{\text{inc}}$ = incident electric field

The vector effective length ℓ_e of the antenna has not yet been defined, and it is introduced in Section 2.15. It is a vector that describes the polarization characteristics of the antenna. Both the PLF and p_e lead to the same answers.

The conjugate (*) is not used in (2-71) or (2-71a) so that a right-hand circularly polarized incident wave (when viewed in its direction of propagation) is matched to right-hand circularly polarized receiving antenna (when its polarization is determined in the transmitting mode). Similarly, a left-hand circularly polarized wave will be matched to a left-hand circularly polarized antenna.

To illustrate the principle of polarization mismatch, two examples are considered.

Example 2.11

The electric field of a linearly polarized electromagnetic wave given by

$$\mathbf{E}_i = \hat{\mathbf{a}}_x E_0(x, y) e^{-jkz}$$

is incident upon a linearly polarized antenna whose electric-field polarization is expressed as

$$\mathbf{E}_a \simeq (\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y) E(r, \theta, \phi)$$

Find the polarization loss factor (PLF).

Solution: For the incident wave

$$\hat{\boldsymbol{\rho}}_w = \hat{\mathbf{a}}_x$$

and for the antenna

$$\hat{\boldsymbol{\rho}}_a = \frac{1}{\sqrt{2}}(\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y)$$

The PLF is then equal to

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = |\hat{\mathbf{a}}_x \cdot \frac{1}{\sqrt{2}}(\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y)|^2 = \tfrac{1}{2}$$

which in dB is equal to

$$\text{PLF (dB)} = 10 \log_{10} \text{PLF (dimensionless)} = 10 \log_{10}(0.5) = -3$$

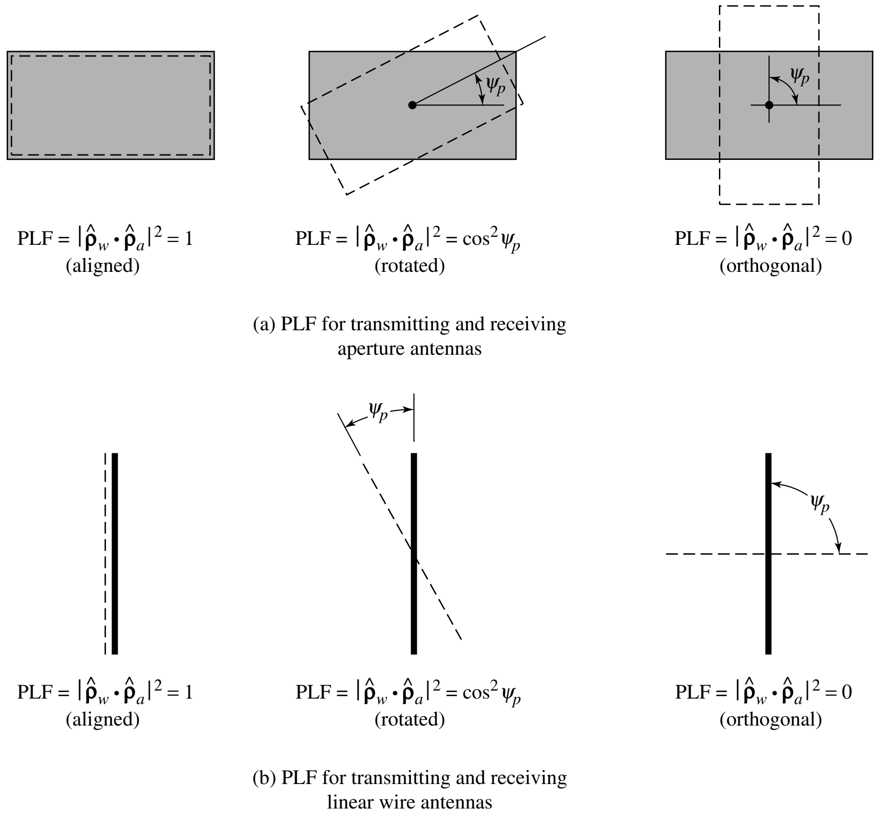


Figure 2.25 Polarization loss factors (PLF) for aperture and linear wire antennas.

Even though in Example 2.11 both the incoming wave and the antenna are linearly polarized, there is a 3-dB loss in extracted power because the polarization of the incoming wave is not aligned with the polarization of the antenna. If the polarization of the incoming wave is orthogonal to the polarization of the antenna, then there will be no power extracted by the antenna from the incoming wave and the PLF will be zero or $-\infty$ dB. In Figures 2.25(a,b) we illustrate the polarization loss factors (PLF) of two types of antennas: wires and apertures.

We now want to consider an example where the polarization of the antenna and the incoming wave are described in terms of complex polarization vectors.

Example 2.12

A right-hand (clockwise) circularly polarized wave radiated by an antenna, placed at some distance away from the origin of a spherical coordinate system, is traveling in the inward radial direction at an angle (θ, ϕ) and it is impinging upon a right-hand circularly polarized receiving antenna placed at the origin (see Figures 2.1 and 17.23 for the geometry of the coordinate system). The polarization of the receiving antenna is defined in the transmitting mode, as desired by the definition of the IEEE. Assuming the polarization of the incident wave is represented by

$$\mathbf{E}_w = (\hat{\mathbf{a}}_\theta + j\hat{\mathbf{a}}_\phi)E(r, \theta, \phi)$$

Determine the polarization loss factor (PLF).

Solution: The polarization of the incident right-hand circularly polarized wave traveling along the $-r$ radial direction is described by the unit vector

$$\hat{\boldsymbol{\rho}}_w = \left(\frac{\hat{\mathbf{a}}_\theta + j\hat{\mathbf{a}}_\phi}{\sqrt{2}}\right)$$

while that of the receiving antenna, in the transmitting mode, is represented by the unit vector

$$\hat{\boldsymbol{\rho}}_a = \left(\frac{\hat{\mathbf{a}}_\theta - j\hat{\mathbf{a}}_\phi}{\sqrt{2}}\right)$$

Therefore the polarization loss factor is

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = \frac{1}{4}|1 + 1|^2 = 1 = 0 \text{ dB}$$

Since the polarization of the incoming wave matches (including the sense of rotation) the polarization of the receiving antenna, there should not be any losses. Obviously the answer matches the expectation.

Based upon the definitions of the wave transmitted and received by an antenna, the polarization of an antenna in the *receiving* mode is related to that in the *transmitting* mode as follows:

1. "In the same plane of polarization, the polarization ellipses have the same axial ratio, the same sense of polarization (rotation) and the same spatial orientation.
2. "Since their senses of polarization and spatial orientation are specified by viewing their polarization ellipses in the respective directions in which they are propagating, one should note that:
   a. Although their senses of polarization are the same, they would appear to be opposite if both waves were viewed in the same direction.
   b. Their tilt angles are such that they are the negative of one another with respect to a common reference."

Since the polarization of an antenna will almost always be defined in its transmitting mode, according to the IEEE Std 145-1983, "the receiving polarization may be used to specify the polarization characteristic of a nonreciprocal antenna which may transmit and receive arbitrarily different polarizations."

The polarization loss must always be taken into account in the link calculations design of a communication system because in some cases it may be a very critical factor. Link calculations of communication systems for outer space explorations are very stringent because of limitations in spacecraft weight. In such cases, power is a

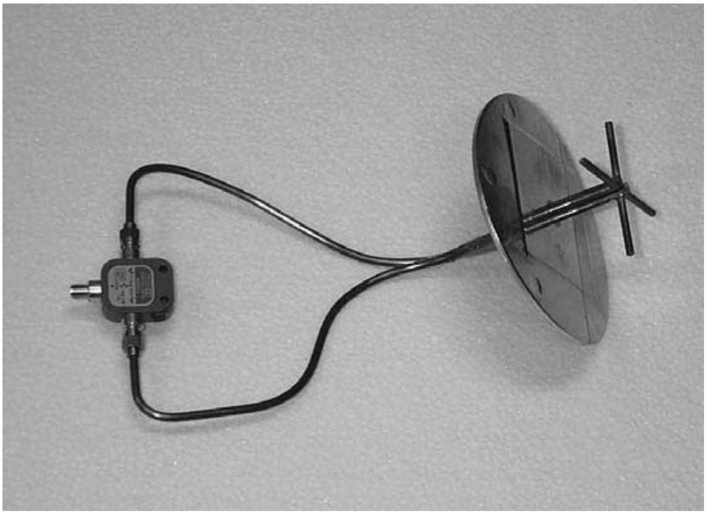


Figure 2.26 Geometry of elliptically polarized cross-dipole antenna.

limiting consideration. The design must properly take into account all loss factors to ensure a successful operation of the system.

An antenna that is elliptically polarized is that composed of two crossed dipoles, as shown in Figure 2.26. The two crossed dipoles provide the two orthogonal field components that are not necessarily of the same field intensity toward all observation angles. If the two dipoles are identical, the field intensity of each along zenith (perpendicular to the plane of the two dipoles) would be of the same intensity. Also, if the two dipoles were fed with a 90° degree time-phase difference (phase quadrature), the polarization along zenith would be circular and elliptical toward other directions. One way to obtain the 90° time-phase difference $\Delta\phi$ between the two orthogonal field components, radiated respectively by the two dipoles, is by feeding one of the two dipoles with a transmission line which is $\lambda/4$ longer or shorter than that of the other $[\Delta\phi = k\Delta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2]$. One of the lengths (longer or shorter) will provide right-hand (CW) rotation while the other will provide left-hand (CCW) rotation.

2.13 INPUT IMPEDANCE

Input impedance is defined as "the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point." In this section we are primarily interested in the input impedance at a pair of terminals which are the input terminals of the antenna. In Figure 2.27(a) these terminals are designated as $a - b$. The ratio of the voltage to current at these terminals, with no load attached, defines the impedance of the antenna as

$$\boxed{Z_A = R_A + jX_A} \tag{2-72}$$

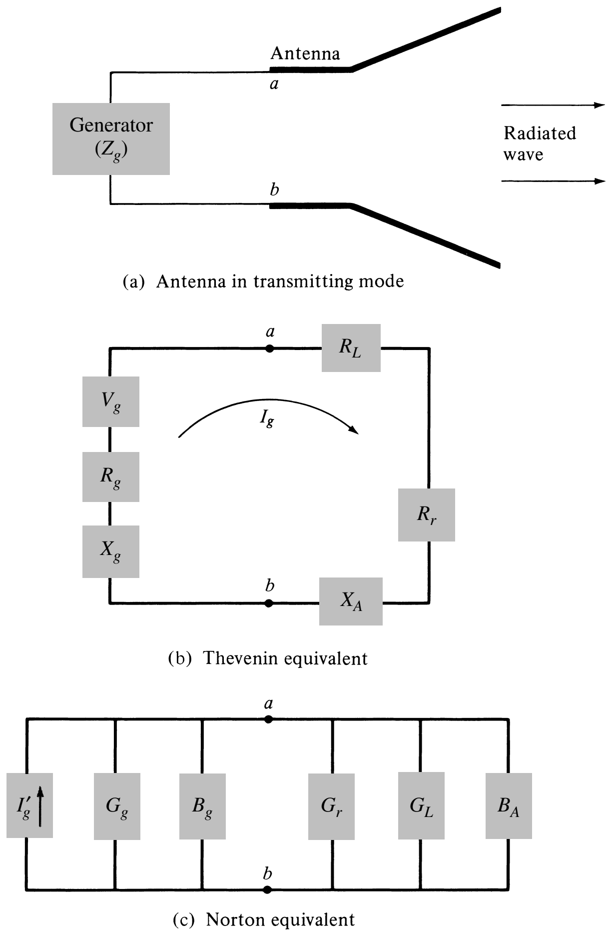


Figure 2.27 Transmitting antenna and its equivalent circuits.

where

Z_A = antenna impedance at terminals a–b (ohms)
R_A = antenna resistance at terminals a–b (ohms)
X_A = antenna reactance at terminals a–b (ohms)

In general the resistive part of (2-72) consists of two components; that is

$$\boxed{R_A = R_r + R_L} \tag{2-73}$$

where

R_r = radiation resistance of the antenna
R_L = loss resistance of the antenna

The radiation resistance will be considered in more detail in later chapters, and it will be illustrated with examples.

If we assume that the antenna is attached to a generator with internal impedance

$$Z_g = R_g + jX_g \tag{2-74}$$

where

R_g = resistance of generator impedance (ohms)
X_g = reactance of generator impedance (ohms)

and the antenna is used in the transmitting mode, we can represent the antenna and generator by an equivalent circuit* shown in Figure 2.27(b). To find the amount of power delivered to R_r for radiation and the amount dissipated in R_L as heat $(I^2R_L/2)$, we first find the current developed within the loop which is given by

$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)} \quad (A) \tag{2-75}$$

and its magnitude by

$$|I_g| = \frac{|V_g|}{[(R_r + R_L + R_g)^2 + (X_A + X_g)^2]^{1/2}} \tag{2-75a}$$

where V_g is the peak generator voltage. The power delivered to the antenna for radiation is given by

$$P_r = \frac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right] \quad (W) \tag{2-76}$$

and that dissipated as heat by

$$P_L = \frac{1}{2}|I_g|^2 R_L = \frac{|V_g|^2}{2}\left[\frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right] \quad (W) \tag{2-77}$$

The remaining power is dissipated as heat on the internal resistance R_g of the generator, and it is given by

$$P_g = \frac{|V_g|^2}{2}\left[\frac{R_g}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right] \quad (W) \tag{2-78}$$

The maximum power delivered to the antenna occurs when we have conjugate matching; that is when

$$R_r + R_L = R_g \tag{2-79}$$

$$X_A = -X_g \tag{2-80}$$

*This circuit can be used to represent small and simple antennas. It cannot be used for antennas with lossy dielectric or antennas over lossy ground because their loss resistance cannot be represented in series with the radiation resistance.

For this case

$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right] \tag{2-81}$$

$$P_L = \frac{|V_g|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right] \tag{2-82}$$

$$P_g = \frac{|V_g|^2}{8}\left[\frac{R_g}{(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{1}{R_r + R_L}\right] = \frac{|V_g|^2}{8R_g} \tag{2-83}$$

From (2-81)–(2-83), it is clear that

$$P_g = P_r + P_L = \frac{|V_g|^2}{8}\left[\frac{R_g}{(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r + R_L}{(R_r + R_L)^2}\right] \tag{2-84}$$

The power supplied by the generator during conjugate matching is

$$P_s = \frac{1}{2}V_g I_g^* = \frac{1}{2}V_g\left[\frac{V_g^*}{2(R_r + R_L)}\right] = \frac{|V_g|^2}{4}\left[\frac{1}{R_r + R_L}\right] \quad (W) \tag{2-85}$$

Of the power that is provided by the generator, half is dissipated as heat in the internal resistance (R_g) of the generator and the other half is delivered to the antenna. This only happens when we have *conjugate matching*. Of the power that is delivered to the antenna, part is radiated through the mechanism provided by the radiation resistance and the other is dissipated as heat which influences part of the overall efficiency of the antenna. If the antenna is lossless and matched to the transmission line ($e_o = 1$), then half of the total power supplied by the generator is radiated by the antenna during conjugate matching, and the other half is dissipated as heat in the generator. Thus, to radiate half of the available power through R_r you must dissipate the other half as heat in the generator through R_g. These two powers are, respectively, analogous to the power transferred to the load and the power scattered by the antenna in the receiving mode. In Figure 2.27 it is assumed that the generator is directly connected to the antenna. If there is a transmission line between the two, which is usually the case, then Z_g represents the equivalent impedance of the generator transferred to the input terminals of the antenna using the impedance transfer equation. If, in addition, the transmission line is lossy, then the available power to be radiated by the antenna will be reduced by the losses of the transmission line. Figure 2.27(c) illustrates the Norton equivalent of the antenna and its source in the transmitting mode.

