Solved Anna University Problems

(NOV/DEC 2010 - NOV/DEC 2019)

EC8651 - Transmission Lines and RF Systems

UNIT 4 - Waveguides

UNIT 4 - WAVEGUIDES

PROBLEMS

PARALLEL PLATE WAVEGUIDE:

1) For a frequency of 10GHz and plane separation of 5cm in air, find the cut off frequency, cut off wavelength, phase velocity and group velocity of the wave. **(EC6503 - NOV/DEC 2019) (13 Marks), (EC6503 - APR/MAY 2019) (13 Marks), (EC6503 - APR/MAY 2016) (16 Marks)**

Solution:

Given: $f=10\times10^{9}$, $a=5\times10^{-2}$, no mode and wave is given so, take dominant mode as TE_{10} or TM_{10}, therefore m=1

1. The cut off frequency, f_c is given by

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2}$$

for air medium, $\frac{1}{\sqrt{\mu\varepsilon}} = c = 3\times10^8$

therefore, $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2} = \frac{mc}{2a} = \frac{1\times3\times10^8}{2\times5\times10^{-2}} = 3\,\text{GHz}$

2. The cutoff wavelength, λc is given by

$$\lambda_c = \frac{c}{f_c} = \frac{3\times10^8}{3\times10^9} = 0.1\text{m}$$

3. The Phase velocity, v_p is given by

$$v_p = \frac{\omega}{\beta_m} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \text{for air medium}\ \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{3\times10^9}{10\times10^9}\right)^2}} = \frac{3\times10^8}{0.9539} = 3.1449\times10^8\,\text{m/sec}$$

v_p=3.1449x10^8 m/sec

4. Group velocity, v_g

$$v_g = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{3.1449\times10^8} = 2.8618\times10^8 \text{ m/sec}$$

v_g=2.8618x10^8 m/sec

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2) A parallel perfectly conducting plates are separated by 7 cm in air and carries a signal with frequency (f) of 6GHz in TE_1 mode. Find

(1) The cut-off frequency (fc),

(2) Phase constant,

(3) Attenuation constant and Phase constant for f = 0.8 fc and

(4) Cut off wavelength. (8) **(EC2503 - APR/MAY 2013)(8 Marks)**

Solution:

Given: a=7cm=0.07m, f=6GHz, m=1

1)The cut off frequency, f_c is given by

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2}$$

for air medium, $\frac{1}{\sqrt{\mu\varepsilon}} = c = 3\times10^8$

therefore, $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2} = \frac{mc}{2a} = \frac{1\times3\times10^8}{2\times7\times10^{-2}} = 2.143\,\text{GHz}$

f_c=2.143GHz

2)The phase constant, β is given by

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\omega = 2\pi f = 2\times3.14\times6\times10^9 = 3.768\times10^{10}$$

$$\omega^2 = 14.198\times10^{20}$$

$$\mu\varepsilon = 11.11\times10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(14.198\times10^{20}\times11.11\times10^{-18}) - \left(\frac{3.14}{0.07}\right)^2}$$

$$\beta_{mn} = \sqrt{15773.78 - 2012.16}$$

$_{mn}$ **= 117.31 read/m**

3)Attenuation constant and Phase constant for f = 0.8 fc,

$f=0.8f_c=0.8x2.143x10^9=1.713x10^9$

$\omega = 2\pi f = 2\times 3.14\times 1.713\times 10^9 = 1.0758\times 10^{10}$

$\omega^2 = 1.15727\times 10^{20}$

$$\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon} = \sqrt{\left(\frac{\pi}{0.07}\right)^2 - (1.15727\times 10^{20}\times 11.11\times 10^{-18})}$$

$$\gamma_{mn} = \sqrt{2012.16 - 1285.72} = 26.92$$

¥$_{mn}$= 26.92

4) The Cut off wavelength λ_c is given by,

$$\lambda_c = \frac{c}{f_c} = \frac{3\times 10^8}{2.143\times 10^9} = 0.13999m\,(Or)\,14cm$$

$^a{}_c$ = 14cm

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3) A wave is propagated in the dominant mode in a parallel plane waveguide. The frequency is 6GHz and the plane separation is 4cm. Calculate the cutoff wavelength and the wavelength in the waveguide. **(EC2503 - APR/MAY 2014)(2 Marks) (EC2503 - APR/MAY 2016)(2 Marks)**

Solution:

Given: a=4cm=0.04m, f=6GHz, mode = Dominant (TE_1 or TM_1,) so, m=1

1) The cutoff wavelength, λc is given by

$$\lambda_c = \frac{c}{f_c} = \frac{3\times 10^8}{3\times 10^9} = 0.1m$$

where, the cut off frequency, f_c is given by

$$f_c = \frac{mc}{2a} = \frac{3\times 10^8}{2\times 0.04} = 3.75\,GHz$$

therefore, $\lambda_c = \frac{c}{f_c} = \frac{3\times 10^8}{3.75\times 10^9} = 0.8m$

2)The guide wavelength λ_g is given by,

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

where, $\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 0.5\text{m}$

$$\lambda_g = \frac{0.5}{\sqrt{1-\left(\frac{0.5}{0.8}\right)^2}} = 0.6405\text{m}$$

$^a{}_g$ = 0.6405m

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4) A wave is propagated in a parallel plane waveguide. The frequency is 6GHz and the plane separation is 3cm. Determine the group and phase velocities for the dominant mode. **EC2305 NOV-DEC 2013 (2 Marks) EC2305 ND2010 (2) EC2305 NOV/DEC 2010 (2)**

Solution:

Given: f=6 GHz, a=3cm=0.03m, mode = Dominant (TE_1 or TM_1,) so, m=1

1) The guide wavelength, λ_g is given by

$$\lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, β_{mn} is given as,

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\omega = 2\pi f = 2\times3.14\times9\times10^9 = 5.652\times10^{10}$$

$$\omega^2 = 31.945\times10^{20}$$

$$\mu\varepsilon = 11.11\times10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(31.945\times10^{20}\times11.11\times10^{-18}) - \left[\left(\frac{3.14}{0.045}\right)^2 + \left(\frac{0}{0.03}\right)^2\right]}$$

$$\beta_{mn} = \sqrt{35.4910\times10^3 - 4868.94}$$

$$\beta_{mn} = 174.99\ \text{rad/m}$$

Therefore, $\lambda_g = \frac{2\pi}{\beta_{mn}} = \frac{2\pi}{174.99} = 0.0359\text{m}$

$^a{}_g$ = 0.0359 m

1) The Phase velocity, v_p is given by

$$v_p = \frac{\omega}{\beta_m}$$

where, β_m is given as,

$$\beta_m = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\omega = 2\pi f = 2\times 3.14\times 6\times 10^9 = 3.768\times 10^{10}$$

$$\omega^2 = 14.198\times 10^{20}$$

$$\mu\varepsilon = 11.11\times 10^{-18}$$

$$\therefore\ \beta_{mn} = \sqrt{(14.198\times 10^{20}\times 11.11\times 10^{-18}) - \left(\frac{3.14}{0.03}\right)^2}$$

$$\beta_{mn} = \sqrt{15.774\times 10^3 - 10955.11}$$

$_m$ = 69.418 rad/m

therefore, $v_p = \frac{\omega}{\beta_m} = \frac{3.768\times 10^{10}}{69.418} = 5.428\times 10^8\ \text{m/sec}$

v_p=5.428x10^8 m/sec

3) Group velocity, v_g

$$v_g = \frac{c^2}{v_p} = \frac{\left(3\times 10^8\right)^2}{5.428\times 10^8} = 1.658\times 10^8\ \text{m/sec}$$

v_g=1.658x10^8 m/sec

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5) A TEM wave at 1MHz propagates in the region between conducting planes which is filled with dielectric material of μ_r=1 and ε_r=2. Find the phase constant and characteristic wave impedance. **(EC2305 - NOV/DEC 2010) (4marks)**

Solution:

Given: f=1GHz, μ_r=1 and ε_r=2

1) where, the phase constant, β_{mn} is given by

$$\beta_{mn} = \omega\sqrt{\mu\varepsilon}$$

$$\beta_{mn}=2\pi f\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

$$\beta_{mn}=2\times 3.14\times 1\times 10^{6}\sqrt{4\times 3.14\times 10^{-7}\times 1\times 8.854\times 10^{-12}\times 2}$$

$$\beta_{mn}=6.28\times 10^{6}\times 4.716\times 10^{-9}$$

$_{mn}$ = 0.0296 rad/m

2) The characteristic impedance for TEM wave is given by,

$$Z_{TEM}=\eta=\sqrt{\frac{\mu}{\varepsilon}}=\sqrt{\frac{\mu_0\ \mu_r}{\varepsilon_0\ \varepsilon r}}=\sqrt{\frac{4\times 3.14\times 10^{-7}\times 1}{8.854\times 10^{-12}\times 2}}=266.32\,\text{ohms}$$

Z_{TEM} = 266.32 ohms

RECTANGULAR WAVEGUIDE

1) A TE_{10} wave at 10GHz propagates in a brass σ_c=1.57x10^7(S/m) rectangular waveguide with inner dimensions a=1.5cm and b=0.6cm, which is filled with ε_r=2.25, μ_r=1, loss tangent=4x10^{-4}. Determine:

1) the phase constant ,

2) the guide wavelength,

3) the phase velocity,

4) the wave impedance,

5) the attenuation constant due to loss in the dielectric and,

6) the attenuation constant due to loss in the guide walls.

(EC6503 - NOV/DEC 2019) (15 Marks) (EC6503 - NOV/DEC 2018) (15 Marks)

Solution:

Given: mode = TE_{10}, f=10GHz, σ_c=1.57x10^7(S/m), a=1.5cm = 1.5x10^{-2} m and b=0.6cm = 0.6x10^{-2} m, ε_r=2.25, μ_r=1, loss tangent, tanδ=4x10^{-4}

1) The phase constant, β is given by

$$\beta_{mn}=\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

where, $$f_c=\frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$f_c=\frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2\sqrt{2.25}}\sqrt{\left(\frac{1}{1.5x10^{-2}}\right)^2 + \left(\frac{0}{0.6x10^{-2}}\right)^2}$$

$$f_c = 6.6667\,\text{GHz}$$

therefore, $\beta_{mn} = 2\pi f\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$\beta_{mn} = 2\times 3.14\times 10\times 10^9\sqrt{4\times 3.14\times 10^{-7}\times 1\times 8.854\times 10^{-12}\times 2.25}\sqrt{1-\left(\frac{6.6667\times 10^9}{10\times 10^9}\right)^2}$$

$$\beta_{mn} = 6.28\times 10^{10}\times 5.0021\times 10^{-9}\times 0.74535$$

$_{mn}$ = 234.139 rad/m

2) The guide wavelength λ_g is given by,

$$\lambda_g = \frac{2\pi}{\beta_{mn}} = \frac{2\times 3.14}{234.139} = 0.02682\,\text{m}$$

$^{a}{}_{g}$ = 0.02682m

3) The Phase velocity, v_p is given by,

$$v_p = \frac{\omega}{\beta_{mn}} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c\text{, and } \mu_r = 1$$

$$v_p = \frac{3\times 10^8}{\sqrt{2.25}\sqrt{1-\left(\frac{6.667\times 10^9}{10\times 10^9}\right)^2}} = \frac{3\times 10^8}{1.118} = 2.6834\times 10^8\,\text{m/sec}$$

v_p=2.6834x10^8 m/sec

4) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}}$$

$$Z_{TE} = \frac{2\times\pi\times f\times\mu_0\mu_r}{\beta_{mn}} = \frac{2\times3.14\times10\times10^9\times4\times3.14\times10^{-7}\times1}{234.139}$$

