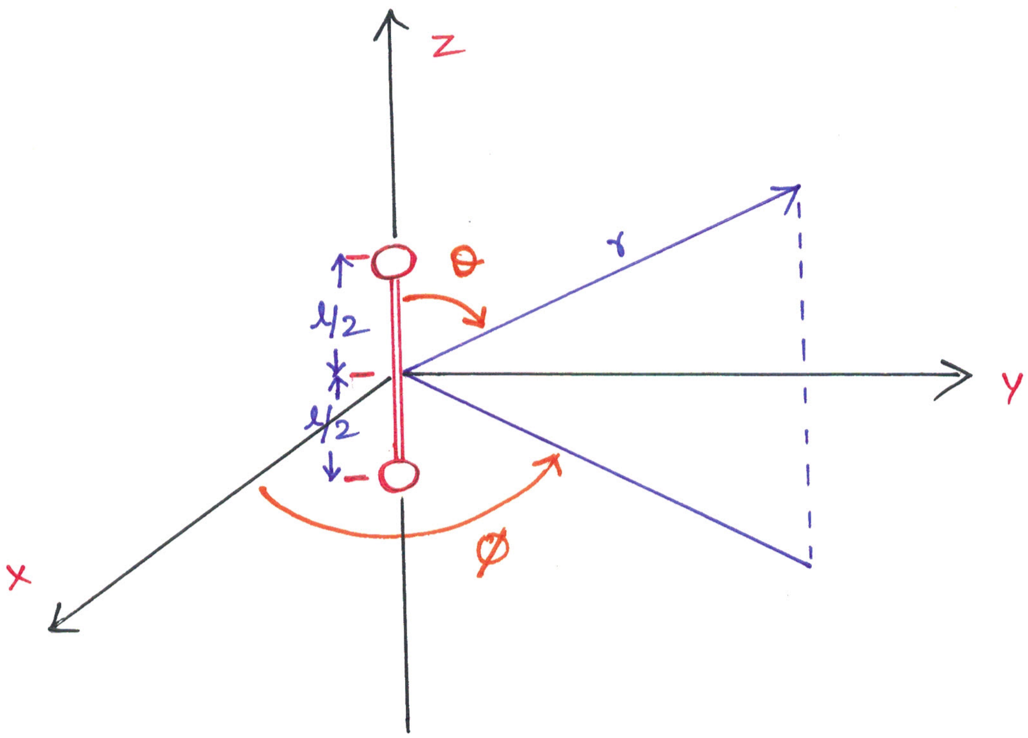
Infinitesimal Dipole :-

* It is also called as Elementary dipole or oscillating electric dipole or short dipole or Hertzian dipole.

* If the dipole is vanishingly short, it is an infinitesimal dipole.



* $l << \lambda \quad (l \leq \lambda/50)$
* They are not very practical
* $x' = y' = z' = 0$
* used to represent capacitor plate antenna
* The end plates are used to provide capacitive loading inorder to maintain uniform current on the dipole.

The end plates are assumed to be very small. so, their radiation is negligible.

The spatial variation of the current is assumed to be constant and is given by,

$$I(z') = \hat{a}_z \cdot I_0 \rightarrow ①$$

Radiation Fields : (E and H)

$$A(x,y,z) = \frac{\mu}{4\pi} \int_C I_e(x',y',z') \cdot \frac{e^{-jkR}}{R} \cdot dl' \rightarrow ②$$

where,

x, y, z → observation point coordinates

x', y', z' → source point coordinates

R → Distance from any point on the source to the observation point

C → Path along the length of the source

$$I_e(x',y',z') = \hat{a}_z I_0 \rightarrow ③$$

$\because$ $x' = y' = z' = 0$ (Infinitesimal dipole)

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$R = r \; ; \; \text{constant} \rightarrow ④$$

$$dl' = dz' \rightarrow (5)$$

$\therefore$ equ (2) can be written as,

$$A(x,y,z) = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \hat{a}_z I_0 \cdot \frac{e^{-jkr}}{r} dz'$$

$$= \frac{\mu}{4\pi} \cdot \hat{a}_z I_0 \cdot \frac{e^{-jkr}}{r} \int_{-l/2}^{l/2} dz'$$

$$A(x,y,z) = \frac{\mu I_0 . l}{4\pi r} . \hat{a}_z \cdot e^{-jkr} \rightarrow (6)$$

Next step is to find magnetic field vector 'H_A' and Electric field vector 'E_A' with $J=0$.