The use of the antenna in the receiving mode is shown in Figure 2.28(a). The incident wave impinges upon the antenna, and it induces a voltage V_T which is analogous to V_g of the transmitting mode. The Thevenin equivalent circuit of the antenna and its load is shown in Figure 2.28(b) and the Norton equivalent in Figure 2.28(c). The discussion for the antenna and its load in the receiving mode parallels that for the transmitting mode, and it will not be repeated here in detail. Some of the results will

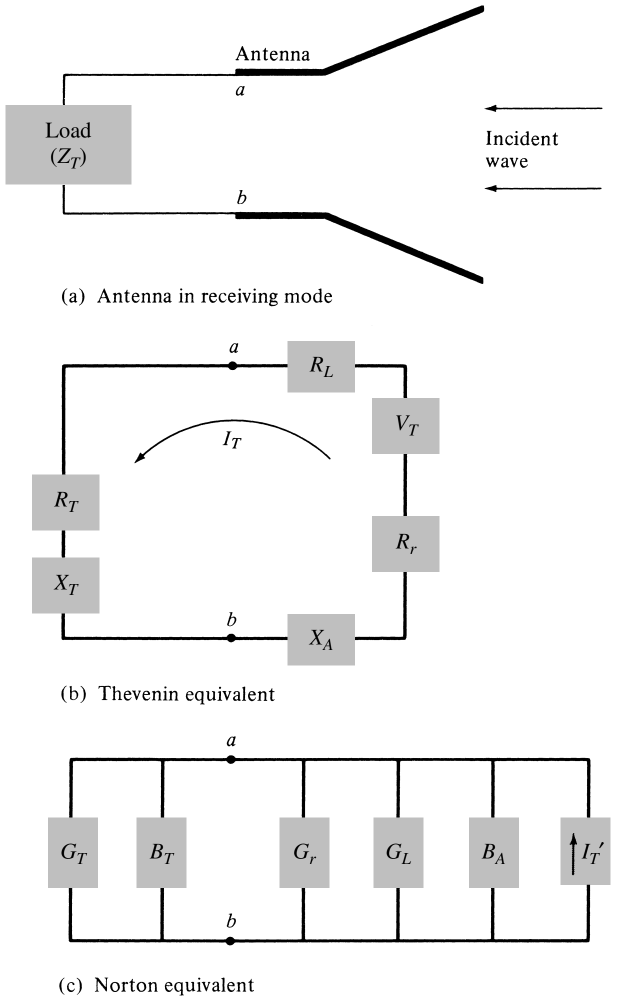


Figure 2.28 Antenna and its equivalent circuits in the receiving mode.

be summarized in order to discuss some subtle points. Following a procedure similar to that for the antenna in the transmitting mode, it can be shown using Figure 2.28 that in the receiving mode under conjugate matching ($R_r + R_L = R_T$ and $X_A = -X_T$) the powers delivered to R_T, R_r, and R_L are given, respectively, by

$$P_T = \frac{|V_T|^2}{8}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8}\left(\frac{1}{R_r + R_L}\right) = \frac{|V_T|^2}{8R_T} \tag{2-86}$$

$$P_r = \frac{|V_T|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right] \tag{2-87}$$

$$P_L = \frac{|V_T|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right] \tag{2-88}$$

while the *induced* (*collected* or *captured*) is

$$P_c = \frac{1}{2} V_T I_T^* = \frac{1}{2} V_T \left[\frac{V_T^*}{2(R_r + R_L)} \right] = \frac{|V_T|^2}{4} \left(\frac{1}{R_r + R_L} \right) \tag{2-89}$$

These are analogous, respectively, to (2-81)–(2-83) and (2-85). The power P_r of (2-87) delivered to R_r is referred to as *scattered* (or *reradiated*) power. It is clear through (2-86)–(2-89) that under conjugate matching of the total power collected or captured [P_c of (2-89)] half is delivered to the load R_T [P_T of (2-86)] and the other half is scattered or reradiated through R_r [P_r of (2-87)] and dissipated as heat through R_L [P_L of (2-88)]. If the losses are zero ($R_L = 0$), then half of the captured power is delivered to the load and the other half is scattered. This indicates that in order to deliver half of the power to the load you must scatter the other half. This becomes important when discussing effective equivalent areas and aperture efficiencies, especially for high directivity aperture antennas such as waveguides, horns, and reflectors with aperture efficiencies as high as 80 to 90%. Aperture efficiency (ϵ_{ap}) is defined by (2-100) and is the ratio of the maximum effective area to the physical area. The effective area is used to determine the power delivered to the load, which under conjugate matching is only one-half of that intercepted; the other half is scattered and dissipated as heat. For a lossless antenna ($R_L = 0$) under conjugate matching, the maximum value of the effective area is equal to the physical area ($\epsilon_{ap} = 1$) and the scattering area is also equal to the physical area. Thus half of the power is delivered to the load and the other half is scattered. Using (2-86) to (2-89) we conclude that even though the aperture efficiencies are higher than 50% (they can be as large as 100%) all of the power that is captured by the antenna is not delivered to the load but it includes that which is scattered plus dissipated as heat by the antenna. The most that can be delivered to the load is only half of that captured and that is only under conjugate matching and lossless transmission line.

The input impedance of an antenna is generally a function of frequency. Thus the antenna will be matched to the interconnecting transmission line and other associated equipment only within a bandwidth. In addition, the input impedance of the antenna depends on many factors including its geometry, its method of excitation, and its proximity to surrounding objects. Because of their complex geometries, only a limited number of practical antennas have been investigated analytically. For many others, the input impedance has been determined experimentally.

2.14 ANTENNA RADIATION EFFICIENCY

The antenna efficiency that takes into account the reflection, conduction, and dielectric losses was discussed in Section 2.8. The conduction and dielectric losses of an antenna are very difficult to compute and in most cases they are measured. Even with measurements, they are difficult to separate and they are usually lumped together to form the e_{cd} efficiency. The resistance R_L is used to represent the conduction-dielectric losses.

The *conduction-dielectric efficiency* e_{cd} is defined as *the ratio of the power delivered to the radiation resistance* R_r *to the power delivered to* R_r *and* R_L. Using (2-76) and

(2-77), the radiation efficiency can be written as

$$\boxed{e_{cd} = \left[\frac{R_r}{R_L + R_r}\right]} \quad \text{(dimensionless)} \tag{2-90}$$

For a metal rod of length l and uniform cross-sectional area A, the dc resistance is given by

$$R_{dc} = \frac{1}{\sigma}\frac{l}{A} \quad \text{(ohms)} \tag{2-90a}$$

If the skin depth $\delta[\delta = \sqrt{2/(\omega\mu_0\sigma)}]$ of the metal is very small compared to the smallest diagonal of the cross section of the rod, the current is confined to a thin layer near the conductor surface. Therefore the high-frequency resistance can be written, based on a *uniform current distribution*, as

$$R_{hf} = \frac{l}{P}R_s = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} \quad \text{(ohms)} \tag{2-90b}$$

where P is the perimeter of the cross section of the rod ($P = C = 2\pi b$ for a circular wire of radius b), R_s is the conductor surface resistance, ω is the angular frequency, μ_0 is the permeability of free-space, and σ is the conductivity of the metal.

Example 2.13

A resonant half-wavelength dipole is made out of copper ($\sigma = 5.7 \times 10^7$ S/m) wire. Determine the conduction-dielectric (radiation) efficiency of the dipole antenna at $f = 100$ MHz if the radius of the wire b is $3 \times 10^{-4}\lambda$, and the radiation resistance of the $\lambda/2$ dipole is 73 ohms.

Solution: At $f = 10^8$ Hz

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{10^8} = 3 \text{ m}$$

$$l = \frac{\lambda}{2} = \frac{3}{2}\text{m}$$

$$C = 2\pi b = 2\pi(3 \times 10^{-4})\lambda = 6\pi \times 10^{-4}\lambda$$

For a $\lambda/2$ dipole with a sinusoidal current distribution $R_L = \frac{1}{2}R_{hf}$ where R_{hf} is given by (2-90b). See Problem 2.52. Therefore,

$$R_L = \frac{1}{2}R_{hf} = \frac{0.25}{6\pi \times 10^{-4}}\sqrt{\frac{\pi(10^8)(4\pi \times 10^{-7})}{5.7 \times 10^7}} = 0.349 \text{ ohms}$$

Thus,

$$e_{cd}(\text{dimensionless}) = \frac{73}{73 + 0.349} = 0.9952 = 99.52\%$$

$$e_{cd}(\text{dB}) = 10\log_{10}(0.9905) = -0.02$$

2.15 ANTENNA VECTOR EFFECTIVE LENGTH AND EQUIVALENT AREAS

An antenna in the receiving mode, whether it is in the form of a wire, horn, aperture, array, dielectric rod, etc., is used to capture (collect) electromagnetic waves and to extract power from them, as shown in Figures 2.29(a) and (b). For each antenna, an equivalent length and a number of equivalent areas can then be defined.

These equivalent quantities are used to describe the receiving characteristics of an antenna, whether it be a linear or an aperture type, when a wave is incident upon the antenna.

2.15.1 Vector Effective Length

The effective length of an antenna, whether it be a linear or an aperture antenna, is a quantity that is used to determine the voltage induced on the open-circuit terminals

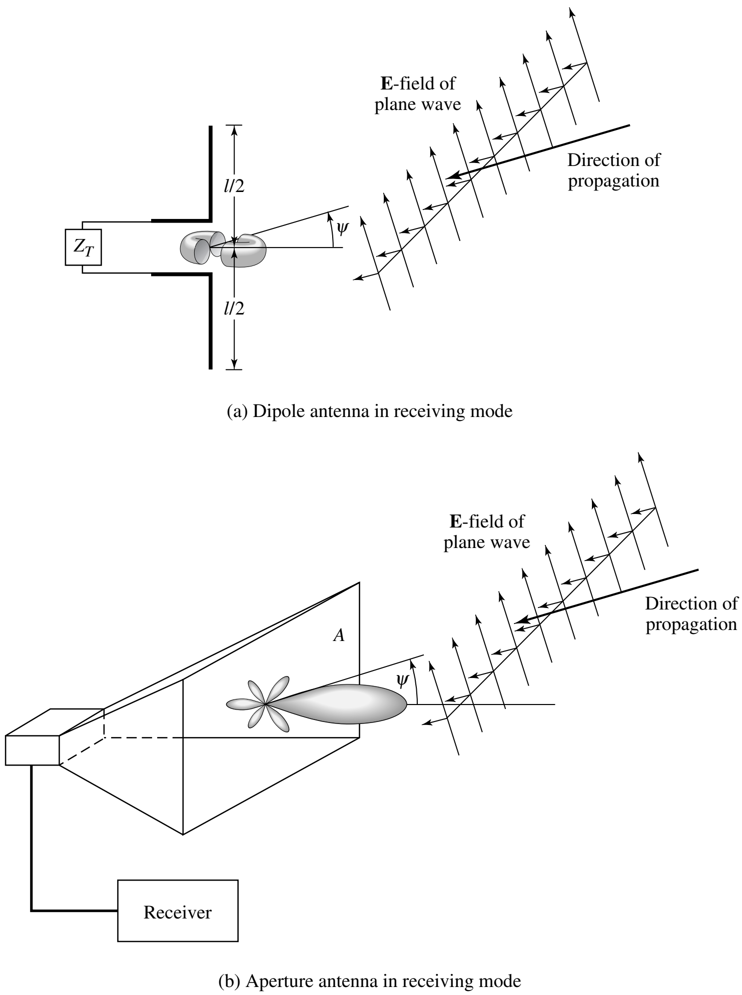


Figure 2.29 Uniform plane wave incident upon dipole and aperture antennas.

of the antenna when a wave impinges upon it. The vector effective length $\boldsymbol{\ell}_e$ for an antenna is usually a complex vector quantity represented by

$$\boldsymbol{\ell}_e(\theta, \phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta, \phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta, \phi) \tag{2-91}$$

It should be noted that it is also referred to as the *effective height*. It is a far-field quantity and it is related to the *far-zone* field $\mathbf{E}_a$ radiated by the antenna, with current I_{in} in its terminals, by [13]–[18]

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi = -j\eta \frac{kI_{in}}{4\pi r} \boldsymbol{\ell}_e e^{-jkr} \tag{2-92}$$

The effective length represents the antenna in its transmitting and receiving modes, and it is particularly useful in relating the open-circuit voltage V_{oc} of receiving antennas. This relation can be expressed as

$$V_{oc} = \mathbf{E}^i \cdot \boldsymbol{\ell}_e \tag{2-93}$$

where

V_{oc} = open-circuit voltage at antenna terminals
$\mathbf{E}^i$ = incident electric field
$\boldsymbol{\ell}_e$ = vector effective length

In (2-93) V_{oc} can be thought of as the voltage induced in a linear antenna of length ℓ_e when $\boldsymbol{\ell}_e$ and $\mathbf{E}^\mathbf{i}$ are linearly polarized [19], [20]. From the relation of (2-93) the *effective length of a linearly polarized antenna receiving a plane wave in a given direction* is defined as "the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization. Alternatively, the effective length is the length of a thin straight conductor oriented perpendicular to the given direction and parallel to the antenna polarization, having a uniform current equal to that at the antenna terminals and producing the same far-field strength as the antenna in that direction."

In addition, as shown in Section 2.12.2, the antenna vector effective length is used to determine the polarization efficiency of the antenna. To illustrate the usefulness of the vector effective length, let us consider an example.

Example 2.14

The far-zone field radiated by a small dipole of length $l < \lambda/10$ and with a triangular current distribution, as shown in Figure 4.4, is derived in Section 4.3 of Chapter 4 and it is given by (4-36a), or

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta j\eta \frac{kI_{in} l e^{-jkr}}{8\pi r} \sin\theta$$

Determine the vector effective length of the antenna.

Solution: According to (2-92), the vector effective length is

$$\boldsymbol{\ell}_e = -\hat{\mathbf{a}}_\theta \frac{l}{2} \sin\theta$$

This indicates, as it should, that the effective length is a function of the direction angle θ, and its maximum occurs when $\theta = 90^\circ$. This tells us that the maximum open-circuit voltage at the dipole terminals occurs when the incident direction of the wave of Figure 2.29(a) impinging upon the small dipole antenna is normal to the axis (length) of the dipole ($\theta = 90^\circ$). This is expected since the dipole has a radiation pattern whose maximum is in the $\theta = 90^\circ$. In addition, the effective length of the dipole to produce the same output open-circuit voltage is only half (50%) of its physical length if it were replaced by a thin conductor having a uniform current distribution (it can be shown that the maximum effective length of an element with an ideal uniform current distribution is equal to its physical length).

2.15.2 Antenna Equivalent Areas

With each antenna, we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. One of these equivalent areas is the *effective area (aperture)*, which in a given direction is defined as "the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna. If the direction is not specified, the direction of maximum radiation intensity is implied." In equation form it is written as

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i} \tag{2-94}$$

where

A_e = effective area (effective aperture) (m^2)
P_T = power delivered to the load (W)
W_i = power density of incident wave (W/m^2)

The effective aperture is the area which when multiplied by the incident power density gives the power delivered to the load. Using the equivalent of Figure 2.28, we can write (2-94) as

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right] \tag{2-95}$$

Under conditions of maximum power transfer (*conjugate matching*), $R_r + R_L = R_T$ and $X_A = -X_T$, the effective area of (2-95) reduces to the maximum effective aperture given by

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right] \tag{2-96}$$

When (2-96) is multiplied by the incident power density, it leads to the maximum power delivered to the load of (2-86).