Z_{TE} = 336.88 ohms

5) The attenuation constant due to dielectric loss is given by,

$$\alpha_d = \frac{\sigma_d\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\text{, where } \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

In this problem, σ_d is not specified, but the loss tangent is given. The loss tangent $\tan\delta$ is

$$\tan\delta = \frac{\sigma_d}{\omega\varepsilon}$$

i.e., $\sigma_d = \omega\varepsilon\tan\delta = 2\pi f\varepsilon_0\varepsilon_r\tan\delta$

$$\text{therefore, } \alpha_d = \frac{\sigma_d\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{2\pi f\varepsilon_0\varepsilon_r\tan\delta\sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\pi f\sqrt{\mu_0}\sqrt{\varepsilon_0\varepsilon_r}\tan\delta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

substituting $\sqrt{\mu_0\varepsilon_0} = \frac{1}{c}$

$$\text{therefore, } \alpha_d = \frac{\pi f\tan\delta}{\frac{c}{\sqrt{\varepsilon_r}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_d = \frac{3.14\times10\times10^9\times4\times10^{-4}}{\frac{3\times10^8}{\sqrt{2.25}}\sqrt{1-\left(\frac{6.6667\times10^9}{10\times10^9}\right)^2}} \quad (\because \tan\delta = 4\text{x}10^{-4}\text{ given})$$

$$\alpha_d = \frac{12.56\times10^6}{149.07\times10^6} = 0.0843\,\text{Np/m}$$

±d = 0.0843 Np/m

6) The attenuation constant σ_c is given by,

σ_c for the TE_{10} mode is,

$$\alpha_c = \frac{R_s\left[\frac{2b}{a}\left(\frac{f_c}{f}\right)^2 + 1\right]}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \text{Np/m}$$

$$\text{where}, R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{3.14 \times 10 \times 10^9 \times 4\pi \times 10^{-7}}{1.57 \times 10^7}}$$

$$R_s = \frac{198.59}{3962.32} = 0.05012 \text{ ohms}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\ \mu_r}{\varepsilon_0\ \varepsilon r}} = \sqrt{\frac{4\pi \times 10^{-7} \times 1}{8.854 \times 10^{-12} \times 2.25}} = 251.0925 \text{ ohms}$$

$$\text{therefore, } \alpha_c = \frac{0.05012\left[\frac{2 \times 0.6 \times 10^{-2}}{1.5 \times 10^{-2}}\left(\frac{6.6667 \times 10^9}{10 \times 10^9}\right)^2 + 1\right]}{0.6 \times 10^{-2} \times 251.0925\sqrt{1-\left(\frac{6.6667 \times 10^9}{10 \times 10^9}\right)^2}} \text{Np/m}$$

$\sim_c$ = 0.0605 Np/m

Total attenuation in the waveguide = $\sim_c + \sim_d$

= 0.0605 +0.0843 = 0.1448 Np/m or 1.258 dB/m

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2) Calculate the cut-off frequency of a rectangular waveguide whose inner dimensions are a=2.5cm and b=1.5cm operating at TE_{10} mode. **(EC6503 - APR/MAY 2017) (2 Marks)**

Solution:

Given: a=2.5cm = $2.5\text{x}10^{-2}$ m and b=1.5cm = $1.5\text{x}10^{-2}$ m, mode = TE_{10}; m=1, n=0

The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2}\sqrt{\left(\frac{1}{2.5 \times 10^{-2}}\right)^2 + \left(\frac{0}{1.5 \times 10^{-2}}\right)^2}$$

f_c =6GHz

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3) An air filled rectangular waveguide of inner dimensions 2.286 x 1.016 in centimeters operates in the dominant TE_{10} modes. Calculate the cut-off frequency and phase velocity of a wave in the guide at a frequency of 7GHz. **(EC6503 - NOV/DEC 2016) (2 Marks)**

Solution:

Given: a= 2.286cm = 0.02286m, b=1.016cm=0.01016m. f=7GHz, mode=TE_{10},

The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{2.286x10^{-2}}\right)^2 + \left(\frac{0}{1.016x10^{-2}}\right)^2}$$

f_c = 6.56GHz

The phase velocity is given by,

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \text{for air medium} \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{6.56\times10^9}{7\times10^9}\right)^2}} = \frac{3\times10^8}{0.3489} = 8.597\times10^8 \text{m/sec}$$

v_p=8.597x10^8 m/sec

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4) A rectangular waveguide of cross section 5cm x 2cm is used to propagate TM_{11} mode at 10 GHz. Determine the cut-off wavelength.

(EC6503 - NOV/DEC 2015) (2marks) (EC2305 - NOV/DEC 2014) (2marks)

Solution:

Given: a=5cm = 0.05m, b=2cm =0.02m, f=10GHz, mode - TM_{11}, m=1, n=1

The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{1}{0.02}\right)^2}$$

$$f_c = 8.078\,\text{GHz}$$

therefore, cut-off wavelength is given by,

$$\lambda_c = \frac{c}{f_c} = \frac{3\times10^8}{8.078\times10^9} = 0.0371\,\text{m}$$

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5) A rectangular air-filled copper waveguide with dimension 0.9 inch x 0.4 inch cross section and 12 inch length is operated at 9.2 GHz with a dominant mode. Find cut-off frequency, guide wavelength, phase velocity, characteristics impedance and the loss. **(EC6503 - NOV/DEC 2015) (16 marks) (EC2305 - NOV/DEC 2011) (16 marks)**

Solution:

Given: a=0.9 inch=0.02286m, b=0.4inch=0.01016m, f=9.2GHz,

l=12inch=0.3048 (**Note**:1 inch=2.54cm), medium=air, dominant mode, TE_{10}

1) The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{2.286x10^{-2}}\right)^2 + \left(\frac{0}{1.016x10^{-2}}\right)^2}$$

f_c = 6.56GHz

2)The phase velocity is given by,

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \text{for air medium} \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{6.56\times10^9}{9.2\times10^9}\right)^2}} = \frac{3\times10^8}{0.7011} = 4.279\times10^8\,\text{m/sec}$$

v_p=4.279x10^8 m/sec

3) The guide wavelength λ_g is

$$\because\ \lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, the phase constant, β_{mn} is given by

$$\beta_{mn} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta_{mn} = 2\pi f\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta_{mn} = 2\times 3.14\times 9.2\times 10^9\sqrt{4\times 3.14\times 10^{-7}\times 1\times 8.854\times 10^{-12}\times 1}\sqrt{1-\left(\frac{6.56\times 10^9}{9.2\times 10^9}\right)^2}$$

$$\beta_{mn} = 5.7776\times 10^{10}\times 3.335\times 10^{-9}\times 0.7011$$

$_{mn}$ = 135.08 rad/m

therefore, the guide wavelength λ_g is given by,

$$\lambda_g = \frac{2\pi}{\beta_{mn}} = \frac{2\times 3.14}{135.08} = 0.0465\,\text{m}$$

$^{a}{}_{g}$ = 0.0465m

4) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}}$$

$$Z_{TE} = \frac{2\times\pi\times f\times\mu_0\mu_r}{\beta_{mn}} = \frac{2\times 3.14\times 9.2\times 10^9\times 4\times 3.14\times 10^{-7}\times 1}{135.08}$$

Z_{TE} = 537.21 ohms

5)The loss due to imperfectly conducting guide walls is found using the formula,

σ_c for the TE_{10} mode is,

$$\alpha_c = \frac{R_s\left[\frac{2b}{a}\left(\frac{f_c}{f}\right)^2+1\right]}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\,\text{Np/m}$$

$$\text{where}, R_s = \sqrt{\frac{\pi f\mu}{\sigma_c}}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{3.14 \times 9.2 \times 10^9 \times 4\pi \times 10^{-7}}{1.57 \times 10^7}}$$

$$R_s = \frac{190.48}{3962.32} = 0.0480\,\text{ohms}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\,\mu_r}{\varepsilon_0\,\varepsilon r}} = 377\,\text{ohms}$$

$$\text{therefore, } \alpha_c = \frac{0.0480\left[\frac{2 \times 0.01016}{0.02286}\left(\frac{6.56 \times 10^9}{9.2 \times 10^9}\right)^2 + 1\right]}{0.01016 \times 377\sqrt{1 - \left(\frac{6.56 \times 10^9}{9.2 \times 10^9}\right)^2}}\,\text{Np/m}$$

$$\alpha_c = \frac{0.0480\left[\frac{2 \times 0.01016}{0.02286}\left(\frac{6.56 \times 10^9}{9.2 \times 10^9}\right)^2 + 1\right]}{0.01016 \times 377\sqrt{1 - \left(\frac{6.56 \times 10^9}{9.2 \times 10^9}\right)^2}} = \frac{0.06969}{2.6855}\,\text{Np/m}$$

~$_c$ = 0.025955 Np/m

therefore, the total loss taking place throughout the waveguide is,

α_c (12 inch) = 0.025955 x 0.3048 = **7.9097x10^{-3}Np/m**

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6) An air filled rectangular waveguide of dimensions a=4.5cm and b=3cm operates in the TM_{11} mode. Find the cutoff wavelength and characteristic wave impedance at a frequency of 9 GHz. **(EC2305 - NOV/DEC 2016) (8marks) (EC2305 - NOV/DEC 2013) (4marks)**

Solution:

Given: a= 4.5cm = 0.045m, b=3cm=0.03m. f=9GHz, mode=TM_{11},

1) The cutoff wavelength, λc is given by

$$\lambda_c = \frac{c}{f_c}$$

where, the cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{0.045}\right)^2 + \left(\frac{1}{0.03}\right)^2}$$

f_c = 6.009GHz

Therefore the cutoff wavelength, λc is

$$\lambda_c = \frac{c}{f_c} = \frac{3\times10^8}{6.009\times10^9} = 0.049925\,\text{m}$$

2) The wave impedance Z_{TM} is given by,

$$Z_{TM} = \frac{\beta_{mn}}{\omega\varepsilon}\,(\text{Or})\,\eta\sqrt{1-\left(\frac{fc}{f}\right)^2}$$

$$Z_{TM} = \eta\sqrt{1-\left(\frac{fc}{f}\right)^2} = 377\sqrt{1-\left(\frac{6.009\times10^9}{9\times10^9}\right)^2}$$

Z_{TM} = 280.66 ohms

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7) An air filled rectangular waveguide of dimensions a=7cm and b=3.5cm operates in the dominant mode. Find the guide wavelength and phase velocity at a frequency of 3.5 GHz. **(EC2305 - MAY/JUNE 2016) (4marks)**

Solution:

Given: a= 7cm = 0.07m, b=3.5cm=0.035m. f=3.5GHz, mode=$TE_{10,}$

1) The guide wavelength, λg is given by

$$\lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, β_{mn} is given as,

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\omega = 2\pi f = 2\times3.14\times3.5\times10^9 = 2.198\times10^{10}$$

$$\omega^2 = 4.8312\times10^{20}$$

$$\mu\varepsilon = 11.11\times10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(4.8312\times10^{20}\times11.11\times10^{-18}) - \left[\left(\frac{3.14}{0.07}\right)^2 + \left(\frac{0}{0.035}\right)^2\right]}$$

$$\beta_{mn} = \sqrt{5.36747\times10^3 - 2012.16}$$

$$\beta_{mn} = 57.925\,\text{rad/m}$$

Therefore, $\lambda_g = \dfrac{2\pi}{\beta_{mn}} = \dfrac{2\pi}{57.925} = 0.1084\text{m}$