To do this, it is often simpler to transform 'A' from rectangular to spherical components and then find E and H.

The transformation between rectangular and spherical components is given by,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \cdot \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_x = A_y = 0 \rightarrow (7)$$

$$\therefore\ A_r = A_z \cos\theta$$

$$\boxed{\begin{aligned} A_r &= \frac{\mu \cdot I_0 l}{4\pi r} \cdot e^{-jkr} \cdot \cos\theta \rightarrow (8) \\ A_\theta &= -A_z \sin\theta \\ &= -\frac{\mu I_0 l}{4\pi r} e^{-jkr} \cdot \sin\theta \rightarrow (9) \\ A_\phi &= 0 \rightarrow (10) \end{aligned}}$$

$$\boxed{\begin{aligned} H &= \frac{1}{\mu} \nabla \times A \rightarrow (11) \\ E &= \frac{1}{j\omega\varepsilon} \nabla \times H \rightarrow (12) \end{aligned}}$$

To Find Magnetic Field, H_ϕ :-

To find magnetic field consider equ (11)

$$H_\phi = \frac{1}{\mu} \cdot \nabla \times A$$

$$= \frac{1}{\mu} \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{bmatrix}$$

$$= \frac{1}{\mu} \frac{1}{\cancel{r^2} \cancel{\sin\theta}} \cdot \cancel{r} \cancel{\sin\theta} \times \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right]$$

$$H_\phi = \frac{1}{\mu r} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right] \rightarrow (13)$$

$$\frac{\partial}{\partial r}(rA_\theta) = \frac{\partial}{\partial r}\cdot \cancel{r}\cdot\left(-\frac{\mu I_0 l}{4\pi \cancel{r}}\, e^{-jkr}.\ \sin\theta\right)$$

$$= -\frac{\mu I_0 l}{4\pi}\ \sin\theta\ \frac{\partial}{\partial r}\ e^{-jkr}$$

$$= -\frac{\mu I_0 l}{4\pi}\ \sin\theta\ e^{-jkr}\cdot(-jk)$$

$$\frac{\partial}{\partial r}(rA_\theta) = \frac{\mu I_0 l}{4\pi}\ e^{-jkr}(jk)\ \sin\theta \rightarrow (14)$$

Illy $\frac{\partial}{\partial \theta}(A_r) = \frac{\partial}{\partial \theta}\ \frac{\mu I_0 l}{4\pi r}\ e^{-jkr}.\ \cos\theta$

$$= -\frac{\mu I_0 l}{4\pi r}\ e^{-jkr}.\ \sin\theta \rightarrow (15)$$

substitute equ. (14) & (15) in (13)

$$H_\phi = \frac{1}{\mu r}\left[\frac{\mu I_0 l}{4\pi}\ e^{-jkr}.\ jk.\ \sin\theta + \frac{\mu I_0 l}{4\pi r}\ e^{-jkr}.\ \sin\theta\right]$$

$$H_\phi = \frac{\cancel{\mu} I_0 l}{\cancel{\mu} r 4\pi}\ e^{-jkr}\ \sin\theta\ \left[jk + \frac{1}{r}\right]$$

$$\boxed{\therefore H_\phi = \frac{jk I_0 l}{4\pi r}\ e^{-jkr}.\ \sin\theta\ \left[1 + \frac{1}{jkr}\right] \qquad H_r = H_\theta = 0} $$

$\rightarrow (16)$

$\rightarrow (17)$

To Find Electric Field; E_r, E_θ :-

To find electric field components consider equ. (12).

$$E = \frac{1}{j\omega\epsilon} \cdot \nabla \times H \rightarrow (12)$$

$$E_r = \frac{1}{j\omega\epsilon} \times \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{bmatrix} \times \frac{1}{r^2\sin\theta}$$

$$E_r = \frac{1}{j\omega\epsilon} \times \frac{1}{r^2\sin\theta} \times \left[\frac{\partial}{\partial\theta}(r\sin\theta H_\phi) - \frac{\partial}{\partial\phi}\underset{0}{(rH_\theta)} \right]$$