All of the power that is intercepted, collected, or captured by an antenna is not delivered to the load, as we have seen using the equivalent circuit of Figure 2.28. In fact, under conjugate matching only half of the captured power is delivered to the load; the other half is scattered and dissipated as heat. Therefore to account for the scattered and dissipated power we need to define, in addition to the effective area, the *scattering, loss* and *capture* equivalent areas. In equation form these can be defined similarly to (2-94)–(2-96) for the effective area.

The *scattering area* is defined as the equivalent area when multiplied by the incident power density is equal to the scattered or reradiated power. Under conjugate matching this is written, similar to (2-96), as

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right] \tag{2-97}$$

which when multiplied by the incident power density gives the scattering power of (2-87).

The *loss area* is defined as the equivalent area, which when multiplied by the incident power density leads to the power dissipated as heat through R_L. Under conjugate matching this is written, similar to (2-96), as

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_L + R_r)^2}\right] \tag{2-98}$$

which when multiplied by the incident power density gives the dissipated power of (2-88).

Finally the *capture area* is defined as the equivalent area, which when multiplied by the incident power density leads to the total power captured, collected, or intercepted by the antenna. Under conjugate matching this is written, similar to (2-96), as

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T + R_r + R_L}{(R_L + R_r)^2}\right] \tag{2-99}$$

When (2-99) is multiplied by the incident power density, it leads to the captured power of (2-89). In general, the total capture area is equal to the sum of the other three, or

$$\textit{Capture Area} = \textit{Effective Area} + \textit{Scattering Area} + \textit{Loss Area}$$

This is apparent under conjugate matching using (2-96)–(2-99). However, it holds even under nonconjugate matching conditions.

Now that the equivalent areas have been defined, let us introduce the *aperture efficiency* ϵ_{ap} of an antenna, which is defined as the ratio of the maximum effective area A_{em} of the antenna to its physical area A_p, or

$$\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}} \tag{2-100}$$

For aperture type antennas, such as waveguides, horns, and reflectors, the maximum effective area cannot exceed the physical area but it can equal it ($A_{em} \leq A_p$ or $0 \leq \epsilon_{ap} \leq 1$). Therefore the maximum value of the aperture efficiency cannot exceed unity (100%). For a lossless antenna ($R_L = 0$) the maximum value of the scattering area is also equal to the physical area. Therefore even though the aperture efficiency is greater than 50%, for a lossless antenna under conjugate matching only half of the captured power is delivered to the load and the other half is scattered.

We can also introduce a *partial effective area* of an antenna *for a given polarization* in a given direction, which is defined as "the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction and with a specified polarization differing from the receiving polarization of the antenna."

The effective area of an antenna is not necessarily the same as the physical aperture. It will be shown in later chapters that aperture antennas with uniform amplitude and phase field distributions have maximum effective areas equal to the physical areas; they are smaller for nonuniform field distributions. In addition, the maximum effective area of wire antennas is greater than the physical area (if taken as the area of a cross section of the wire when split lengthwise along its diameter). Thus the wire antenna can capture much more power than is intercepted by its physical size! This should not come as a surprise. If the wire antenna would only capture the power incident on its physical size, it would be almost useless. So electrically, the wire antenna looks much bigger than its physical stature.

To illustrate the concept of effective area, especially as applied to a wire antenna, let us consider an example. In later chapters, we will consider examples of aperture antennas.

Example 2.15

A uniform plane wave is incident upon a very short lossless dipole ($l \ll \lambda$), as shown in Figure 2.29(a). Find the maximum effective area assuming that the radiation resistance of the dipole is $R_r = 80(\pi l/\lambda)^2$, and the incident field is linearly polarized along the axis of the dipole.

Solution: For $R_L = 0$, the maximum effective area of (2-96) reduces to

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right]$$

Since the dipole is very short, the induced current can be assumed to be constant and of uniform phase. The induced voltage is

$$V_T = El$$

where

$V_T =$ induced voltage on the dipole
$E =$ electric field of incident wave
$l =$ length of dipole

For a uniform plane wave, the incident power density can be written as

$$W_i = \frac{E^2}{2\eta}$$

where η is the intrinsic impedance of the medium ($\simeq 120\pi$ ohms for a free-space medium). Thus

$$A_{em} = \frac{(El)^2}{8(E^2/2\eta)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

The above value is only valid for a lossless antenna (the losses of a short dipole are usually significant). If the loss resistance is equal to the radiation resistance ($R_L = R_r$) and the sum of the two is equal to the load (receiver) resistance ($R_T = R_r + R_L = 2R_r$), then the effective area is only one-half of the maximum effective area given above.

Let us now examine the significance of the effective area. From Example 2.15, the maximum effective area of a short dipole with $l \ll \lambda$ was equal to $A_{em} = 0.119\lambda^2$. Typical antennas that fall under this category are dipoles whose lengths are $l \leq \lambda/50$. For the purpose of demonstration, let us assume that $l = \lambda/50$. Because $A_{em} = 0.119\lambda^2 = lw_e = (\lambda/50)w_e$, the maximum effective electrical width of this dipole is $w_e = 5.95\lambda$. Typical physical diameters (widths) of wires used for dipoles may be about $w_p = \lambda/300$. Thus the maximum effective width w_e is about 1,785 times larger than its physical width.

2.16 MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

To derive the relationship between directivity and maximum effective area, the geometrical arrangement of Figure 2.30 is chosen. Antenna 1 is used as a transmitter and 2 as a receiver. The effective areas and directivities of each are designated as A_t, A_r and D_t, D_r. If antenna 1 were isotropic, its radiated power density at a distance R would be

$$W_0 = \frac{P_t}{4\pi R^2} \tag{2-101}$$

where P_t is the total radiated power. Because of the directive properties of the antenna, its actual density is

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2} \tag{2-102}$$

The power collected (received) by the antenna and transferred to the load would be

$$P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2} \tag{2-103}$$

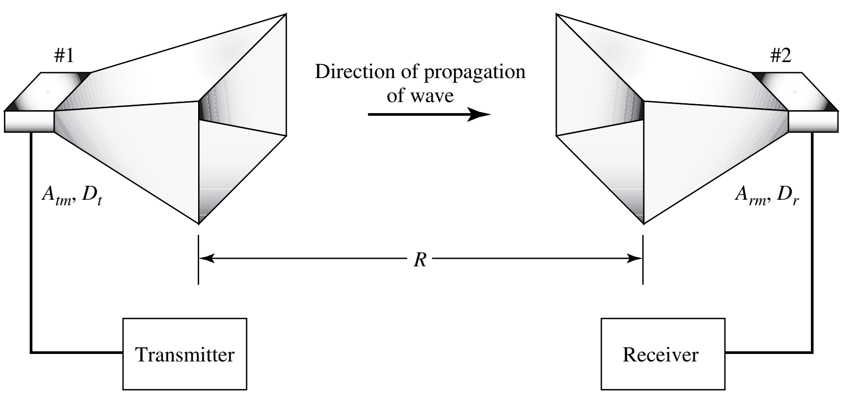


Figure 2.30 Two antennas separated by a distance R.

or

$$D_t A_r = \frac{P_r}{P_t}(4\pi R^2) \tag{2-103a}$$

If antenna 2 is used as a transmitter, 1 as a receiver, and the intervening medium is linear, passive, and isotropic, we can write that

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \tag{2-104}$$

Equating (2-103a) and (2-104) reduces to

$$\frac{D_t}{A_t} = \frac{D_r}{A_r} \tag{2-105}$$

Increasing the directivity of an antenna increases its effective area in direct proportion. Thus, (2-105) can be written as

$$\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}} \tag{2-106}$$

where A_{tm} and A_{rm} (D_{0t} and D_{0r}) are the *maximum* effective areas (directivities) of antennas 1 and 2, respectively.

If antenna 1 is isotropic, then $D_{0t} = 1$ and its maximum effective area can be expressed as

$$A_{tm} = \frac{A_{rm}}{D_{0r}} \tag{2-107}$$

Equation (2-107) states that the maximum effective area of an isotropic source is equal to the ratio of the maximum effective area to the maximum directivity of any other source. For example, let the other antenna be a very short ($l \ll \lambda$) dipole whose effective area ($0.119\lambda^2$ from Example 2.15) and maximum directivity (1.5) are known.

The maximum effective area of the isotropic source is then equal to

$$A_{tm} = \frac{A_{rm}}{D_{0r}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi} \tag{2-108}$$

Using (2-108), we can write (2-107) as

$$A_{rm} = D_{0r} A_{tm} = D_{0r}\left(\frac{\lambda^2}{4\pi}\right) \tag{2-109}$$

In general then, the *maximum effective aperture* (A_{em}) *of any antenna is related to its maximum directivity* (D_0) *by*

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_0} \tag{2-110}$$

Thus, when (2-110) is multiplied by the power density of the incident wave it leads to the maximum power that can be delivered to the load. This assumes that there are no conduction-dielectric losses (radiation efficiency e_{cd} is unity), the antenna is matched to the load (reflection efficiency e_r is unity), and the polarization of the impinging wave matches that of the antenna (polarization loss factor PLF and polarization efficiency p_e are unity). If there are losses associated with an antenna, its maximum effective aperture of (2-110) must be modified to account for conduction-dielectric losses (radiation efficiency). Thus,

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right)D_0 \tag{2-111}$$

The maximum value of (2-111) assumes that the antenna is matched to the load and the incoming wave is polarization-matched to the antenna. If reflection and polarization losses are also included, then the maximum effective area of (2-111) is represented by

$$\boxed{\begin{aligned} A_{em} &= e_0\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 \\ &= e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 \end{aligned}} \tag{2-112}$$

2.17 FRIIS TRANSMISSION EQUATION AND RADAR RANGE EQUATION

The analysis and design of radar and communications systems often require the use of the *Friis Transmission Equation* and the *Radar Range Equation*. Because of the importance [21] of the two equations, a few pages will be devoted for their derivation.

2.17.1 Friis Transmission Equation

The Friis Transmission Equation relates the power received to the power transmitted between two antennas separated by a distance $R > 2D^2/\lambda$, where D is the largest dimension of either antenna. Referring to Figure 2.31, let us assume that the transmitting antenna is initially isotropic. If the input power at the terminals of the transmitting antenna is P_t, then its isotropic power density W_0 at distance R from the antenna is

$$W_0 = e_t\frac{P_t}{4\pi R^2} \tag{2-113}$$

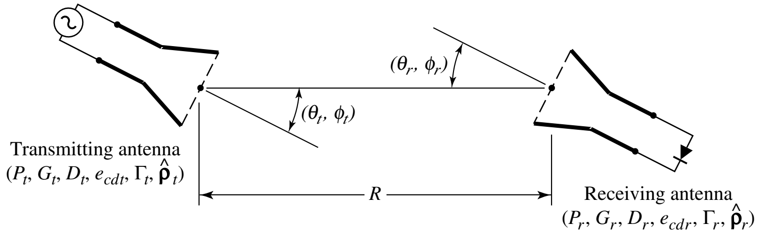


Figure 2.31 Geometrical orientation of transmitting and receiving antennas for Friis transmission equation.

where e_t is the radiation efficiency of the transmitting antenna. For a nonisotropic transmitting antenna, the power density of (2-113) in the direction θ_t, ϕ_t can be written as

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2} \tag{2-114}$$

where $G_t(\theta_t, \phi_t)$ is the gain and $D_t(\theta_t, \phi_t)$ is the directivity of the transmitting antenna in the direction θ_t, ϕ_t. Since the effective area A_r of the receiving antenna is related to its efficiency e_r and directivity D_r by

$$A_r = e_r D_r(\theta_r, \phi_r) \left(\frac{\lambda^2}{4\pi} \right) \tag{2-115}$$

the amount of power P_r collected by the receiving antenna can be written, using (2-114) and (2-115), as

$$P_r = e_r D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi} W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t}{(4\pi R)^2} |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \tag{2-116}$$

or the ratio of the received to the input power as

$$\boxed{\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2}} \tag{2-117}$$

The power received based on (2-117) assumes that the transmitting and receiving antennas are matched to their respective lines or loads (reflection efficiencies are unity) and the polarization of the receiving antenna is polarization-matched to the impinging wave (polarization loss factor and polarization efficiency are unity). If these two factors are also included, then the ratio of the received to the input power of (2-117) is represented by

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2} \tag{2-118}$$

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception, (2-118) reduces to

$$\boxed{\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R} \right)^2 G_{0t} G_{0r}} \tag{2-119}$$

Equations (2-117), (2-118), or (2-119) are known as the *Friis Transmission Equation*, and it relates the power P_r (delivered to the receiver load) to the input power of the

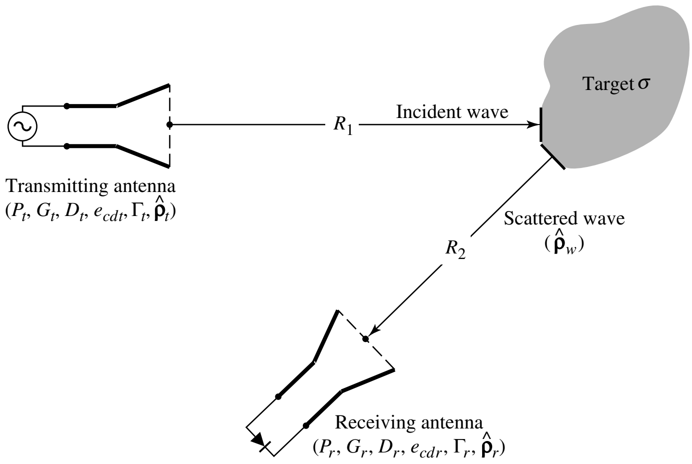


Figure 2.32 Geometrical arrangement of transmitter, target, and receiver for radar range equation.

transmitting antenna P_t. The term $(\lambda/4\pi R)^2$ is called the *free-space loss factor*, and it takes into account the losses due to the spherical spreading of the energy by the antenna.

2.17.2 Radar Range Equation

Now let us assume that the transmitted power is incident upon a target, as shown in Figure 2.32. We now introduce a quantity known as the *radar cross section* or *echo area* (σ) of a target which is defined as *the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target* [13]. In equation form

$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s \tag{2-120}$$

or

$$\boxed{\begin{aligned}\sigma &= \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\mathbf{E}^s|^2}{|\mathbf{E}^i|^2}\right]\\ &= \lim_{R\to\infty}\left[4\pi R^2 \frac{|\mathbf{H}^s|^2}{|\mathbf{H}^i|^2}\right]\end{aligned}} \tag{2-120a}$$

where

σ = radar cross section or echo area (m^2)
R = observation distance from target (m)
W_i = incident power density (W/m^2)
W_s = scattered power density (W/m^2)
$\mathbf{E}^i$ ($\mathbf{E}^s$) = incident (scattered) electric field (V/m)
$\mathbf{H}^i$ ($\mathbf{H}^s$) = incident (scattered) magnetic field (A/m)

Any of the definitions in (2-120a) can be used to derive the radar cross section of any antenna or target. For some polarization one of the definitions based either on the power density, electric field, or magnetic field may simplify the derivation, although all should give the same answers [13].

