SMALL CIRCULAR LOOP :-

The loop antenna is placed on the x-y plane at $z=0$. The wire is assumed to be very thin and the current spatial distribution is given by,

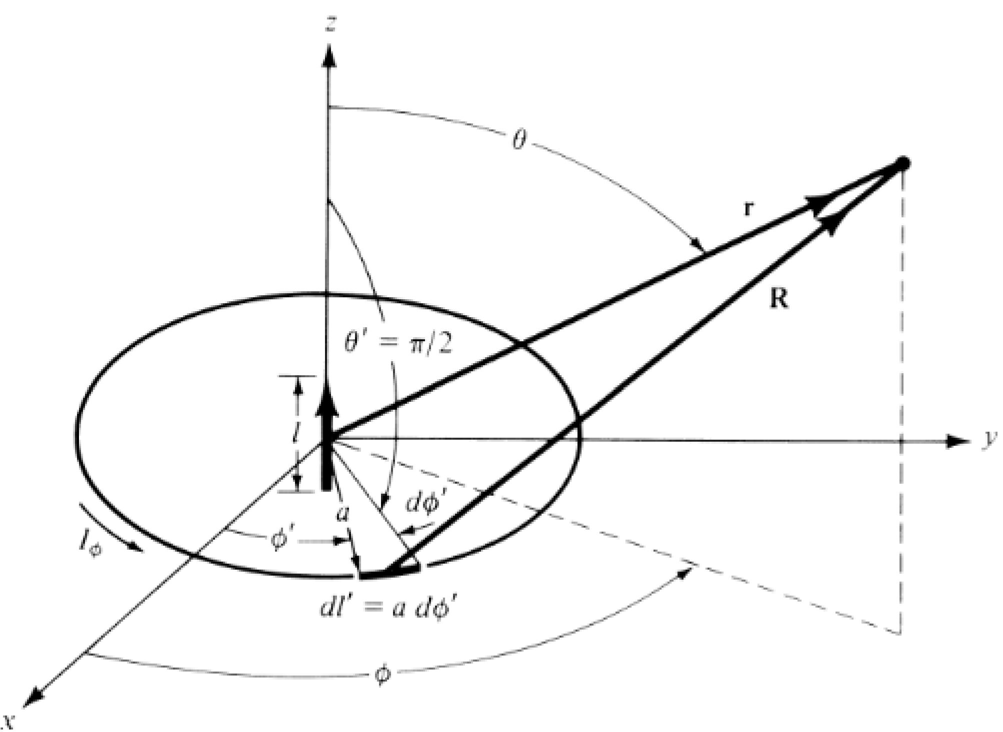
$$I_\phi = I_0 \rightarrow (1)$$

To find the fields distributed by the loop, the same procedure is followed as the linear dipole.

The potential function 'A' is given by,

$$A(x,y,z) = \frac{\mu}{4\pi} \int_C I_e(x',y',z') \frac{e^{-jKR}}{R} \cdot dl' \rightarrow (2)$$

$dl \rightarrow$ Infinitesimal section of the Loop



(a) Geometry for circular loop

The current spatial distribution can be given as,

$$I_e(x', y', z') = a_x \cdot I_x(x', y', z') + a_y I_y(x', y', z') + a_z I_z(x', y', z') \rightarrow (3)$$

For circular loop antenna, it will be more convenient to write the rectangular components in terms of cylindrical components.

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \cos\phi' & -\sin\phi' & 0 \\ \sin\phi' & \cos\phi' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_\rho \\ I_\phi \\ I_z \end{bmatrix} \rightarrow (4)$$

$$\left.\begin{aligned} I_x &= I_\rho \cos\phi' - I_\phi \sin\phi' \\ I_y &= I_\rho \cdot \sin\phi' + I_\phi \cos\phi' \\ I_z &= I_z \end{aligned}\right\} \rightarrow (5)$$

Since the radiated fields are usually determined in spherical components, the rectangular unit vectors are transformed to spherical unit vectors.