λ_g = 0.108 m

2)The phase velocity v_p is given by,

$$v_p = \frac{\omega}{\beta_m} = \frac{2.198 \times 10^{10}}{57.925} = 3.79456 \times 10^8 \text{ m/sec}$$

v_p = 3.7945 x 10^8 m/sec

----

8) A rectangular wave guide with dimensions a = 2.5 cm, b= 1 cm is to operate below 15 GHz. How many TE and TM modes can the waveguide transmit if the guide is filled with a medium characterized by α=0, $\varepsilon = 4\,\varepsilon_o$, μ_r=1? Calculate the cutoff frequencies of the modes. **(EC2305 - APR/MAY 2011) (16marks)**

Solution:

Given: a = 2.5 cm = 0.025m, b= 1cm= 0.01m, f=15 GHz, α=0, $\varepsilon = 4\,\varepsilon_o$, μ_r=1

The cut-off frequency is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3 \times 10^8}{2\sqrt{4}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = 75 \times 10^6 \sqrt{\left(\frac{m}{0.025}\right)^2 + \left(\frac{n}{0.01}\right)^2}$$

Now, let us consider different values of m and n

For TE_{01} mode: m=0, n=1

$$f_{c01} = 75 \times 10^6 \sqrt{\left(\frac{0}{0.025}\right)^2 + \left(\frac{1}{0.01}\right)^2} = 7.5\,\text{GHz}$$

For TE_{02} mode: m=0, n=2

$$f_{c02} = 75 \times 10^6 \sqrt{\left(\frac{0}{0.025}\right)^2 + \left(\frac{2}{0.01}\right)^2} = 15\,\text{GHz}$$

For TE_{03} mode: m=0, n=3

$$f_{c03} = 75\times10^6\sqrt{\left(\frac{0}{0.025}\right)^2+\left(\frac{3}{0.01}\right)^2} = 22.5\,\text{GHz}$$

Thus for TE_{03} mode, $f_{c03} > f$, i.e., $f_{c03} > 15$ GHz. Hence the modes with n>2 cannot propagate through guide.

For TE_{10} mode: m=1, n=0

$$f_{c10} = 75\times10^6\sqrt{\left(\frac{1}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 3\,\text{GHz}$$

For TE_{20} mode: m=2, n=0

$$f_{c20} = 75\times10^6\sqrt{\left(\frac{2}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 6\,\text{GHz}$$

For TE_{30} mode: m=3, n=0

$$f_{c30} = 75\times10^6\sqrt{\left(\frac{3}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 9\,\text{GHz}$$

For TE_{40} mode: m=4, n=0

$$f_{c40} = 75\times10^6\sqrt{\left(\frac{4}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 12\,\text{GHz}$$

For TE_{50} mode: m=5, n=0

$$f_{c50} = 75\times10^6\sqrt{\left(\frac{5}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 15\text{GHz}$$

For TE_{60} mode: m=6, n=0

$$f_{c60} = 75\times10^6\sqrt{\left(\frac{6}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2} = 18\,\text{GHz}$$

Thus for TE_{60} mode, $f_{c60} > f$, i.e., $f_{c60} > 15$ GHz. Hence the modes with m>5 cannot propagate through guide.

For TE_{11} , TM_{11} modes (Degenerate modes): m=1, n=1

$$f_{c11} = 75\times10^6\sqrt{\left(\frac{1}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2} = 8.078\,\text{GHz}$$

For TE_{12} , TM_{12} modes (Degenerate modes): m=1, n=2

$$f_{c12} = 75\times10^6\sqrt{\left(\frac{1}{0.025}\right)^2+\left(\frac{2}{0.01}\right)^2} = 15.3\,\text{GHz}$$

For TE_{21} , TM_{21} modes (Degenerate modes): m=2, n=1

$$f_{c21} = 75\times10^6\sqrt{\left(\frac{2}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2} = 9.6\,\text{GHz}$$

For TE_{31} , TM_{31} modes (Degenerate modes): m=3, n=1

$$f_{c31} = 75\times10^6\sqrt{\left(\frac{3}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2} = 11.72\,\text{GHz}$$

For TE_{41} , TM_{41} modes (Degenerate modes): m=4, n=1

$$f_{c41} = 75\times10^6\sqrt{\left(\frac{4}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2} = 14.14\,\text{GHz}$$

For TE_{51} , TM_{51} modes (Degenerate modes): m=5, n=1

$$f_{c51} = 75\times10^6\sqrt{\left(\frac{5}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2} = 16.77\,\text{GHz}$$

Thus the modes whose cut-off frequency is less than or equal to 15 GHZ will be propagated or transmitted. This in all total 11 TE modes and 4 AM modes are transmitted by the rectangular guide.

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9) An air filled rectangular waveguide of dimension a=6cm and b=4cm operates in the TM_{11} mode. Find the cutoff frequency, guide wavelength and phase velocity at a frequency of 3GHz. APR-MAY2014 (6 Marks)

Solution:

Given: a=6cm = 0.06m, b=4cm = 0.04m, mode-TM_{11}, m=1, n=1, f=3GHz

The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{0.06}\right)^2+\left(\frac{1}{0.04}\right)^2}$$

f_c = 4.5069 GHz

2) The guide wavelength λ_g, is given by,

$$\lambda_g = \frac{2\pi}{\beta_{mn}}$$

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\omega = 2\pi f = 2\times 3.14\times 3\times 10^9 = 1.884\times 10^{10}$$

$$\omega^2 = 3.549\times 10^{20}$$

$$\mu\varepsilon = 11.11\times 10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(3.549\times 10^{20}\times 11.11\times 10^{-18}) - \left[\left(\frac{3.14}{0.06}\right)^2 + \left(\frac{3.14}{0.04}\right)^2\right]}$$

$$\beta_{mn} = \sqrt{3.943\times 10^3 - 8901.02}$$

β_{mn} value is imaginary. So, no wave propagates and guide wavelength cannot be calculated.