Using the definition of radar cross section, we can consider that the transmitted power incident upon the target is initially captured and then it is reradiated isotropically, insofar as the receiver is concerned. The amount of captured power P_c is obtained by multiplying the incident power density of (2-114) by the radar cross section σ, or

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^2} = e_t \sigma \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R_1^2} \tag{2-121}$$

The power captured by the target is reradiated isotropically, and the scattered power density can be written as

$$W_s = \frac{P_c}{4\pi R_2^2} = e_{cdt} \sigma \frac{P_t D_t(\theta_t, \phi_t)}{(4\pi R_1 R_2)^2} \tag{2-122}$$

The amount of power delivered to the receiver load is given by

$$P_r = A_r W_s = e_{cdt} e_{cdr} \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \tag{2-123}$$

where A_r is the effective area of the receiving antenna as defined by (2-115).

Equation (2-123) can be written as the ratio of the received power to the input power, or

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2} \tag{2-124}$$

Expression (2-124) is used to relate the received power to the input power, and it takes into account only conduction-dielectric losses (radiation efficiency) of the transmitting and receiving antennas. It does not include reflection losses (reflection efficiency) and polarization losses (polarization loss factor or polarization efficiency). If these two losses are also included, then (2-124) must be expressed as

$$\boxed{\begin{aligned} \frac{P_r}{P_t} &= e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \\ &\quad \times \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_r|^2 \end{aligned}} \tag{2-125}$$

where

$\hat{\boldsymbol{\rho}}_w$ = polarization unit vector of the scattered waves
$\hat{\boldsymbol{\rho}}_r$ = polarization unit vector of the receiving antenna

For polarization-matched antennas aligned for maximum directional radiation and reception, (2-125) reduces to

$$\boxed{\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left[\frac{\lambda}{4\pi R_1 R_2} \right]^2} \tag{2-126}$$

Equation (2-124), or (2-125) or (2-126) is known as the *Radar Range Equation*. It relates the power P_r (delivered to the receiver load) to the input power P_t transmitted by an antenna, after it has been scattered by a target with a radar cross section (echo area) of σ.

Example 2.16

Two *lossless* *X*-band (8.2–12.4 GHz) horn antennas are separated by a distance of 100λ. The reflection coefficients at the terminals of the transmitting and receiving antennas are 0.1 and 0.2, respectively. The maximum directivities of the transmitting and receiving antennas (over isotropic) are 16 dB and 20 dB, respectively. Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2W, and the antennas are aligned for maximum radiation between them and are polarization-matched, find the power delivered to the load of the receiver.

Solution: For this problem

$$\begin{aligned} e_{cdt} &= e_{cdr} = 1 \text{ because antennas are lossless.} \\ |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 &= 1 \text{ because antennas are polarization-matched} \\ \left.\begin{aligned} D_t &= D_{0t} \\ D_r &= D_{0r} \end{aligned}\right\} &\text{ because antennas are aligned for maximum radiation between them} \\ D_{0t} &= 16 \text{ dB} \Rightarrow 39.81 \text{ (dimensionless)} \\ D_{0r} &= 20 \text{ dB} \Rightarrow 100 \text{ (dimensionless)} \end{aligned}$$

Using (2-118), we can write

$$\begin{aligned} P_r &= [1 - (0.1)^2][1 - (0.2)^2][\lambda/4\pi(100\lambda)]^2(39.81)(100)(2) \\ &= 4.777 \text{ mW} \end{aligned}$$

2.17.3 Antenna Radar Cross Section

The radar cross section, usually referred to as RCS, is a far-field parameter, which is used to characterize the scattering properties of a radar target. For a target, there is *monostatic* or *backscattering RCS* when the transmitter and receiver of Figure 2.32 are at the same location, and a *bistatic RCS* when the transmitter and receiver are not at the same location. In designing low-observable or low-profile (stealth) targets,

TABLE 2.2 RCS of Some Typical Targets

	Typical RCSs [22]	
Object	**RCS** (m^2)	**RCS** (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

it is the parameter that you attempt to minimize. For complex targets (such as aircraft, spacecraft, missiles, ships, tanks, automobiles) it is a complex parameter to derive. In general, the RCS of a target is a function of the polarization of the incident wave, the angle of incidence, the angle of observation, the geometry of the target, the electrical properties of the target, and the frequency of operation. The units of RCS of three-dimensional targets are meters squared (m^2) or for normalized values decibels per squared meter (dBsm) or RCS per squared wavelength in decibels (RCS/λ^2 in dB). Representative values of some typical targets are shown in Table 2.2 [22]. Although the frequency was not stated [22], these numbers could be representative at X-band.

The RCS of a target can be controlled using primarily two basic methods: *shaping* and the use of *materials*. Shaping is used to attempt to direct the scattered energy toward directions other than the desired. However, for many targets shaping has to be compromised in order to meet other requirements, such as aerodynamic specifications for flying targets. Materials is used to trap the incident energy within the target and to dissipate part of the energy as heat or to direct it toward directions other than the desired. Usually both methods, shaping and materials, are used together in order to optimize the performance of a radar target. One of the "golden rules" to observe in order to achieve low RCS is to "*round corners, avoid flat and concave surfaces, and use material treatment in flare spots*."

There are many methods of analysis to predict the RCS of a target [13], [22]–[33]. Some of them are full-wave methods, others are designated as asymptotic methods, either low-frequency or high-frequency, and some are considered as numerical methods. The methods of analysis are often contingent upon the shape, size, and material composition of the target. Some targets, because of their geometrical complexity, are often simplified and are decomposed into a number of basic shapes (such as strips, plates, cylinders, cones, wedges) which when put together represent a very good replica of the actual target. This has been used extensively and proved to be a very good approach. The topic is very extensive to be treated here in any detail, and the reader is referred to the literature [13], [22]–[33]. There is a plethora of references but because of space limitations, only a limited number is included here to get the reader started on the subject.

Antennas individually are radar targets which many exhibit large radar cross section. In many applications, antennas are mounted on the surface of other complex targets (such as aircraft, spacecraft, satellites, missiles, automobiles), and become part of the overall radar target. In such configurations, many antennas, especially aperture types (such as waveguides, horns) become large contributors to the total RCS, monostatic or bistatic, of the target. Therefore in designing low-observable targets, the antenna type, location and contributions become an important consideration of the overall design.

The scattering and transmitting (radiation) characteristics of an antenna are related [34]–[36]. There are various methods which can be used to analyze the fields scattered by an antenna. The presentation here parallels that in [23], [37]–[40]. In general the electric field scattered by an antenna with a load impedance Z_L can be expressed by

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t}\frac{Z_L}{Z_L + Z_A}\mathbf{E}^t \tag{2-127}$$

where

$\mathbf{E}^s(Z_L)$ = electric field scattered by antenna with a load Z_L
$\mathbf{E}^s(0)$ = electric field scattered by short-circuited antenna ($Z_L = 0$)
I_s = short-circuited current induced by the incident field on the antenna with $Z_L = 0$
I_t = antenna current in transmitting mode
$Z_A = R_A + jX_A$ = antenna input impedance
$\mathbf{E}^t$ = electric field radiated by the antenna in transmitting mode

By defining an antenna reflection coefficient of

$$\Gamma_A = \frac{Z_L - Z_A}{Z_L + Z_A} \tag{2-128}$$

the scattered field of (2-127) can be written as

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(0) - \frac{I_s}{I_t}\frac{1}{2}(1 + \mathbf{\Gamma}_A)\mathbf{E}^t \tag{2-129}$$

Therefore according to (2-129) the scattered field by an antenna with a load Z_L is equal to the scattered field when the antenna is short-circuited ($Z_L = 0$) minus a term related to the reflection coefficient and the field transmitted by the antenna.

Green has expressed the field scattered by an antenna terminated with a load Z_L in a more convenient form which allows it to be separated into the *structural* and *antenna mode* scattering terms [23], [37]–[40]. This is accomplished by assuming that the antenna is loaded with a conjugate-matched impedance ($Z_L = Z_A^*$). Doing this generates using

(2-127) another equation for the field scattered by the antenna with a load $Z_L = Z_A^*$. When this new equation is subtracted from (2-127) it eliminates the short-circuited scattered field, and we can write that the field scattered by the antenna with a load Z_L is

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_s}{I_t}\frac{\boldsymbol{\Gamma}^* Z_A}{2R_A}\mathbf{E}^t \tag{2-130}$$

$$\boldsymbol{\Gamma}^* = \frac{Z_L - Z_A^*}{Z_L + Z_A^*} \tag{2-130a}$$

where

$\mathbf{E}^s(Z_L)$ = electric field scattered by the antenna with load Z_L
$\mathbf{E}^s(Z_A^*)$ = electric field scattered by the antenna with a conjugate-matched load
$I(Z_A^*)$ = current induced by the incident wave at the terminals matched with a conjugate impedance load
Γ^* = conjugate-matched reflection coefficient
Z_L = load impedance attached to antenna terminals

For the short-circuited case and the conjugate-matched transmitting (radiating) case, the product of their currents and antenna impedance are related by [34]

$$I_s Z_A = I_m^*(Z_A + Z_A^*) = 2R_A I_m^* \tag{2-131}$$

where I_m^* is the scattering current when the antenna is conjugate-matched ($Z_L = Z_A^*$). Substituting (2-131) into (2-130) for I_s reduces (2-130) to

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A^*) - \frac{I_m^*}{I_t}\Gamma^*\mathbf{E}^t \tag{2-132}$$

It can also be shown that if the antenna is matched with a load Z_A (instead of Z_A^*), then (2-132) can be written as

$$\mathbf{E}^s(Z_L) = \mathbf{E}^s(Z_A) - \frac{I_m}{I_t}\Gamma_A\mathbf{E}^t \tag{2-133}$$

Therefore the field scattered by an antenna loaded with an impedance Z_L is related to the field radiated by the antenna in the transmitting mode in three different ways, as shown by (2-129), (2-132), and (2-133). According to (2-129) the field scattered by an antenna when it is loaded with an impedance Z_L is equal to the field scattered by the antenna when it is short-circuited ($Z_L = 0$) minus a term related to the antenna reflection coefficient and the field transmitted by the antenna. In addition, according to (2-132), the field scattered by an antenna when it is terminated with an impedance Z_L is equal to the field scattered by the antenna when it is conjugate-matched with an impedance Z_A^* minus the field transmitted (radiated) times the conjugate reflection coefficient. The second term is weighted by the two currents. Alternatively, according to (2-133), the field scattered by the antenna when it is terminated with an impedance Z_L is equal to the field scattered by the antenna when it is matched with an impedance Z_A minus the field transmitted (radiated) times the reflection coefficient weighted by the two currents.

In (2-132) the first term consists of the *structural* scattering term and the second of the *antenna mode* scattering term. The *structural* scattering term is introduced by the currents induced on the surface of the antenna by the incident field when the antenna is conjugate-matched, and it is independent of the load impedance. The *antenna mode* scattering term is only a function of the radiation characteristics of the antenna, and its scattering pattern is the square of the antenna radiation pattern. The antenna mode depends on the power absorbed in the load of a lossless antenna and the power which is radiated by the antenna due to a load mismatch. This term vanishes when the antenna is conjugate-matched.

From the scattered field expression of (2-129), it can be shown that the total radar cross section of the antenna terminated with a load Z_L can be written as [40]

$$\sigma = |\sqrt{\sigma^s} - (1 + \Gamma_A)\sqrt{\sigma^a}e^{j\phi_r}|^2 \tag{2-134}$$

where

σ = total RCS with antenna terminated with Z_L
σ^s = RCS due to structural term
σ^a = RCS due to antenna mode term
ϕ_r = relative phase between the structural and antenna mode terms

If the antenna is short-circuited ($\Gamma_A = -1$), then according to (2-134)

$$\sigma_{\text{short}} = \sigma^s \tag{2-135}$$

If the antenna is open-circuited ($\Gamma_A = +1$), then according to (2-134)

$$\sigma_{\text{open}} = |\sqrt{\sigma^s} - 2\sqrt{\sigma^a}e^{j\phi_r}|^2 = \sigma_{\text{residual}} \tag{2-136}$$

Lastly, if the antenna is matched $Z_L = Z_A(\Gamma_A = 0)$, then according to (2-134)

$$\sigma_{\text{match}} = |\sqrt{\sigma^s} - \sqrt{\sigma^a}e^{j\phi_r}|^2 \tag{2-137}$$

Therefore under matched conditions, according to (2-137), the range of values (minimum to maximum) of the radar cross section is

$$|\sigma^s - \sigma^a| \leq \sigma \leq |\sigma^s + \sigma^a| \tag{2-138}$$

The minimum value occurring when the two RCSs are in phase while the maximum occurs when they are out of phase.

Example 2.17

The structural RCS of a resonant wire dipole is in phase and in magnitude slightly greater than four times that of the antenna mode. Relate the short-circuited, open-circuited, and matched RCSs to that of the antenna mode.

Solution: Using (2-135)

$$\sigma_{\text{short}} = 4\sigma_{\text{antenna}}$$

Using (2-136)

$$\sigma_{\text{open}} = 2\sigma_{\text{antenna}}(0) = 0 \text{ or very small}$$

The matched value is obtained using (2-137), or

$$\sigma_{\text{match}} = \sigma_{\text{antenna}}$$

To produce a zero RCS, (2-134) must vanish. This is accomplished if

$$Re(\Gamma_A) = -1 + \cos\phi_r \sqrt{\sigma^s/\sigma^a} \tag{2-139a}$$

$$Im(\Gamma_A) = -\sin\phi_r \sqrt{\sigma^s/\sigma^a} \tag{2-139b}$$

Assuming positive values of resistances, the real value of Γ_A cannot be greater than unity. Therefore there are some cases where the RCS cannot be reduced to zero by choosing Z_L. Because Z_A can be complex, there is no limit on the imaginary part of Γ_A.

In general, the structural and antenna mode scattering terms are very difficult to predict and usually require that the antenna is solved as a boundary-value problem. However, these two terms have been obtained experimentally utilizing the Smith chart [37]–[39].

For a monostatic system the receiving and transmitting antennas are collocated. In addition, if the antennas are identical ($G_{0r} = G_{0t} = G_0$) and are polarization-matched ($P_r = P_t = 1$), the total radar cross section of the antenna for backscattering can be written as

$$\sigma = \frac{\lambda_0^2}{4\pi} G_0^2 |A - \Gamma^*|^2 \tag{2-140}$$

where A is a complex parameter independent of the load.

If the antenna is a thin dipole, then $A \simeq 1$ and (2-140) reduces to

$$\begin{aligned}\sigma &\simeq \frac{\lambda_0^2}{4\pi} G_0^2 |1 - \Gamma^*|^2 = \frac{\lambda_0^2}{4\pi} G_0^2 \left|1 - \frac{Z_L - Z_A^*}{Z_L + Z_A}\right| \\ &= \frac{\lambda_0^2}{4\pi} G_0^2 \left|\frac{2R_A}{Z_L + Z_A}\right|^2\end{aligned} \tag{2-141}$$

If in addition we assume that the dipole length is $l = \lambda_0/2$ and is short-circuited ($Z_L = 0$), then the normalized radar cross section of (2-141) is equal to

$$\frac{\sigma}{\lambda_0^2} \simeq \frac{G_0^2}{\pi} = \frac{(1.643)^2}{\pi} = 0.8593 \simeq 0.86 \tag{2-142}$$

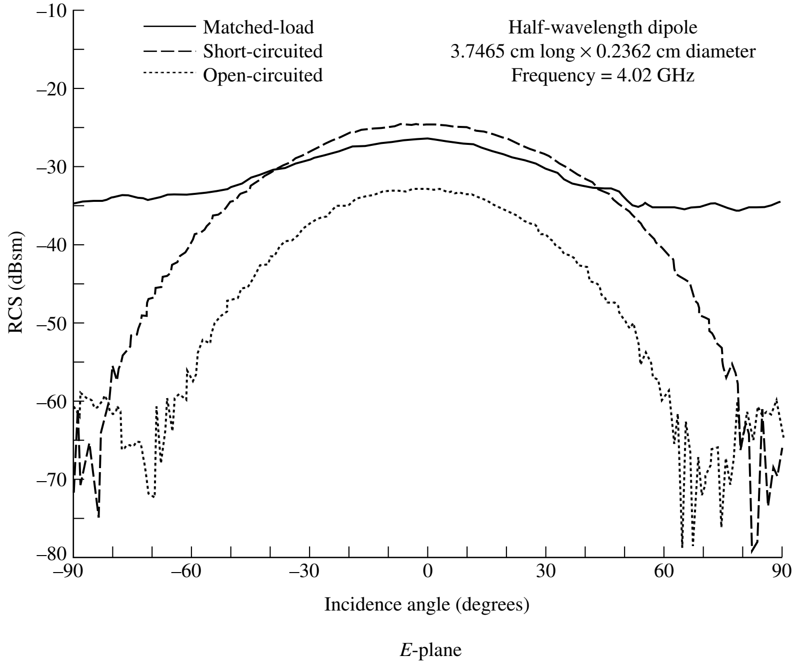


Figure 2.33 E-plane monostatic RCS ($\sigma_{\theta\theta}$) versus incidence angle for a half-wavelength dipole.