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$$

$$\left.\begin{aligned} \hat{a}_x &= \hat{a}_r \sin\theta\cos\phi + \hat{a}_\theta \cos\theta\cos\phi - \hat{a}_\phi \sin\phi \\ \hat{a}_y &= \hat{a}_r \sin\theta\sin\phi + \hat{a}_\theta \cos\theta\sin\phi + \hat{a}_\phi \cos\phi \\ \hat{a}_z &= \hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta \end{aligned}\right\} \rightarrow (6)$$

substitute (5) and (6) in (3)

$$I_e = (\hat{a}_r \sin\theta \cos\phi + \hat{a}_\theta \cos\theta \cos\phi - \hat{a}_\phi \sin\phi) \times (I_\rho \cos\phi' - I_\phi \sin\phi')$$
$$+ (\hat{a}_r \sin\theta \sin\phi + \hat{a}_\theta \cos\theta \sin\phi + \hat{a}_\phi \cos\phi) \times (I_\rho \sin\phi' + I_\phi \cos\phi')$$
$$+ (\hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta) I_z$$

$$I_e = \hat{a}_r \sin\theta \cos\phi \, I_\rho \cos\phi' - \hat{a}_r \sin\theta \cos\phi \, I_\phi \sin\phi'$$
$$+ \hat{a}_\theta \cos\theta \cos\phi \, I_\rho \cos\phi' - \hat{a}_\theta \cos\theta \cos\phi \, I_\phi \sin\phi'$$
$$- \hat{a}_\phi \sin\phi \, I_\rho \cos\phi' + \hat{a}_\phi \sin\phi \, I_\phi \sin\phi'$$
$$+ \hat{a}_r \sin\theta \sin\phi \, I_\rho \sin\phi' + \hat{a}_r \sin\theta \sin\phi \, I_\phi \cos\phi'$$
$$+ \hat{a}_\theta \cos\theta \sin\phi \, I_\rho \sin\phi' + \hat{a}_\theta \cos\theta \sin\phi \, I_\phi \cos\phi'$$
$$+ \hat{a}_\phi \cos\phi \, I_\rho \sin\phi' + \hat{a}_\phi \cos\phi \cdot I_\phi \cos\phi'$$
$$+ \hat{a}_r \cos\theta \, I_z - \hat{a}_\theta \sin\theta \, I_z .$$

$$I_e = \hat{a}_r [I_\rho \sin\theta \cos\phi \cos\phi' - I_\phi \sin\theta \cos\phi \cdot \sin\phi'$$
$$+ I_\rho \sin\theta \sin\phi \sin\phi' + I_\phi \sin\theta \sin\phi \cos\phi'$$
$$+ I_z \cos\theta]$$
$$+ \hat{a}_\theta [I_\rho \cos\theta \cos\phi \cos\phi' - I_\phi \cos\theta \cos\phi \sin\phi'$$
$$+ I_\rho \cos\theta \sin\phi \sin\phi' + I_\phi \cos\theta \sin\phi \cos\phi'$$
$$- I_z \sin\theta]$$
$$+ \hat{a}_\phi [-I_\rho \sin\phi \cos\phi' + I_\phi \sin\phi \sin\phi'$$
$$+ I_\rho \cos\phi \sin\phi' + I_\phi \cos\phi \cos\phi']$$

$$I_e = \hat{a}_r \left[I_\rho \sin\theta \cos(\phi-\phi') + I_\phi \sin\theta . \sin(\phi-\phi') + I_z \cos\theta \right]$$
$$+ \hat{a}_\theta \left[I_\rho \cos\theta \cos(\phi-\phi') + I_\phi \cos\theta \sin(\phi-\phi') - I_z \sin\theta \right]$$
$$+ \hat{a}_\phi \left[-I_\rho \sin(\phi-\phi') + I_\phi \cos(\phi-\phi') \right] \rightarrow (7)$$

The source coordinates are designated as Primed (ρ', ϕ', z') and the observation coordinates are designated as unprimed (r, θ, ϕ)

For circular loop, the current is flowing in ϕ direction. so, equ (7) reduces to

$$I_e = \hat{a}_r I_\phi \sin\theta \sin(\phi-\phi') + \hat{a}_\theta I_\phi \cos\theta \sin(\phi-\phi') + \hat{a}_\phi I_\phi \cos(\phi-\phi') \rightarrow (8)$$

The distance R, from any point on the loop to the observation point can be written as,