$$E_r = \frac{1}{j\omega\epsilon} \cdot \frac{1}{r^2\sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\cancel{r}\sin\theta \cdot \frac{jKI_0 l}{4\pi \cancel{r}} \sin\theta\left[1+\frac{1}{jKr}\right] \cdot e^{-jKr}\right)$$

$$E_r = \frac{1}{\cancel{j}\omega\epsilon r^2\sin\theta} \cdot \frac{\cancel{j}KI_0 l}{4\pi}\left(1+\frac{1}{jKr}\right) e^{-jKr} \cdot \frac{\partial}{\partial\theta}\sin^2\theta$$

$$E_r = \frac{\overset{\eta}{K} I_0 l}{\omega\epsilon\, r^2 \cancel{\sin\theta}\, \underset{2}{\cancel{4}}\pi}\left(1+\frac{1}{jKr}\right) \cdot e^{-jKr} \cdot \cancel{2}\cancel{\sin\theta}\cos\theta$$

$$\boxed{\therefore E_r = \frac{\eta I_0 l \cos\theta}{2\pi r^2}\left[1+\frac{1}{jKr}\right] \cdot e^{-jKr} \rightarrow (18)}$$

$$E_\theta = \frac{1}{j\omega\varepsilon} \times \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{bmatrix}$$

$$E_\theta = \frac{1}{j\omega\varepsilon} \times \frac{-1}{r^{\cancel{2}}\sin\theta} \cdot \cancel{r} \left[\frac{\partial}{\partial r}\left(r\sin\theta\, H_\phi\right) - \frac{\partial}{\partial \phi}\underset{\uparrow \atop 0}{\left(H_r\right)} \right]$$

$$E_\theta = \frac{-1}{j\omega\varepsilon} \cdot \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial r}\left(r\sin\theta\, H_\phi\right)$$

$$= \frac{-1}{j\omega\varepsilon} \cdot \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial r} \cancel{r}\sin\theta \cdot \frac{jkI_0 l}{4\pi \cancel{r}} e^{-jkr} \cdot \sin\theta\left(1 + \frac{1}{jkr}\right)$$

$$= -\frac{\cancel{j}kI_0 l}{\cancel{j}\omega\varepsilon\, r\, \sin\cancel{\theta}} \cdot \frac{\sin^{\cancel{2}}\theta}{4\pi} \cdot \frac{\partial}{\partial r}\left(1 + \frac{1}{jkr}\right) e^{-jkr}$$

$\eta \rightarrow$

$$E_\theta = -\frac{\eta I_0 l}{r 4\pi} \sin\theta \frac{\partial}{\partial r}\left(e^{-jkr} + \frac{e^{-jkr}}{jkr}\right)$$

$$= -\frac{\eta I_0 l}{4\pi r} \sin\theta \left[e^{-jkr} \cdot (-jk) + \frac{(\cancel{j}kr)(-jk)e^{-jkr} - e^{-jkr} \cdot (\cancel{j}k)}{(\cancel{j}kr)(jkr)} \right]$$

$$= -\frac{\eta I_0 l}{4\pi r} \sin\theta \left[-jk \cdot e^{-jkr} + \frac{-jkr\, e^{-jkr} - e^{-jkr}}{jkr^2} \right]$$

$$E_\theta = \frac{-\eta I_0 l}{4\pi r} \sin\theta \left[-jke^{-jkr} \quad \frac{-jkre^{-jkr}}{jkr^2} \quad - \frac{e^{-jkr}}{jkr^2} \right]$$

$$E_\theta = \frac{-\eta I_0 l}{4\pi r} \sin\theta \times -jk e^{-jkr} \left[1 + \frac{\cancel{r}}{jkr^{\cancel{2}}} + \frac{1}{jkr^2 (jk)} \right]$$

$$\boxed{E_\theta = \frac{j\eta I_0 l k \sin\theta}{4\pi r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] \cdot e^{-jkr} \rightarrow (19)}$$

$$E_\phi = 0$$

Summary :-

$$H_r = H_\theta = E_\phi = 0$$

$$H_\phi = \frac{jkI_0 l \sin\theta}{4\pi r} \left[1 + \frac{1}{jkr} \right] \cdot e^{-jkr}$$

$$E_r = \frac{\eta I_0 l \cos\theta}{2\pi r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

$$E_\theta = \frac{j\eta k I_0 l \sin\theta}{4\pi r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] \cdot e^{-jkr}$$