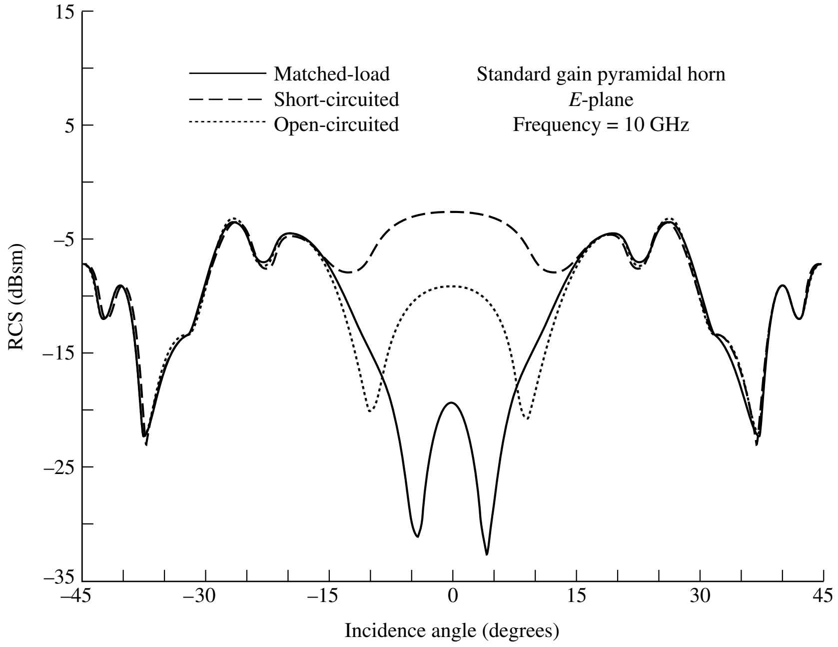
which agrees with experimental corresponding maximum monostatic value of Figure 2.33 and those reported in the literature [41], [42].

Shown in Figure 2.33 is the measured E-plane monostatic RCS of a half-wavelength dipole when it is matched to a load, short-circuited (straight wire) and open-circuited (gap at the feed). The aspect angle is measured from the normal to the wire. As expected, the RCS is a function of the observation (aspect) angle. Also it is apparent that there are appreciable differences between the three responses. For the short-circuited case, the maximum value is approximately −24 dBsm which closely agrees with the computed value of −22.5 dBsm using (2-142). Similar responses for the monostatic RCS of a pyramidal horn are shown in Figure 2.34(a) for the E-plane and in Figure 2.34(b) for the H-plane. The antenna is a commercial X-band (8.2-12.4 GHz) 20-dB standard gain horn with aperture dimension of 9.2 cm by 12.4 cm. The length of the horn is 25.6 cm. As for the dipole, there are differences between the three responses for each plane. It is seen that the short-circuited response exhibits the largest return.

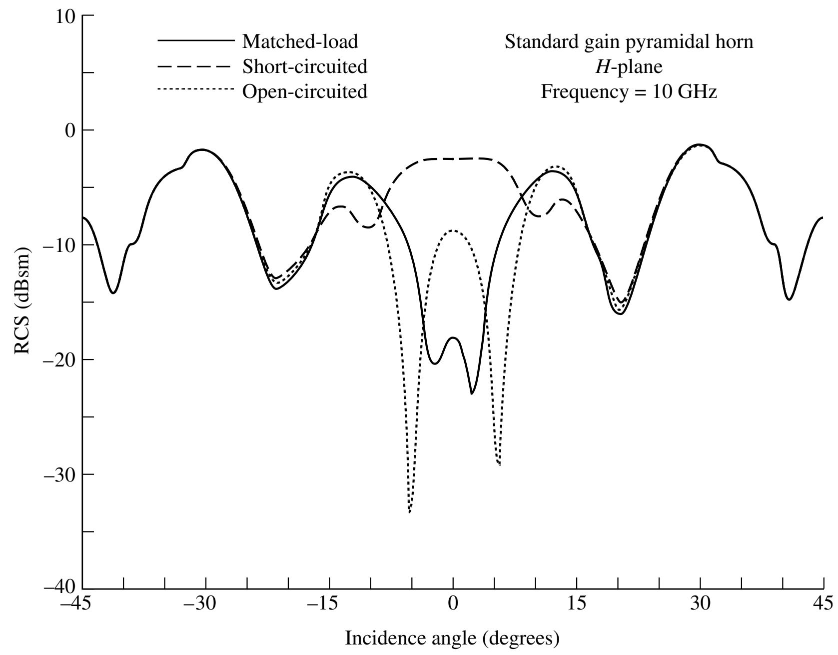
Antenna RCS from model measurements [43] and microstrip patches [44], [45] have been reported.

2.18 ANTENNA TEMPERATURE

Every object with a physical temperature above absolute zero (0 K = −273°C) radiates energy [6]. The amount of energy radiated is usually represented by an equivalent



(a) E-plane ($\sigma_{\theta\theta}$)



(b) H-plane ($\sigma_{\phi\phi}$)

Figure 2.34 E- and H-plane monostatic RCS versus incidence angle for a pyramidal horn antenna.

temperature T_B, better known as brightness temperature, and it is defined as

$$T_B(\theta, \phi) = \epsilon(\theta, \phi)T_m = (1 - |\Gamma|^2)T_m \tag{2-143}$$

where

T_B = brightness temperature (equivalent temperature; K)
ϵ = emissivity (dimensionless)
T_m = molecular (physical) temperature (K)
$\Gamma(\theta, \phi)$ = reflection coefficient of the surface for the polarization of the wave

Since the values of emissivity are $0 \leq \epsilon \leq 1$, the maximum value the brightness temperature can achieve is equal to the molecular temperature. Usually the emissivity is a function of the frequency of operation, polarization of the emitted energy, and molecular structure of the object. Some of the better natural emitters of energy at microwave frequencies are (a) the ground with equivalent temperature of about 300 K and (b) the sky with equivalent temperature of about 5 K when looking toward zenith and about 100–150 K toward the horizon.

The brightness temperature emitted by the different sources is intercepted by antennas, and it appears at their terminals as an antenna temperature. The temperature appearing at the terminals of an antenna is that given by (2-143), after it is weighted by the gain pattern of the antenna. In equation form, this can be written as

$$T_A = \frac{\displaystyle\int_0^{2\pi}\int_0^{\pi} T_B(\theta, \phi)G(\theta, \phi)\sin\theta\, d\theta\, d\phi}{\displaystyle\int_0^{2\pi}\int_0^{\pi} G(\theta, \phi)\sin\theta\, d\theta\, d\phi} \tag{2-144}$$

where

T_A = antenna temperature (effective noise temperature of the antenna radiation resistance; K)
$G(\theta, \phi)$ = gain (power) pattern of the antenna

Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by

$$P_r = kT_A\Delta f \tag{2-145}$$

where

P_r = antenna noise power (W)
k = Boltzmann's constant (1.38×10^{-23} J/K)
T_A = antenna temperature (K)
Δf = bandwidth (Hz)

If the antenna and transmission line are maintained at certain physical temperatures, and the transmission line between the antenna and receiver is lossy, the antenna temperature T_A as seen by the receiver through (2-145) must be modified to include the other contributions and the line losses. If the antenna itself is maintained at a certain

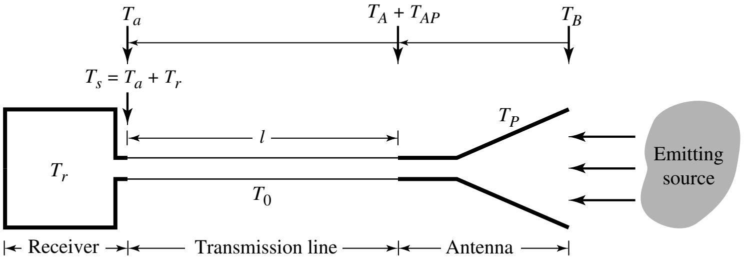


Figure 2.35 Antenna, transmission line, and receiver arrangement for system noise power calculation.

physical temperature T_p and a transmission line of length l, constant physical temperature T_0 throughout its length, and uniform attenuation of α (Np/unit length) is used to connect an antenna to a receiver, as shown in Figure 2.35, the effective antenna temperature at the receiver terminals is given by

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0(1 - e^{-2\alpha l}) \tag{2-146}$$

where

$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_p \tag{2-146a}$$

T_a = antenna temperature at the receiver terminals (K)
T_A = antenna noise temperature at the antenna terminals (2-144) (K)
T_{AP} = antenna temperature at the antenna terminals due to physical temperature (2-146a) (K)
T_p = antenna physical temperature (K)
α = attenuation coefficient of transmission line (Np/m)
e_A = thermal efficiency of antenna (dimensionless)
l = length of transmission line (m)
T_0 = physical temperature of the transmission line (K)

The antenna noise power of (2-145) must also be modified and written as

$$P_r = kT_a \Delta f \tag{2-147}$$

where T_a is the antenna temperature at the receiver input as given by (2-146).

If the receiver itself has a certain noise temperature T_r (due to thermal noise in the receiver components), the *system noise power at the receiver terminals* is given by

$$P_s = k(T_a + T_r)\Delta f = kT_s \Delta f \tag{2-148}$$

where

P_s = system noise power (at receiver terminals)
T_a = antenna noise temperature (at receiver terminals)
T_r = receiver noise temperature (at receiver terminals)
$T_s = T_a + T_r$ = effective system noise temperature (at receiver terminals)

A graphical relation of all the parameters is shown in Figure 2.35. The effective system noise temperature T_s of radio astronomy antennas and receivers varies from very few degrees (typically $\simeq 10$ K) to thousands of Kelvins depending upon the type of antenna, receiver, and frequency of operation. Antenna temperature changes at the antenna terminals, due to variations in the target emissions, may be as small as a fraction of one degree. To detect such changes, the receiver must be very sensitive and be able to differentiate changes of a fraction of a degree.

Example 2.18

The effective antenna temperature of a target at the input terminals of the antenna is 150 K. Assuming that the antenna is maintained at a thermal temperature of 300 K and has a thermal efficiency of 99% and it is connected to a receiver through an X-band (8.2–12.4 GHz) rectangular waveguide of 10 m (loss of waveguide = 0.13 dB/m) and at a temperature of 300 K, find the effective antenna temperature at the receiver terminals.

Solution: We first convert the attenuation coefficient from dB to Np by $\alpha(\text{dB/m}) = 20(\log_{10} e)\alpha(\text{Np/m}) = 20(0.434)\alpha(\text{Np/m}) = 8.68\alpha(\text{Np/m})$. Thus $\alpha(\text{Np/m}) = \alpha(\text{dB/m})/8.68 = 0.13/8.68 = 0.0149$. The effective antenna temperature at the receiver terminals can be written, using (2-146a) and (2-146), as

$$T_{AP} = 300\left(\frac{1}{0.99} - 1\right) = 3.03$$

$$T_a = 150e^{-0.149(2)} + 3.03e^{-0.149(2)} + 300[1 - e^{-0.149(2)}]$$

$$= 111.345 + 2.249 + 77.31 = 190.904 \text{ K}$$

The results of the above example illustrate that the antenna temperature at the input terminals of the antenna and at the terminals of the receiver can differ by quite a few degrees. For a smaller transmission line or a transmission line with much smaller losses, the difference can be reduced appreciably and can be as small as a fraction of a degree.

A summary of the pertinent parameters and associated formulas and equation numbers for this chapter are listed in Table 2.3.

2.19 MULTIMEDIA

In the CD that is part of the book, the following multimedia resources are included for the review, understanding, and visualization of the material of this chapter:

a. **Java**-based **interactive questionnaire**, with answers.
b. **Java**-based **applet** for computing and displaying graphically the directivity of an antenna.
c. **Matlab** and **Fortran** computer program, designated ***Directivity***, for computing the directivity of an antenna. A description of this program is in the READ ME file of the attached CD.

d. **Matlab** plotting computer programs:
   - **2-D Polar** (designated as ***Polar***). This program can be used to plot the two-dimensional patterns, in both polar and semipolar form (*in linear or dB scale*), of an antenna.
   - **3-D Spherical**. This program (designated as ***Spherical***) can be used to plot the three-dimensional pattern (*in linear or dB scale*) of an antenna in spherical form.

   A description of these programs is in the corresponding READ ME files of the attached CD.
e. **Power Point (PPT)** viewgraphs, in multicolor.