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \rightarrow (9)$$

since,

$$\left.\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \\ x^2+y^2+z^2 &= r^2 \end{aligned}\right\} \rightarrow (10a)$$

$$\left.\begin{aligned} x' &= a\cos\phi' \\ y' &= a\sin\phi' \\ z' &= 0 \\ x'^2 + y'^2 + z'^2 &= a^2 \end{aligned}\right\} \rightarrow (10b)$$

substitute (10a) and (10b) in (9)

$$R = \sqrt{(r\sin\theta\cos\phi - a\cos\phi')^2 + (r\sin\theta\sin\phi - a\sin\phi')^2 + (r\cos\theta - 0)^2}$$

$$= \sqrt{r^2\sin^2\theta\cos^2\phi + a^2\cos^2\phi' - 2ar\sin\theta\cos\phi.\cos\phi' + r^2\sin^2\theta\sin^2\phi + a^2\sin^2\phi' - 2ar\sin\theta\sin\phi\sin\phi' + r^2\cos^2\theta}$$

$$= \sqrt{r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + a^2(\cos^2\phi' + \sin^2\phi') - 2ar\sin\theta(\cos\phi\cos\phi' + \sin\phi\sin\phi') + r^2\cos^2\theta}$$

$$= \sqrt{r^2\sin^2\theta + a^2 - 2ar\sin\theta\cos(\phi - \phi') + r^2\cos^2\theta}$$

$$R = \sqrt{r^2(\sin^2\theta + \cos^2\theta) + a^2 - 2ar\sin\theta\cos(\phi - \phi')}$$

$$R = \sqrt{r^2 + a^2 - 2ar\sin\theta\cos(\phi - \phi')} \rightarrow (11)$$

The differential element length is given by

$$dl' = a\,d\phi' \rightarrow (12)$$

substitute equs. (8), (11) and (12) in (2)

$$A_\phi = \frac{\mu a}{4\pi}\int_0^{2\pi} \frac{I_\phi \cos(\phi-\phi') \cdot e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar\sin\theta.\cos(\phi-\phi')}} \cdot d\phi \quad \rightarrow (13)$$

for simplicity, $\phi = 0$.

$$A_\phi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi} \frac{\cos\phi' \cdot e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}} \cdot d\phi \quad \rightarrow (14)$$

For small loops, the function

$$f = \frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta.\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}} \quad \rightarrow (15)$$

The above function can be expanded by Maclaurin series in 'a' using

$$f = f(0) + \frac{1}{1!} f'(0).a + \frac{1}{2!} f''(0).a^2 + \cdots + \frac{1}{(n-1)!} f^{(n-1)}(0).a^{n-1} + \cdots \quad \rightarrow (16)$$

where,

$$f'(0) = \left.\frac{\partial f}{\partial a}\right|_{a=0} \quad \rightarrow (17)$$

$$f''(0) = \left.\frac{\partial^2 f}{\partial a^2}\right|_{a=0} \quad \rightarrow (18)$$

Taking into account only the 1st 2 terms of equ (16)

$$f(0) = \frac{e^{-jkr}}{r} \rightarrow \quad (19)$$

equ. (15) can be written as,

$$f = \frac{e^{-jk(r^2+a^2-2ar\sin\theta\cos\phi')^{1/2}}}{(r^2+a^2-2ar\sin\theta\cos\phi')^{1/2}}$$

$$f'(0) = \frac{\left[(r^2+a^2-2ar\sin\theta\cos\phi')^{1/2} \cdot e^{-jk(r^2+a^2-2ar\sin\theta\cos\phi')^{1/2}} \cdot \left(\frac{-jk}{2}\right)(r^2+a^2-2ar\sin\theta\cos\phi')^{-1/2} \cdot (2a-2r\sin\theta\cos\phi')\right] - \left[e^{-jk(r^2+a^2-2ar\sin\theta\cos\phi')^{1/2}} \cdot \frac{1}{2}(r^2+a^2-2ar\sin\theta\cos\phi')^{-1/2} \cdot (2a-2r\sin\theta\cos\phi')\right]}{(r^2+a^2-2ar\sin\theta\cos\phi')}$$

$$f'(0)\Big|_{a=0} = \frac{\left[(r^2)^{1/2} \cdot e^{-jk(r^2)^{1/2}} \cdot \left(\frac{-jk}{2}\right)(r^2)^{-1/2} \cdot (-2r\sin\theta\cos\phi')\right] - \left[e^{-jk(r^2)^{1/2}} \cdot \frac{1}{2}(r^2)^{-1/2} \cdot (-2r\sin\theta\cos\phi')\right]}{r^2}$$