2)The phase velocity v_p is given by,

$$v_p = \frac{\omega}{\beta_m}$$

The wave does not propagate.

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10) An air filled rectangular waveguide has a=6cm and b=4cm. The signal frequency is 3GHz. Determine the cutoff frequency for TM_{11} mode.

(EC2305 - APR/MAY 2016) (2 marks)

Solution:

Given: a=6cm = 0.06m, b=4cm = 0.04m, mode-TM_{11}, m=1, n=1, f=3GHz

The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{0.06}\right)^2 + \left(\frac{1}{0.04}\right)^2}$$

f_c = 4.5069 GHz

---

11) A rectangular waveguide of cross section 5cm x 2cm is used to propagate TM_{11} mode at 10GHz. Determine the cutoff wave length.

(EC2305 - NOV/DEC 2015) (2 marks), (EC2305 - NOV/DEC 2011) (2 marks)

Solution:

Given: a=5cm = 0.05m, b=2cm = 0.02m, mode-TM_{11}, m=1, n=1, f=10GHz

The cutoff wavelength, λc is given by

$$\lambda_c = \frac{c}{f_c}$$

where, the cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{1}{0.02}\right)^2}$$

f_c = 8.078 GHz

Therefore the cutoff wavelength, λc is

$$\lambda_c = \frac{c}{f_c} = \frac{3 \times 10^8}{8.078 \times 10^9} = 0.0371 \text{ m}$$

$^a{}_c$=0.0371 m

----

12) A rectangular air-filled copper waveguide with a=2.28 am and b=1.01 am cross section and l=30.45 cm is operated at 9.2 GHz with a dominant mode. Find cut-off frequency, guide wavelength, phase velocity, and characteristics impedance. **(EC2305 - NOV/DEC 2014) (16 marks)**

Solution:

Given: a=2.28 cm = 0.0228 m, b=1.01inch=0.0101m, f= 9.2GHz,

l=30.45cm=0.3045m, medium=air, dominant mode, TE_{10}

1) The cut-off frequency f_c is given by,

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3x10^8}{2}\sqrt{\left(\frac{1}{2.28x10^{-2}}\right)^2 + \left(\frac{0}{1.01x10^{-2}}\right)^2}$$

f_c = 6.579GHz

2)The phase velocity is given by,

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \text{for air medium} \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{6.579\times10^9}{9.2\times10^9}\right)^2}} = \frac{3\times10^8}{0.699} = 4.292\times10^8 \text{m/sec}$$

v_p=4.292x10⁸ m/sec

3) The guide wavelength λ_g is

$$\because \lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, the phase constant, β_{mn} is given by

$$\beta_{mn} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta_{mn} = 2\pi f\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta_{mn} = 2\times3.14\times9.2\times10^9\sqrt{4\times3.14\times10^{-7}\times1\times8.854\times10^{-12}\times1}\sqrt{1-\left(\frac{6.579\times10^9}{9.2\times10^9}\right)^2}$$

$$\beta_{mn} = 5.7776\times10^{10}\times3.335\times10^{-9}\times0.699$$

$_{mn}$ = 134.69 rad/m

therefore, the guide wavelength λ_g is given by,

$$\lambda_g = \frac{2\pi}{\beta_{mn}} = \frac{2\times3.14}{134.69} = 0.0466\,\text{m}$$

$^a{}_g$ = 0.0466m

4) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}}$$

$$Z_{TE} = \frac{2\times\pi\times f\times\mu_0\mu_r}{\beta_{mn}} = \frac{2\times3.14\times9.2\times10^9\times4\times3.14\times10^{-7}\times1}{134.69}$$

Z_{TE} = 538.77 ohms

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13) A rectangular waveguide with a=7cm and b=3.5cm is used to propagate TM_{10} at 3.5GHZ. Determine the guided wavelength.

(EC2305 - NOV/DEC 2013) (2marks)

Solution:

Given: a= 7cm = 0.07m, b=3.5cm=0.035m. f=3.5GHz, mode=TM_{10},

1) The guide wavelength, λg is given by

$$\lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, β_{mn} is given as,

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\omega = 2\pi f = 2\times 3.14\times 3.5\times 10^9 = 2.198\times 10^{10}$$

$$\omega^2 = 4.8312\times 10^{20}$$

$$\mu\varepsilon = 11.11\times 10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(4.8312\times 10^{20}\times 11.11\times 10^{-18}) - \left[\left(\frac{3.14}{0.07}\right)^2 + \left(\frac{0}{0.035}\right)^2\right]}$$

$$\beta_{mn} = \sqrt{5.36747\times 10^3 - 2012.16}$$

$$\beta_{mn} = 57.925\,\mathrm{rad/m}$$

Therefore, $\lambda_g = \frac{2\pi}{\beta_{mn}} = \frac{2\pi}{57.925} = 0.1084\mathrm{m}$

a_g = 0.108 m

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14) A rectangular waveguide measuring a=4.5cm and b=3cm internally has a 9GHZ signal propagated in it. Calculate the guide wavelength, phase and group velocities and characteristic impedance for the dominant mode.