TABLE 2.3 Summary of Important Parameters and Associated Formulas and Equation Numbers

Parameter	Formula	Equation Number
Infinitesimal area of sphere	$dA = r^2 \sin\theta \, d\theta \, d\phi$	(2-1)
Elemental solid angle of sphere	$d\Omega = \sin\theta \, d\theta \, d\phi$	(2-2)
Average power density	$\mathbf{W}_{av} = \frac{1}{2}\text{Re}[\mathbf{E} \times \mathbf{H}^*]$	(2-8)
Radiated power/average radiated power	$P_{rad} = P_{av} = \oiint_S \mathbf{W}_{av} \cdot d\mathbf{s} = \frac{1}{2} \oiint_S \text{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot d\mathbf{s}$	(2-9)
Radiation density of isotropic radiator	$W_0 = \frac{P_{rad}}{4\pi r^2}$	(2-11)
Radiation intensity (far field)	$U = r^2 W_{rad} = B_0 F(\theta, \phi) \simeq \frac{r^2}{2\eta} \times \left[\lvert E_\theta(r, \theta, \phi)\rvert^2 + \lvert E_\phi(r, \theta, \phi)\rvert^2\right]$	(2-12), (2-12a)
Directivity $D(\theta, \phi)$	$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} = \frac{4\pi}{\Omega_A}$	(2-16), (2-23)
Beam solid angle Ω_A	$\Omega_A = \int_0^{2\pi} \int_0^{\pi} F_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$ $F_n(\theta, \phi) = \frac{F(\theta, \phi)}{\lvert F(\theta, \phi)\rvert_{\max}}$	(2-24) (2-25)

(*continued overleaf*)

TABLE 2.3 (*continued*)

Parameter	Formula	Equation Number
Maximum directivity D_0	$D_{\max} = D_0 = \dfrac{U_{\max}}{U_0} = \dfrac{4\pi U_{\max}}{P_{rad}}$	(2-16a)
Partial directivities D_θ, D_ϕ	$D_0 = D_\theta + D_\phi$ $D_\theta = \dfrac{4\pi U_\theta}{P_{rad}} = \dfrac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi}$ $D_\phi = \dfrac{4\pi U_\phi}{P_{rad}} = \dfrac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$	(2-17) (2-17a) (2-17b)
Approximate maximum directivity (*one main lobe pattern*)	$D_0 \simeq \dfrac{4\pi}{\Theta_{1r}\Theta_{2r}} = \dfrac{41{,}253}{\Theta_{1d}\Theta_{2d}}$ (Kraus) $D_0 \simeq \dfrac{32\ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \dfrac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2} = \dfrac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2}$ (Tai-Pereira)	(2-26), (2-27) (2-30), (2-30a), (2-30b)
Approximate maximum directivity (*omnidirectional pattern*)	$D_0 \simeq \dfrac{101}{\text{HPBW(degrees)} - 0.0027[\text{HPBW(degrees)}]^2}$ (McDonald) $D_0 \simeq -172.4 + 191\sqrt{0.818 + \dfrac{1}{\text{HPBW(degrees)}}}$ (Pozar)	(2-33a) (2-33b)
Gain $G(\theta, \phi)$	$G = \dfrac{4\pi U(\theta,\phi)}{P_{in}} = e_{cd}\left[\dfrac{4\pi U(\theta,\phi)}{P_{rad}}\right] = e_{cd} D(\theta,\phi)$ $P_{rad} = e_{cd} P_{in}$	(2-46), (2-47), (2-49)
Antenna radiation efficiency e_{cd}	$e_{cd} = \dfrac{R_r}{R_r + R_L}$	(2-90)
Loss resistance R_L (*straight wire/uniform current*)	$R_L = R_{hf} = \dfrac{l}{P}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$	(2-90b)
Loss resistance R_L (*straight wire/$\lambda/2$ dipole*)	$R_L = \dfrac{l}{2P}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$	

TABLE 2.3 (*continued*)

Parameter	Formula	Equation Number
Maximum gain G_0	$G_0 = e_{cd} D_{\max} = e_{cd} D_0$	(2-49a)
Partial gains G_θ, G_ϕ	$G_0 = G_\theta + G_\phi$ $G_\theta = \dfrac{4\pi U_\theta}{P_{in}}, \quad G_\phi = \dfrac{4\pi U_\phi}{P_{in}}$	(2-50) (2-50a), (2-50b)
Absolute gain G_{abs}	$G_{abs} = e_r G(\theta,\phi) = e_r e_{cd} D(\theta,\phi) = (1-\|\Gamma\|^2) e_{cd} D(\theta,\phi)$ $= e_0 D(\theta,\phi)$	(2-49a) (2-49b)
Total antenna efficiency e_0	$e_0 = e_r e_c e_d = e_r e_{cd} = (1-\|\Gamma\|^2) e_{cd}$	(2-52)
Reflection efficiency e_r	$e_r = (1-\|\Gamma\|^2)$	(2-45)
Beam efficiency BE	$\mathrm{BE} = \dfrac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}$	(2-54)
Polarization loss factor (PLF)	$\mathrm{PLF} = \|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a\|^2$	(2-71)
Vector effective length $\boldsymbol{\ell}_e(\theta,\phi)$	$\boldsymbol{\ell}_e(\theta,\phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta,\phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta,\phi)$	(2-91)
Polarization efficiency p_e	$p_e = \dfrac{\|\boldsymbol{\ell}_e \cdot \mathbf{E}^{inc}\|^2}{\|\boldsymbol{\ell}_e\|^2 \|\mathbf{E}^{inc}\|^2}$	(2-71a)
Antenna impedance Z_A	$Z_A = R_A + jX_A = (R_r + R_L) + jX_A$	(2-72), (2-73)
Maximum effective area A_{em}	$A_{em} = \dfrac{\|V_T\|^2}{8W_i}\left[\dfrac{1}{R_r + R_L}\right] = e_{cd}\left(\dfrac{\lambda^2}{4\pi}\right) D_0 \|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a\|^2$ $= \left(\dfrac{\lambda^2}{4\pi}\right) G_0 \|\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a\|^2$	(2-96), (2-111), (2-112)
Aperture efficiency ε_{ap}	$\varepsilon_{ap} = \dfrac{A_{em}}{A_p} = \dfrac{\text{maximum effective area}}{\text{physical area}}$	(2-100)

(*continued overleaf*)

TABLE 2.3 (*continued*)

Parameter	**Formula**	**Equation Number**
Friis transmission equation	$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r} \lvert\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r\rvert^2$	(2-118), (2-119)
Radar range equation	$\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left[\frac{\lambda}{4\pi R_1 R_2}\right]^2 \lvert\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_r\rvert^2$	(2-125), (2-126)
Radar cross section (RCS)	$\sigma = \lim_{R\to\infty} \left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty} \left[4\pi R^2 \frac{\lvert\mathbf{E}^s\rvert^2}{\lvert\mathbf{E}^i\rvert^2}\right] = \lim_{R\to\infty} \left[4\pi R^2 \frac{\lvert\mathbf{H}^s\rvert^2}{\lvert\mathbf{H}^i\rvert^2}\right]$	(2-120a)
Brightness temperature $T_B(\theta, \phi)$	$T_B(\theta, \phi) = \varepsilon(\theta, \phi) T_m = (1 - \lvert\Gamma\rvert^2) T_m$	(2-144)
Antenna temperature T_A	$T_A = \frac{\int_0^{2\pi} \int_0^{\pi} T_B(\theta, \phi) G(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_{0\pi}^{2\pi} \int_0^{\pi} G(\theta, \phi) \sin\theta \, d\theta \, d\phi}$	(2-145)

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PROBLEMS

2.1. An antenna has a beam solid angle that is equivalent to a *trapezoidal* patch (*patch with 4 sides, 2 of which are parallel to each other*) on the surface of a sphere of radius r. The angular space of the patch on the surface of the sphere extends between $\pi/6 \leq \theta \leq \pi/3 (30^\circ \leq \theta \leq 60^\circ)$ in latitude and $\pi/4 \leq \phi \leq \pi/3 (45^\circ \leq \phi \leq 60^\circ)$ in longitude. Find the following:

(a) Equivalent *beam solid angle* [which is equal to number of *square radians/steradians* **or** $(degrees)^2$] of the patch [*in square radians/steradians* ***and*** *in* $(degrees)^2$].

- Exact.
- Approximate using $\Omega_A = \Delta\Theta \cdot \Delta\Phi = (\theta_2 - \theta_1) \cdot (\phi_2 - \phi_1)$. Compare with the exact.

(b) Corresponding antenna *maximum directivities* of part *a* (*dimensionless* ***and*** *in dB*).

2.2. Derive (2-7) given the definitions of (2-5) and (2-6)

2.3. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the

(a) power density (W_{rad})

(b) power radiated (P_{rad})

2.4. Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), *in radians and degrees*, for the following normalized radiation intensities:

$$\left.\begin{array}{ll} \text{(a) } U(\theta) = \cos\theta & \text{(b) } U(\theta) = \cos^2\theta \\ \text{(c) } U(\theta) = \cos(2\theta) & \text{(d) } U(\theta) = \cos^2(2\theta) \\ \text{(e) } U(\theta) = \cos(3\theta) & \text{(f) } U(\theta) = \cos^2(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

2.5. Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), *in radians and degrees*, for the following normalized radiation intensities:

$$\left.\begin{array}{l} \text{(a) } U(\theta) = \cos\theta\cos(2\theta) \\ \text{(b) } U(\theta) = \cos^2\theta\cos^2(2\theta) \\ \text{(c) } U(\theta) = \cos(\theta)\cos(3\theta) \\ \text{(d) } U(\theta) = \cos^2(\theta)\cos^2(3\theta) \\ \text{(e) } U(\theta) = \cos(2\theta)\cos(3\theta) \\ \text{(f) } U(\theta) = \cos^2(2\theta)\cos^2(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

2.6. The maximum radiation intensity of a 90% efficiency antenna is 200 mW/unit solid angle. Find the directivity and gain (dimensionless and in dB) when the

(a) input power is 125.66 mW

(b) radiated power is 125.66 mW

2.7. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$$\left.\begin{array}{l} \text{(a) } U = B_o\cos^2\theta \\ \text{(b) } U = B_o\cos^3\theta \end{array}\right\} \quad \begin{array}{l} \text{(watts/unit solid angle)} \\ (0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi) \end{array}$$

For each, find the

(a) maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.

(b) exact and approximate beam solid angle Ω_A.

(c) directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) gain, exact and approximate, of the antenna (dimensionless and in dB).

2.8. You are an antenna engineer and you are asked to design a high directivity/gain antenna for a space-borne communication system operating at 10 GHz. The specifications of the antenna are such that its pattern consists basically of *one major lobe* and, for simplicity, *no minor lobes* (*if there are any minor lobes they are of such very low intensity and you can assume they are negligible/zero*). Also it is desired that the pattern is symmetrical in the azimuthal plane. In order to meet the desired objectives, the main lobe of the pattern should have a *half-power beamwidth* of *10 degrees*. In order to expedite the design, it is assumed that the major lobe of the normalized radiation intensity of the antenna is approximated by

$$U(\theta, \phi) = \cos^n(\theta)$$

and it exists only in the upper hemisphere $(0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi)$. Determine the:

(a) Value of n (*not necessarily an integer*) to meet the specifications of the major lobe. *Keep 5 significant figures in your calculations.*

(b) *Exact* maximum directivity of the antenna (*dimensionless* and *in dB*).

(c) *Approximate* maximum directivity of the antenna *based on Kraus'* formula (*dimensionless* and *in dB*).

(d) *Approximate* maximum directivity of the antenna based on *Tai & Pereira's* formula (*dimensionless* and *in dB*).

2.9. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta,\phi) = \left\{\begin{array}{cc} 1 & 0^\circ \le \theta < 20^\circ \\ 0.342\csc(\theta) & 20^\circ \le \theta < 60^\circ \\ 0 & 60^\circ \le \theta \le 180^\circ \end{array}\right\} 0^\circ \le \phi \le 360^\circ$$

Find the directivity (in dB) using the exact formula.

2.10. A beam antenna has half-power beamwidths of 30° and 35° in perpendicular planes intersecting at the maximum of the mainbeam. Find its approximate maximum effective aperture (in λ^2) using (a) Kraus' and (b) Tai and Pereira's formulas. The minor lobes are very small and can be neglected.

2.11. The normalized radiation intensity of a given antenna is given by

(a) $U = \sin\theta\sin\phi$ (b) $U = \sin\theta\sin^2\phi$

(c) $U = \sin\theta\sin^3\phi$ (d) $U = \sin^2\theta\sin\phi$

(e) $U = \sin^2\theta\sin^2\phi$ (f) $U = \sin^2\theta\sin^3\phi$

The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the

(a) exact directivity (dimensionless and in dB).

(b) azimuthal and elevation plane half-power beamwidths (in degrees).

2.12. Find the directivity (dimensionless and in dB) for the antenna of Problem 2.11 using

(a) Kraus' approximate formula (2-26)

(b) Tai and Pereira's approximate formula (2-30a)

2.13. For Problem 2.5, determine the approximate directivity (in dB) using

(a) Kraus' formula

(b) Tai and Pereira's formula.

2.14. The normalized radiation intensity of an antenna is rotationally symmetric in ϕ, and it is represented by

$$U = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ 0.5 & 30^\circ \le \theta < 60^\circ \\ 0.1 & 60^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?

(b) What is the directivity (above an infinitesimal dipole) of the antenna (in dB)?

2.15. The radiation intensity of an antenna is given by

$$U(\theta, \phi) = \cos^4\theta \sin^2\phi$$

for $0 \le \theta \le \pi/2$ and $0 \le \phi \le 2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.
Find the
(a) exact directivity (dimensionless and in dB)
(b) elevation plane half-power beamwidth (in degrees)

2.16. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by

$$U(\theta) = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ \dfrac{\cos(\theta)}{0.866} & 30^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

and it is independent of ϕ. Find the
(a) exact directivity by integrating the function
(b) approximate directivity using Kraus' formula

2.17. The maximum gain of a horn antenna is +20 dB, while the gain of its first sidelobe is −15 dB. What is the difference in gain between the maximum and first sidelobe:
(a) in dB
(b) as a ratio of the field intensities.

2.18. The normalized radiation intensity of an antenna is approximated by

$$U = \sin\theta$$

where $0 \le \theta \le \pi$, and $0 \le \phi \le 2\pi$. Determine the directivity using the
(a) exact formula
(b) formulas of (2-33a) by McDonald and (2-33b) by Pozar
(c) computer program ***Directivity*** of this chapter.

2.19. Repeat Problem 2.18 for a $\lambda/2$ dipole whose normalized intensity is approximated by

$$U \simeq \sin^3\theta$$

Compare the value with that of (4-91) or 1.643 (2.156 dB).

2.20. The radiation intensity of a circular loop of radius a and of constant current is given by

$$U = J_1^2(ka\sin\theta), \quad 0 \le \theta \le \pi \quad \text{and} \quad 0 \le \phi \le 2\pi$$

where $J_1(x)$ is the Bessel function of order 1. For a loop with radii of $a = \lambda/10$ and $\lambda/20$, determine the directivity using the:

(a) formulas (2-33a) by McDonald and (2-33b) by Pozar.

(b) computer program ***Directivity*** of this chapter.

Compare the answers with that of a very small loop represented by 1.5 or 1.76 dB.

2.21. Find the directivity (dimensionless and in dB) for the antenna of Problem 2.11 using numerical techniques with 10° uniform divisions and with the field evaluated at the

(a) midpoint

(b) trailing edge of each division.

2.22. Compute the directivity values of Problem 2.11 using the ***Directivity*** computer program of this chapter.

2.23. The far-zone electric-field intensity (array factor) of an end-fire two-element array antenna, placed along the z-axis and radiating into free-space, is given by

$$E = \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\frac{e^{-jkr}}{r}, \qquad 0 \le \theta \le \pi$$

Find the directivity using

(a) Kraus' approximate formula

(b) the ***Directivity*** computer program of this chapter.

2.24. Repeat Problem 2.23 when

$$E = \cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]\frac{e^{-jkr}}{r}, \qquad 0 \le \theta \le \pi$$

2.25. The radiation intensity is represented by

$$U = \begin{cases} U_0 \sin(\pi \sin\theta), & 0 \le \theta \le \pi/2 \text{ and } 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity

(a) exactly

(b) using the computer program ***Directivity*** of this chapter.

2.26. The radiation intensity of an aperture antenna, mounted on an infinite ground plane with z perpendicular to the aperture, is rotationally symmetric (not a function of ϕ), and it is given by

$$U = \left[\frac{\sin(\pi \sin\theta)}{\pi \sin\theta}\right]^2$$

Find the approximate directivity (dimensionless and in dB) using

(a) numerical integration. Use the ***Directivity*** computer program of this chapter.

(b) Kraus' formula

(c) Tai and Pereira's formula.

2.27. The normalized far-zone field pattern of an antenna is given by

$$E = \begin{cases} (\sin\theta\cos^2\phi)^{1/2} & 0 \le \theta \le \pi \text{ and } 0 \le \phi \le \pi/2, 3\pi/2 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity using

(a) the exact expression

(b) Kraus' approximate formula

(c) Tai and Pereira's approximate formula

(d) the computer program ***Directivity*** of this chapter

2.28. The normalized field pattern of the main beam of a conical horn antenna, mounted on an infinite ground plane with z perpendicular to the aperture, is given by

$$\frac{J_1(ka\sin\theta)}{\sin\theta}$$

where a is its radius at the aperture. Assuming that $a = \lambda$, find the

(a) half-power beamwidth

(b) directivity using Kraus' approximate formula

2.29. A base station cellular communication systems *lossless* antenna has a *maximum gain* of 16 dB (above isotropic) at 1,900 MHz. Assuming the *input power* to the antenna is 8 watts, what is the *maximum* radiated power density (in watts/cm^2) at a distance of 100 meters? This will determine the safe level for human exposure to electromagnetic radiation.