(EC2305 - NOV/DEC 2010) (6marks)

Solution:

Given: a= 4.5cm = 0.045m, b=3cm=0.03m. f=9GHz, mode=TE_{10},

1) The guide wavelength, λ_g is given by

$$\lambda_g = \frac{2\pi}{\beta_{mn}}$$

where, β_{mn} is given as,

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\omega = 2\pi f = 2\times 3.14\times 9\times 10^9 = 5.652\times 10^{10}$$

$$\omega^2 = 31.945\times 10^{20}$$

$$\mu\varepsilon = 11.11\times 10^{-18}$$

$$\therefore \beta_{mn} = \sqrt{(31.945\times 10^{20}\times 11.11\times 10^{-18}) - \left[\left(\frac{3.14}{0.045}\right)^2 + \left(\frac{0}{0.03}\right)^2\right]}$$

$$\beta_{mn} = \sqrt{35.4910\times 10^3 - 4868.94}$$

$$\beta_{mn} = 174.99\ \text{rad/m}$$

Therefore, $\lambda_g = \dfrac{2\pi}{\beta_{mn}} = \dfrac{2\pi}{174.99} = 0.0359\text{m}$

$^a{}_g$ = 0.0359 m

2) The Phase velocity, v_p is given by

$$v_p = \frac{\omega}{\beta_m} = \frac{5.652\times 10^{10}}{174.99} = 3.23\times 10^8\ \text{m/sec}$$

v_p=3.23x10^8 m/sec

3) Group velocity, v_g

$$v_g = \frac{c^2}{v_p} = \frac{(3\times 10^8)^2}{3.23\times 10^8} = 2.786\times 10^8\ \text{m/sec}$$

v_g=2.786x10^8 m/sec

4)The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}}$$

$$Z_{TE} = \frac{2\times\pi\times f\times\mu_0\mu_r}{\beta_{mn}} = \frac{2\times 3.14\times 9\times 10^9\times 4\times 3.14\times 10^{-7}\times 1}{174.99}$$

Z_{TE} = 405.675 ohms

CIRCULAR WAVEGUIDE

1) A TE_{11} wave is propagating through a circular waveguide. The diameter of the guide is 10cm and the guide is air-filled. Given X_{11} = 1.842.

(1) Find the cut-off frequency. (3)

(2) Find the wavelength in the guide for a frequency of 3 GHz. (2)

(3) Determine the wave impedance in the guide. (3)

(EC6503 - NOV/DEC 2016) (8 Marks) (EC2305 - NOV/DEC 2018) (8 Marks)

Solution:

Given: diameter = 10cm, a=5cm = 0.05m, Mode - TE_{11}, m=1, n=1

(1) The cut off frequency f_c is given by,

$$f_c = \frac{(ha)_{mn}}{2\pi a\sqrt{\mu\varepsilon}} = \frac{(ha)_{mn}\,c}{2\pi a} \qquad \because \frac{1}{\sqrt{\mu\varepsilon}} = c; \quad (\text{for air medium})$$

$$f_c = \frac{(ha)_{mn}\,c}{2\pi a} = \frac{(ha)_{11}\,c}{2\pi a}$$

$$f_c = \frac{1.842 \times 3 \times 10^8}{2\pi \times 5 \times 10^{-2}} = 1.7599\,\text{GHz}; \qquad \because (ha)_{11} = 1.842\ ;\text{given value}$$

f_c = 1.7599 GHz

(2) The wavelength in the guide for a frequency of 3 GHz is given by,

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{3\times 10^9} = 0.1\text{m}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{0.1}{\sqrt{1-\left(\frac{1.7599\times 10^9}{3\times 10^9}\right)^2}} = \frac{0.1}{0.8099} = 0.1234\text{m}$$

$^a{}_g$ = 0.1234 m

(c) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}} \text{ (or) } Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{1.7599\times10^9}{3\times10^9}\right)^2}}$$

$$Z_{TE} = \frac{377}{0.8099} = 465.49 \text{ ohms}; \qquad \because \eta = 120\pi \text{ or } 377 \text{ ohms for air medium}$$

Z_{TE} = 465.49 ohms

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2) An air filled circular waveguide having an inner radius of 1cm is excited in dominant mode at 10GHz. Find (a) the cut off frequency of the dominant mode at 10GHz. (b) the guide wavelength and (c) wave impedance. Also find the bandwidth for operation in the dominant mode only.

(EC6503 - APR/MAY 2018) (13 Marks) (EC2305 - APR/MAY 2019) (16 Marks) (EC2305 - NOV/DEC 2011) (8 marks)

Solution:

Given: Dominant mode (TE_{11}), inner radius, a= 1cm=1×10^{-2}m, f=10GHz

(a) the cut off frequency f_c is given by,

$$f_c = \frac{h_{mn}}{2\pi\sqrt{\mu\varepsilon}} = \frac{(ha)_{mn}}{2\pi a\sqrt{\mu\varepsilon}} = \frac{(ha)_{mn}\,c}{2\pi a} \qquad \because hmn = \frac{(ha)_{mn}}{a}, \frac{1}{\sqrt{\mu\varepsilon}} = c; \quad \text{(for air medium)}$$

$$f_c = \frac{(ha)_{mn}\,c}{2\pi a} = \frac{(ha)_{11}\,c}{2\pi a}$$

$$f_c = \frac{1.841\times3\times10^8}{2\pi\times1\times10^{-2}} = 8.795\,\text{GHz}; \qquad \because (ha)_{11} = 1.841$$

f_c = 8.795 GHz

(b) the guide wavelength λ_g is given by,

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 3\times10^{-2}\,\text{m}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^{-2}}{\sqrt{1-\left(\frac{8.795\times10^9}{10\times10^9}\right)^2}} = \frac{3\times10^{-2}}{0.4759} = 0.063\text{m}$$

$^a{}_g$ **= 0.063 m**

(c) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}} \text{ (or) } Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{8.795\times10^9}{10\times10^9}\right)^2}}$$

$$Z_{TE} = \frac{377}{0.4759} = 792.185 \text{ ohms}; \qquad \because \eta = 120\pi \text{ or } 377 \text{ ohms for air medium}$$