2.30. A uniform plane wave, of a form similar to (2-55), is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle τ (in degrees) when

(a) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = 0$

(b) $E_x \neq E_y$, $\Delta\phi = \phi_y - \phi_x = 0$

(c) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/2$

(d) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$

(e) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/4$

(f) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/4$

(g) $E_x = 0.5E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/2$

(h) $E_x = 0.5E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$

In all cases, justify the answer.

2.31. Derive (2-66), (2-67), and (2-68).

2.32. Write a general expression for the polarization loss factor (PLF) of two linearly polarized antennas if

(a) both lie in the same plane

(b) both do not lie in the same plane

2.33. A linearly polarized wave traveling in the positive z-direction is incident upon a circularly polarized antenna. Find the polarization loss factor PLF (dimensionless and in dB) when the antenna is (based upon its transmission mode operation)

(a) right-handed (CW)
(b) left-handed (CCW)

2.34. A 300 MHz uniform plane wave, traveling along the *x-axis in the negative x direction*, whose electric field is given by

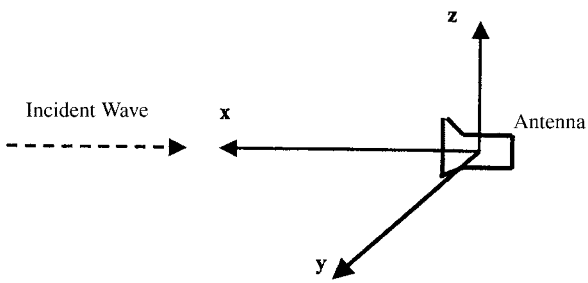
$$\mathbf{E}_w = E_o(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}$$

where E_o is a real constant, impinges upon a dipole antenna that is placed at the origin and whose electric field radiated toward the *x-axis in the positive x direction* is given by

$$\mathbf{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}$$

where E_a is a real constant. Determine the following:

(a) Polarization of the incident wave (*including axial ratio and sense of rotation, if any*). You must justify (state why?).
(b) Polarization of the antenna (*including axial ratio and sense of rotation, if any*). You must justify (state why?).
(c) Polarization loss factor (*dimensionless* and *in dB*).



2.35. The electric field of a uniform plane wave traveling along the *negative z* direction is given by

$$\mathbf{E}^i_w = (\hat{a}_x + j\hat{a}_y)E_o e^{+jkz}$$

and is incident upon a receiving antenna placed at the origin and *whose radiated electric field, toward the incident wave*, is given by

$$\mathbf{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{e^{-jkr}}{r}$$

Determine the following:

(a) Polarization of the *incident wave*, and why?
(b) Sense of rotation of the *incident wave*.
(c) Polarization *of the antenna*, and why?
(d) Sense of rotation of the *antenna polarization*.
(e) Losses (*dimensionless* and *in dB*) due to polarization mismatch between the incident wave and the antenna.

2.36. A ground-based helical antenna is placed at the origin of a coordinate system and it is used as a receiving antenna. The normalized far-zone electric-field pattern of the helical antenna in the transmitting mode is represented in the direction θ_o, ϕ_o by

$$\mathbf{E}_a = E_o(j\hat{a}_\theta + 2\hat{a}_\phi)f_o(\theta_o, \phi_o)\frac{e^{-jkr}}{r}$$

The far-zone electric field transmitted by an antenna on a flying aircraft towards θ_o, ϕ_o, which is received by the ground-based helical antenna, is represented by

$$\mathbf{E}_w = E_1(2\hat{a}_\theta + j\hat{a}_\phi)f_1(\theta_o, \phi_o)\frac{e^{+jkr}}{r}$$

Determine the following:

(a) Polarization (*linear, circular, or elliptical*) of the helical antenna in the transmitting mode. *State also the sense of rotation, if any.*

(b) Polarization (*linear, circular, or elliptical*) of the incoming wave that impinges upon the helical antenna. *State also the sense of rotation, if any.*

(c) Polarization loss (*dimensionless and in dB*) due to match/mismatch of the polarizations of the antenna and incoming wave.

2.37. A circularly polarized wave, traveling in the positive z-direction, is incident upon a circularly polarized antenna. Find the polarization loss factor PLF (dimensionless and in dB) for right-hand (CW) and left-hand (CCW) wave and antenna.

2.38. The electric field radiated by a rectangular aperture, mounted on an infinite ground plane with z perpendicular to the aperture, is given by

$$\mathbf{E} = [\hat{\mathbf{a}}_\theta \cos\phi - \hat{\mathbf{a}}_\phi \sin\phi\cos\theta]f(r, \theta, \phi)$$

where $f(r, \theta, \phi)$ is a scalar function which describes the field variation of the antenna. Assuming that the receiving antenna is linearly polarized along the x-axis, find the polarization loss factor (PLF).

2.39. A circularly polarized wave, traveling in the $+z$-direction, is received by an elliptically polarized antenna whose reception characteristics near the main lobe are given approximately by

$$\mathbf{E}_a \simeq [2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y]f(r, \theta, \phi)$$

Find the polarization loss factor PLF (dimensionless and in dB) when the incident wave is

(a) right-hand (CW)

(b) left-hand (CCW)

circularly polarized. Repeat the problem when

$$\mathbf{E}_a \simeq [2\hat{\mathbf{a}}_x - j\hat{\mathbf{a}}_y]f(r, \theta, \phi)$$

In each case, what is the polarization of the antenna? How does it match with that of the wave?

2.40. A linearly polarized wave traveling in the negative z-direction has a tilt angle (τ) of 45°. It is incident upon an antenna whose polarization characteristics are given by

$$\hat{\boldsymbol{\rho}}_a = \frac{4\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y}{\sqrt{17}}$$

Find the polarization loss factor PLF (dimensionless and db).

2.41. An elliptically polarized wave traveling in the negative z-direction is received by a circularly polarized antenna whose main lobe is along the $\theta = 0$ direction. The unit vector describing the polarization of the incident wave is given by

$$\hat{\boldsymbol{\rho}}_w = \frac{2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y}{\sqrt{5}}$$

Find the polarization loss factor PLF (dimensionless and in dB) when the wave that would be transmitted by the antenna is

(a) right-hand CP
(b) left-hand CP

2.42. A CW circularly polarized uniform plane wave is traveling in the $+z$ direction. Find the polarization loss factor PLF (dimensionless and in dB) assuming the receiving antenna (in its transmitting mode) is

(a) CW circularly polarized
(b) CCW circularly polarized

2.43. A linearly polarized uniform plane wave traveling in the $+z$ direction, with a power density of 10 milliwatts per square meter, is incident upon a CW circularly polarized antenna whose gain is 10 dB at 10 GHz. Find the

(a) maximum effective area of the antenna (in square meters)
(b) power (in watts) that will be delivered to a load attached directly to the terminals of the antenna.

2.44. A linearly polarized plane wave traveling along the negative z-axis is incident upon an elliptically polarized antenna (either CW or CCW). The axial ratio of the antenna polarization ellipse is 2:1 and its major axis coincides with the principal x-axis. Find the polarization loss factor (PLF) assuming the incident wave is linearly polarized in the

(a) x-direction
(b) y-direction

2.45. A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of **E** given by:

$$\mathscr{E}'_y = 3\cos\omega t$$

$$\mathscr{E}'_x = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

and the other with components given by:

$$\mathscr{E}_y'' = 2\cos\omega t$$

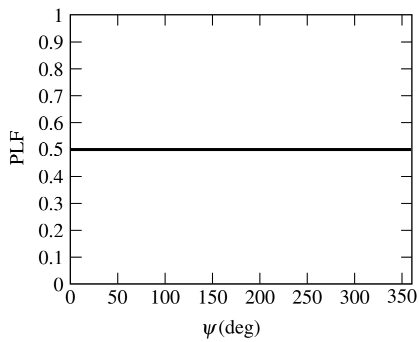
$$\mathscr{E}_x'' = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

(a) What is the axial ratio of the resultant wave?

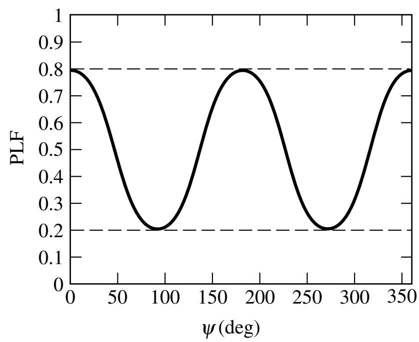
(b) Does the resultant vector **E** rotate clockwise or counterclockwise?

2.46. A linearly polarized antenna lying in the x-y plane is used to determine the polarization axial ratio of incoming plane waves traveling in the negative z-direction. The polarization of the antenna is described by the unit vector

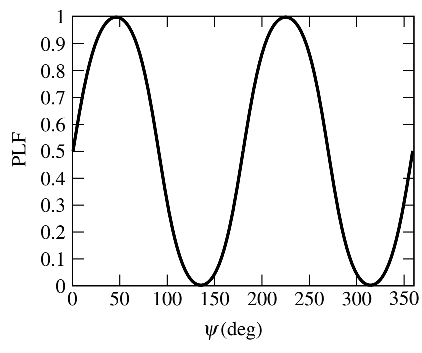
$$\hat{\boldsymbol{\rho}}_a = \hat{\mathbf{a}}_x \cos\psi + \hat{\mathbf{a}}_y \sin\psi$$



(a) PLF versus ψ



(b) PLF versus ψ



(c) PLF versus ψ

where ψ is an angle describing the orientation in the x-y plane of the receiving antenna. Above are the polarization loss factor (PLF) versus receiving antenna orientation curves obtained for three different incident plane waves. For each curve determine the axial ratio of the incident plane wave.

2.47. A $\lambda/2$ dipole, with a total loss resistance of 1 ohm, is connected to a generator whose internal impedance is $50 + j25$ ohms. Assuming that the peak voltage of the generator is 2 V and the impedance of the dipole, excluding the loss resistance, is $73 + j42.5$ ohms, find the power

(a) supplied by the source (real)

(b) radiated by the antenna

(c) dissipated by the antenna

2.48. The antenna and generator of Problem 2.47 are connected via a 50-ohm $\lambda/2$-long lossless transmission line. Find the power

(a) supplied by the source (real)

(b) radiated by the antenna

(c) dissipated by the antenna

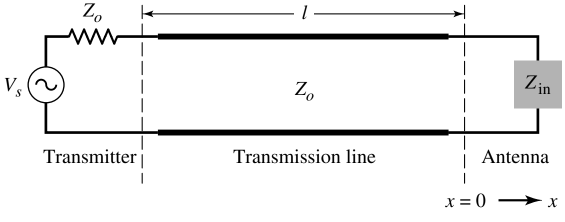
2.49. An antenna with a radiation resistance of 48 ohms, a loss resistance of 2 ohms, and a reactance of 50 ohms is connected to a generator with open-circuit voltage of 10 V and internal impedance of 50 ohms via a $\lambda/4$-long transmission line with characteristic impedance of 100 ohms.

(a) Draw the equivalent circuit

(b) Determine the power supplied by the generator

(c) Determine the power radiated by the antenna

2.50. A transmitter, with an internal impedance Z_0 (real), is connected to an antenna through a lossless transmission line of length l and characteristic impedance Z_0. Find a *simple* expression for the ratio between the antenna gain and its realized gain.



$V(x) = A\,[e^{-jkx} + \Gamma(0)e^{+jkx}]$

$I(x) = \dfrac{A}{Z_0}\,[e^{-jkx} - \Gamma(0)e^{+jkx}]$

V_s = strength of voltage source

$Z_{in} = R_{in} + jX_{in}$ = input impedance of the antenna

$Z_0 = R_0$ = characteristic impedance of the line

P_{accepted} = power accepted by the antenna $\{P_{\text{accepted}} = \text{Re}[V(0)I^*(0)]\}$

$P_{\text{available}}$ = power delivered to a matched load [i.e., $Z_{in} = Z_0^* = Z_0$]

2.51. The input reactance of an infinitesimal linear dipole of length $\lambda/60$ and radius $a = \lambda/200$ is given by

$$X_{in} \simeq -120\frac{[\ln(\ell/2a) - 1]}{\tan(k\ell/2)}$$

Assuming the wire of the dipole is copper with a conductivity of 5.7×10^7 S/m, determine at $f = 1$ GHz the

(a) loss resistance

(b) radiation resistance

(c) radiation efficiency
(d) VSWR when the antenna is connected to a 50-ohm line

2.52. A dipole antenna consists of a circular wire of length l. Assuming the current distribution on the wire is cosinusoidal, i.e.,

$$I_z(z) = I_0 \cos\left(\frac{\pi}{l} z'\right) \qquad -l/2 \le z' \le l/2$$

where I_0 is a constant, derive an expression for the loss resistance R_L, which is one-half of (2-90b).

2.53. The E-field pattern of an antenna, independent of ϕ, varies as follows:

$$E = \begin{cases} 1 & 0^\circ \le \theta \le 45^\circ \\ 0 & 45^\circ < \theta \le 90^\circ \\ \frac{1}{2} & 90^\circ < \theta \le 180^\circ \end{cases}$$

(a) What is the directivity of this antenna?
(b) What is the radiation resistance of the antenna at 200 m from it if the field is equal to 10 V/m (rms) for $\theta = 0^\circ$ at that distance and the terminal current is 5 A (rms)?

2.54. The far-zone field radiated by a rectangular aperture mounted on a ground plane, with dimensions a and b and uniform aperture distribution, is given by (see Table 12.1)

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= C \sin\phi \frac{\sin X}{X} \frac{\sin Y}{Y} \\ E_\phi &= C \cos\theta \cos\phi \frac{\sin X}{X} \frac{\sin Y}{Y} \end{aligned}\right\} \begin{aligned} X &= \frac{ka}{2} \sin\theta \cos\phi; \quad 0 \le \theta \le 90^\circ \\ Y &= \frac{kb}{2} \sin\theta \sin\phi; \quad 0 \le \phi \le 180^\circ \end{aligned}$$

where C is a constant and $0 \le \theta \le 90^\circ$ and $0 \le \phi \le 180^\circ$. For an aperture with $a = 3\lambda$, $b = 2\lambda$, determine the

(a) maximum partial directivities D_θ, D_ϕ (*dimensionless* and *in dB*) and
(b) total maximum directivity D_o (*dimensionless* and *in dB*). Compare with that computed using the equation in Table 12.1.

Use the computer program ***Directivity*** of this chapter.

2.55. Repeat Problem 2.54 when the aperture distribution is that of the dominant TE_{10} mode of a rectangular waveguide, or from Table 12.1

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= -\frac{\pi}{2} C \sin\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \\ E_\phi &= -\frac{\pi}{2} C \cos\theta \cos\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \end{aligned}\right\} \begin{aligned} X &= \frac{ka}{2} \sin\theta \cos\phi \\ Y &= \frac{kb}{2} \sin\theta \sin\phi \end{aligned}$$

2.56. Repeat Problem 2.55 when the aperture dimensions are those of an X-band rectangular waveguide with $a = 2.286$ cm (0.9 in.), $b = 1.016$ cm (0.4 in.) and frequency of operation is 10 GHz.