Z_{TE} = 792.185 ohms

(d) The bandwidth is given by,

Bandwidth = cutoff frequency of TM_{01} - cutoff frequency of TE_{11}

cutoff frequency of TM_{01} is $f_c = \frac{(ha)_{mn}c}{2\pi a} = \frac{(ha)_{01}c}{2\pi a}$

$$f_c = \frac{2.405\times3\times10^8}{2\pi\times1\times10^{-2}} = 11.4889\,\text{GHz}; \qquad \because (ha)_{01} = 2.405$$

f_c = 11.4889 GHz

f_c for TM_{01}= 11.4889 GHz and f_c for TE_{01}= 8.795 GHz

Therefore Bandwidth = (11.4889 - 8.795)GHz = 2.695 GHz

Bandwidth = 2.695 GHz

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3) When dominant mode is transmitted through a circular waveguide, the wavelength measured is to be 13.33cm. The frequency of the microwave signal is 3.75GHz. Calculate the cut-off frequency, inner radius of guide, phase velocity, group velocity, phase constant, wave impedance, bandwidth for operation in dominant mode only. **(EC6503 - APR/MAY 2017) (2 Marks)**

Solution:

Given: Dominant mode (TE_{11}), λ_g=13.33cm = 13.33x10^{-2} m, f=3.75 GHz

1) The Cut-off frequency f_c is given by

$$f_c = \frac{(ha)_{mn}c}{2\pi a} = \frac{(ha)_{11}c}{2\pi a}$$

Note: inner radius of the circular guide is not given. so, we cannot use the above formula. To find f_c, first we have to find λc and then using λc we have to find f_c.

W.K.T $\frac{1}{\lambda^2{}_0} = \frac{1}{\lambda^2{}_c} + \frac{1}{\lambda^2{}_g}$

where, $\lambda_0 = \frac{c}{f} = \frac{3x10^8}{3.75x10^9} = 0.08m$

$$\frac{1}{(0.08)^2} = \frac{1}{\lambda^2{}_c} + \frac{1}{(0.01333)^2}$$

$^a{}_c$=0.1m

Therefore, f_c = $f_c = \frac{c}{\lambda_c} = \frac{3\times10^8}{0.1} = 3\,GHz$

f_c = 3GHz

2) The inner radius can be found using the formula,

$$f_c = \frac{(ha)_{mn}\,c}{2\pi a} = \frac{(ha)_{11}\,c}{2\pi a}$$

$$a = \frac{(ha)_{11}\,c}{2\pi f_c} = \frac{1.841\times3\times10^8}{2\times3.14\times3\times10^9} = 0.02932\,m$$

a=0.02932m

3) The Phase velocity, v_p is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \qquad \because \text{ for air medium } \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{3\times10^9}{3.75\times10^9}\right)^2}} = \frac{3\times10^8}{0.6} = 5.0\times10^8\,m/sec$$

v_p=5.0x10^8 m/sec

Group velocity, v_g

$$v_g = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{5\times10^8} = 2.8618\times10^8\,m/sec$$

v_g=1.8x10^8 m/sec

$$\because \lambda_g = \frac{2\pi}{\beta_{mn}}$$

$$\beta_{mn} = \frac{2\pi}{\lambda_g} = \frac{2\times 3.14}{0.01333} = 471.118\,\text{m}$$

(c) The wave impedance Z_{TE} is given by,

$$Z_{TE} = \frac{\omega\mu}{\beta_{mn}} \text{ (or) } Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{3\times 10^9}{3.75\times 10^9}\right)^2}}$$

$$Z_{TE} = \frac{377}{0.6} = 628.33 \text{ ohms;} \qquad \because \eta = 120\pi \text{ or } 377 \text{ ohms for air medium}$$

Z_{TE} = 628.33 ohms

(d) The bandwidth is given by,

Bandwidth = cutoff frequency of TM_{01} - cutoff frequency of TE_{11}

cutoff frequency of TM_{01} is $f_c = \frac{(ha)_{mn}\,c}{2\pi a} = \frac{(ha)_{01}\,c}{2\pi a}$

$$f_c = \frac{2.405\times 3\times 10^8}{2\pi\times 0.02932\times 10^{-2}} = 3.9184\,\text{GHz}; \qquad \because (ha)_{01} = 2.405$$

f_c = 3.9184 GHz

f_c for TM_{01}= 3.9184 GHz and f_c for TE_{01}= 3 GHz

Therefore Bandwidth = (3.9184 - 3)GHz = 0.9184 GHz

Bandwidth = 0.9184 GHz

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4) A 10 GHZ signal is to be transmitted inside a hollow circular conducting pipe. Determine the inside diameter of the pipe such that its lowest cut off frequency is 20% below this signal frequency. (6) **[AU-MAY 2012]**

Solution:

Given: f=10 GHz, f_c = 20%f

The lowest cutoff frequency for the dominant mode is given by,

$$f_c = \frac{(ha)_{mn}\, c}{2\pi a} = \frac{(ha)_{11}\, c}{2\pi a}$$

$$f_c = \frac{(ha)_{11}\, c}{2\pi a} = \frac{1.841 \times 3 \times 10^8}{2 \times 3.14 \times 3 \times 10^9} = \frac{0.293c}{a}$$

But the lowest cut-off frequency fc_{11} is 20% less than the operating frequency of 10 GHz. Hence we can write,

fc_{11} = 10 GHz - (20% of 10 GHZ)

= $10x10^9$ - $2x10^9$

= 8 GHz

therefore, $a = \frac{0.293c}{f_c} = \frac{0.293 \times 3 \times 10^8}{8 \times 10^9} = 1.098\,\text{cm}$

a=1.095 am

Hence the inner diameter of the pipe is 2a = 2.197cm