2.57. Repeat Problem 2.54 for a circular aperture with a uniform distribution and whose far-zone fields are, from Table 12.2

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= jC_1 \sin\phi \frac{J_1(Z)}{Z} \\ E_\phi &= jC_1 \cos\theta \cos\phi \frac{J_1(Z)}{Z} \end{aligned}\right\} \quad Z = ka\sin\theta; \quad \begin{aligned} &0 \le \theta \le 90^\circ \\ &0 \le \phi \le 180^\circ \end{aligned}$$

where C_1 is a constant and $J_1(Z)$ is the Bessel function of the first kind. Assume $a = 1.5\lambda$.

2.58. Repeat Problem 2.57 when the aperture distribution is that of the dominant TE_{11} mode of a circular waveguide, or from Table 12.2

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= C_2 \sin\phi \frac{J_1(Z)}{Z} \\ E_\phi &= C_2 \cos\theta \cos\phi \frac{J_z'(Z)}{(1) - \left(\dfrac{Z}{\chi_{11}'}\right)^2} \end{aligned}\right\} \quad \begin{aligned} &Z = ka\sin\theta; && 0 \le \theta \le 90^\circ \\ &J_z'(Z) = J_o(Z) && 0 \le \phi \le 180^\circ \\ &\quad - J_1(Z)/Z; && \end{aligned}$$

where C_2 is a constant, $J_1'(Z)$ is the derivative of $J_1(Z)$, $\chi_{11}' = 1.841$ is the first zero of $J_1'(Z)$, and $J_o(Z)$ is the Bessel function of the first kind of order zero.

2.59. Repeat 2.58 when the radius of the aperture is $a = 1.143$ cm (0.45 in.) and the frequency of operation is 10 GHz.

2.60. A 1-m long dipole antenna is driven by a 150 MHz source having a source resistance of 50 ohms and a voltage of 100 V. If the ohmic resistance of the antennas is given by $R_L = 0.625$ ohms, find the:

(a) Current going into the antenna (I_{ant})
(b) Power dissipated by the antenna
(c) Power radiated by the antenna
(d) Radiation efficiency of the antenna

2.61. The field radiated by an infinitesimal dipole of very small length ($\ell \le \lambda/50$), and of uniform current distribution I_o, is given by (4-26a) or

$$\mathbf{E} = \hat{a}_\theta E_\theta \approx \hat{a}_\theta j\eta \frac{kI_o\ell}{4\pi r} e^{-jkr} \sin\theta$$

Determine the

(a) vector effective length
(b) maximum value of the vector effective length. Specify the angle.

(c) ratio of the maximum effective length to the physical length ℓ.

2.62. The field radiated by a half-wavelength dipole ($\ell = \lambda/2$), with a sinusoidal current distribution, is given by (4-84) or

$$\mathbf{E} = \hat{a}_\theta E_\theta \approx \hat{a}_\theta j\eta \frac{I_o}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

where I_o is the maximum current. Determine the

(a) vector effective length

(b) maximum value of the vector effective length. Specify the angle.

(c) ratio of the maximum effective length to the physical length ℓ.

2.63. A uniform plane wave, of 10^{-3}watts/cm^2 power density, is incident upon an infinitesimal dipole of length $\ell = \lambda/50$ and uniform current distribution, as shown in Figure 2.29(a). For a frequency of 10 GHz, determine the maximum open-circuited voltage at the terminals of the antenna. See Problem 2.61.

2.64. Repeat Problem 2.63 for a small dipole with triangular current distribution and length $\ell = \lambda/10$. See Example 2.14.

2.65. Repeat Problem 2.63 for a half-wavelength dipole ($\ell = \lambda/2$) with sinusoidal current distribution. See Problem 2.62.

2.66. Show that the effective length of a linear antenna can be written as

$$l_e = \sqrt{\frac{A_e |Z_t|^2}{\eta R_T}}$$

which for a lossless antenna and maximum power transfer reduces to

$$l_e = 2\sqrt{\frac{A_{em} R_r}{\eta}}$$

A_e and A_{em} represent, respectively, the effective and maximum effective apertures of the antenna while η is the intrinsic impedance of the medium.

2.67. An antenna has a maximum effective aperture of 2.147 m^2 at its operating frequency of 100 MHz. It has no conduction or dielectric losses. The input impedance of the antenna itself is 75 ohms, and it is connected to a 50-ohm transmission line. Find the directivity of the antenna system ("system" meaning includes any effects of connection to the transmission line). Assume no polarization losses.

2.68. A small circular parabolic reflector, often referred to as dish, is now being advertised as a TV antenna for direct broadcast. Assuming the diameter of the antenna is 1 meter, the frequency of operation is 3 GHz, and its aperture efficiency is 68%, determine the following:

(a) Physical area of the reflector (*in* m^2).

(b) Maximum effective area of the antenna (*in* m^2).

(c) Maximum directivity (*dimensionless* and *in dB*).

(d) Maximum power (*in watts*) that can be delivered to the TV if the power density of the wave incident upon the antenna is *10* μ*watts*/m^2. Assume *no losses* between the incident wave and the receiver (TV).

2.69. An incoming wave, with a uniform power density equal to 10^{-3} W/m^2 is incident normally upon a lossless horn antenna whose directivity is 20 dB. At a frequency of 10 GHz, determine the very maximum possible power that can be expected to be delivered to a receiver or a load connected to the horn antenna. There are no losses between the antenna and the receiver or load.

2.70. A linearly polarized aperture antenna, with a uniform field distribution over its area, is used as a receiving antenna. The antenna physical area over its aperture is 10 cm^2, and it is operating at 10 GHz. The antenna is illuminated with a circularly polarized plane wave whose incident power density is 10 mwatts/cm^2. Assuming the antenna element itself is lossless, determine its

(a) gain (*dimensionless* and *in dB*).

(b) maximum power (*in watts*) that can be delivered to a load connected to the antenna. Assume no other losses between the antenna and the load.

2.71. The *far-zone power density* radiated by a helical antenna can be approximated by

$$\mathbf{W}_{rad} = \mathbf{W}_{ave} \approx \hat{a}_r C_o \frac{1}{r^2} \cos^4 \theta$$

The radiated power density is symmetrical with respect to ϕ, and *it exists only in the upper hemisphere* $(0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi)$; C_o is a constant. Determine the following:

(a) Power radiated by the antenna (*in watts*).

(b) Maximum directivity of the antenna (*dimensionless* and *in dB*)

(c) Direction (*in degrees*) along which the maximum directivity occurs.

(d) Maximum effective area (*in* m^2) at 1 GHz.

(e) Maximum power (*in watts*) received by the antenna, assuming no losses, at 1 GHz when the antenna is used as a receiver and the incident power density is *10 mwatts/*m^2.

2.72. For an X-band (8.2–12.4 GHz) rectangular horn, with aperture dimensions of 5.5 cm and 7.4 cm, find its maximum effective aperture (*in* cm^2) when its gain (over isotropic) is

(a) 14.8 dB at 8.2 GHz

(b) 16.5 dB at 10.3 GHz

(c) 18.0 dB at 12.4 GHz

2.73. For Problem 2.54 compute the

(a) maximum effective area (in λ^2) using the computer program ***Directivity*** of this chapter. Compare with that computed using the equation in Table 12.1.

(b) aperture efficiencies of part (a). Are they smaller or larger than unity and why?

2.74. Repeat Problem 2.73 for Problem 2.55.

2.75. Repeat Problem 2.73 for Problem 2.56.

2.76. Repeat Problem 2.73 for Problem 2.57. Compare with those in Table 12.2.

2.77. Repeat Problem 2.73 for Problem 2.58. Compare with those in Table 12.2.

2.78. Repeat Problem 2.73 for Problem 2.59. Compare with those in Table 12.2.

2.79. A 30-dB, right-circularly polarized antenna in a radio link radiates 5 W of power at 2 GHz. The receiving antenna has an impedance mismatch at its terminals, which leads to a VSWR of 2. The receiving antenna is about 95% efficient and has a field pattern near the beam maximum given by $\mathbf{E}_r = (2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y)F_r(\theta, \phi)$. The distance between the two antennas is 4,000 km, and the receiving antenna is required to deliver 10^{-14} W to the receiver. Determine the maximum effective aperture of the receiving antenna.

2.80. The radiation intensity of an antenna can be approximated by

$$U(\theta, \phi) = \begin{cases} \cos^4(\theta) & 0^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases} \quad \text{with } 0^\circ \le \phi \le 360^\circ$$

Determine the maximum effective aperture (*in* m^2) of the antenna if its frequency of operation is $f = 10$ GHz.

2.81. A communication satellite is in stationary (synchronous) orbit about the earth (assume altitude of 22,300 statute miles). Its transmitter generates 8.0 W. Assume the transmitting antenna is isotropic. Its signal is received by the 210-ft diameter tracking paraboloidal antenna on the earth at the NASA tracking station at Goldstone, California. Also assume no resistive losses in either antenna, perfect polarization match, and perfect impedance match at both antennas. At a frequency of 2 GHz, determine the:

(a) power density (*in watts/m*^{2}) incident on the receiving antenna.

(b) power received by the ground-based antenna whose gain is 60 dB.

2.82. A lossless ($e_{cd} = 1$) antenna is operating at 100 MHz and its maximum effective aperture is 0.7162 m^2 at this frequency. The input impedance of this antenna is 75 ohms, and it is attached to a 50-ohm transmission line. Find the directivity (dimensionless) of this antenna if it is polarization-matched.

2.83. A resonant, lossless ($e_{cd} = 1.0$) half-wavelength dipole antenna, having a directivity of 2.156 dB, has an input impedance of 73 ohms and is connected to a lossless, 50 ohms transmission line. A wave, having the same polarization as the antenna, is incident upon the antenna with a power density of 5 W/m^2 at a frequency of 10 MHz. Find the received power available at the end of the transmission line.

2.84. Two X-band (8.2–12.4 GHz) rectangular horns, with aperture dimensions of 5.5 cm and 7.4 cm and each with a gain of 16.3 dB (over isotropic) at 10 GHz,

are used as transmitting and receiving antennas. Assuming that the input power is 200 mW, the VSWR of each is 1.1, the conduction-dielectric efficiency is 100%, and the antennas are polarization-matched, find the maximum received power when the horns are separated in air by
(a) 5 m (b) 50 m (c) 500 m

2.85. Transmitting and receiving antennas operating at 1 GHz with gains (over isotropic) of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the maximum power delivered to the load when the input power is 150 W. Assume that the

(a) antennas are polarization-matched
(b) transmitting antenna is circularly polarized (either right- or left-hand) and the receiving antenna is linearly polarized.

2.86. Two lossless, polarization-matched antennas are aligned for maximum radiation between them, and are separated by a distance of 50λ. The antennas are matched to their transmission lines and have directivities of 20 dB. Assuming that the power at the input terminals of the transmitting antenna is 10 W, find the power at the terminals of the receiving antenna.

2.87. Repeat Problem 2.86 for two antennas with 30 dB directivities and separated by 100λ. The power at the input terminals is 20 W.

2.88. Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the power delivered to the load when the input power is 150 W. Assume the PLF = 1.

2.89. A series of microwave repeater links operating at 10 GHz are used to relay television signals into a valley that is surrounded by steep mountain ranges. Each repeater consists of a receiver, transmitter, antennas, and associated equipment. The transmitting and receiving antennas are identical horns, each having gain over isotropic of 15 dB. The repeaters are separated in distance by 10 km. For acceptable signal-to-noise ratio, the power received at each repeater must be greater than 10 nW. Loss due to polarization mismatch is not expected to exceed 3 dB. Assume matched loads and free-space propagation conditions. Determine the minimum transmitter power that should be used.

2.90. A one-way communication system, operating at 100 MHz, uses two identical $\lambda/2$ vertical, resonant, and lossless dipole antennas as transmitting and receiving elements separated by 10 km. In order for the signal to be detected by the receiver, the power level at the receiver terminals must be at least 1 μW. Each antenna is connected to the transmitter and receiver by a lossless 50-Ω transmission line. Assuming the antennas are polarization-matched and are aligned so that the maximum intensity of one is directed toward the maximum radiation intensity of the other, determine the minimum power that must be generated by the transmitter so that the signal will be detected by the receiver. Account for the proper losses from the transmitter to the receiver.

2.91. In a long-range microwave communication system operating at 9 GHz, the transmitting and receiving antennas are identical, and they are separated by

10,000 m. To meet the signal-to-noise ratio of the receiver, the received power must be at least 10 μW. Assuming the two antennas are aligned for maximum reception to each other, including being polarization-matched, what should the gains (in dB) of the transmitting and receiving antennas be when the input power to the transmitting antenna is 10 W?

2.92. A mobile wireless communication system operating at 2 GHz utilizes two antennas, one at the base station and the other at the mobile unit, which are separated by *16 kilometers*. The transmitting antenna, at the base station, is circularly-polarized while the receiving antenna, at the mobile station, is linearly polarized. The *maximum gain of the transmitting antenna is 20 dB* while the gain of the receiving antennas is unknown. The input power to the transmitting antenna is *100 watts* and the power received at the receiver, which is connected to the receiving antenna, is *5 nanowatts*. Assuming that the two antennas are aligned so that the maximum of one is directed toward the maximum of the other, *and also assuming no reflection/mismatch losses at the transmitter or the receiver*, what is the maximum gain of the receiving antenna (*dimensions* and *in dB*)?

2.93. A rectangular X-band horn, with aperture dimensions of 5.5 cm and 7.4 cm and a gain of 16.3 dB (over isotropic) at 10 GHz, is used to transmit and receive energy scattered from a perfectly conducting sphere of radius $a = 5\lambda$. Find the maximum scattered power delivered to the load when the distance between the horn and the sphere is
(a) 200λ (b) 500λ
Assume that the input power is 200 mW, and the radar cross section is equal to the geometrical cross section.

2.94. A radar antenna, used for both transmitting and receiving, has a gain of 150 (dimensionless) at its operating frequency of 5 GHz. It transmits 100 kW, and is aligned for maximum directional radiation and reception to a target 1 km away having a radar cross section of 3 m^2. The received signal matches the polarization of the transmitted signal. Find the received power.

2.95. In an experiment to determine the radar cross section of a Tomahawk cruise missile, a 1,000 W, 300 MHz signal was transmitted toward the target, and the received power was measured to be 0.1425 mW. The same antenna, whose gain was 75 (*dimensionless*), was used for both transmitting and receiving. The polarizations of both signals were identical (PLF = 1), and the distance between the antenna and missile was 500 m. What is the radar cross section of the cruise missile?

2.96. Repeat Problem 2.95 for a radar system with 1,000 W, 100 MHz transmitted signal, 0.01 W received signal, an antenna with a gain of 75 (*dimensionless*), and separation between the antenna and target of 700 m.

2.97. The maximum radar cross section of a resonant linear $\lambda/2$ dipole is approximately $0.86\lambda^2$. For a monostatic system (i.e., transmitter and receiver at the same location), find the received power (in W) if the transmitted power is 100 W, the distance of the dipole from the transmitting and receiving antennas is 100 m, the gain of the transmitting and receiving antennas is 15 dB each, and the frequency of operation is 3 GHz. Assume a polarization loss factor of −1 dB.

2.98. The effective antenna temperature of an antenna looking toward zenith is approximately 5 K. Assuming that the temperature of the transmission line (waveguide) is 72 °F, find the effective temperature at the receiver terminals when the attenuation of the transmission line is 4 dB/100 ft and its length is
(a) 2 ft (b) 100 ft
Compare it to a receiver noise temperature of about 54 K.

2.99. Derive (2-146). Begin with an expression that assumes that the physical temperature and the attenuation of the transmission line are not constant.