$$f'(0)\Big|_{a=0} = \frac{r \cdot e^{-jkr} \cdot \frac{jk}{\cancel{2r}} \cdot \cancel{2r}\sin\theta\cos\phi' + \frac{e^{-jkr}}{\cancel{2r}} \cdot \cancel{2r}\sin\theta\cos\phi'}{r^2}$$

$$f'(0)\Big|_{a=0} = \frac{jkr\, e^{-jkr}.\ \sin\theta\cos\phi' + e^{-jkr}.\ \sin\theta\cos\phi'}{r^2}$$

$$f'(0)\Big|_{a=0} = \left(\frac{jk}{r} + \frac{1}{r^2}\right) e^{-jkr}.\ \sin\theta\cos\phi' \rightarrow (20)$$

sub (19) & (20) in (16)

$$\therefore f \simeq \left[\frac{1}{r} + a\left(\frac{jk}{r} + \frac{1}{r^2}\right)\sin\theta\cos\phi'\right] e^{-jkr} \rightarrow (21)$$

Equation (14) reduces to

$$A\phi \simeq \frac{a\mu I_0}{4\pi}\int_0^{2\pi}\cos\phi'\left[\frac{1}{r} + a\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\cos\phi'\right].\ e^{-jkr}.\,d\phi'$$

$$A\phi \simeq \frac{a\mu I_0}{4\pi}e^{-jkr}\left[\int_0^{2\pi}\frac{\cos\phi'}{r}.\,d\phi' + \int_0^{2\pi} a\left(\frac{jk}{r}+\frac{1}{r^2}\right).\ \sin\theta\cos^2\phi'\,d\phi\right]$$

$$A\phi \simeq \frac{a\mu I_0}{4\pi}e^{-jkr}\left[\frac{1}{r}\left[\sin\phi'\right]_0^{2\pi} + a\left(\frac{jk}{r}+\frac{1}{r^2}\right).\ \sin\theta\int_0^{2\pi}\frac{1-\cos 2\phi}{2}.\,d\phi\right]$$

$$A\phi \simeq \frac{a\mu I_0}{4\pi}e^{-jkr}\left[0 + \frac{a}{2}\left(\frac{jk}{r}+\frac{1}{r^2}\right).\ \sin\theta\left[\phi - \frac{\sin 2\phi}{2}\right]_0^{2\pi}\right]$$

$$A\phi \simeq \frac{a\mu I_0.\,e^{-jkr}}{4\pi}.\ \frac{a}{2}\left(\frac{jk}{r}+\frac{1}{r^2}\right).\ \sin\theta\Big[2\pi - \underset{0}{\underset{\uparrow}{\sin 8\pi}} - 0 + \underset{0}{\underset{\uparrow}{\sin 0}}\Big]$$

$$A\phi \simeq \frac{a\mu I_0.\,e^{-jkr}}{4\cancel{\pi}}.\ \frac{a}{\cancel{2}}\left(\frac{jk}{r}+\frac{1}{r^2}\right).\ \sin\theta.\ \cancel{2\pi}$$

$$\therefore A_\phi \simeq \frac{a^2 \mu I_0}{4} \cdot e^{-jkr} \cdot \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta \rightarrow (22)$$

similarly A_r and A_θ can be obtained but which when integrated reduces to zero. Thus.

$$A \simeq \hat{a}_\phi A_\phi = \hat{a}_\phi \cdot \frac{a^2 \mu I_0}{4} \cdot e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta$$

$$A \simeq \hat{a}_\phi \frac{jk\mu a^2 I_0 \sin\theta}{4r} \left(1 + \frac{1}{jkr} \right) e^{-jkr} \rightarrow (23)$$

To Find Magnetic Field :-

$$H = \frac{1}{\mu} \cdot \nabla \times A$$

$$H = \frac{1}{\mu} \cdot \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{bmatrix} \rightarrow (24)$$

$$H_r = \frac{1}{\mu r^2 \sin\theta} \cdot 1 \cdot \left[\frac{\partial}{\partial \theta} r\sin\theta A_\phi - \frac{\partial}{\partial \phi} \cdot r \cdot \underset{\substack{\uparrow \\ 0}}{A_\theta} \right]$$

$$\therefore H_r = \frac{1}{\mu r^2 \sin\theta} \cdot \frac{\partial}{\partial \theta} r\sin\theta A_\phi$$

$$H_r = \frac{1}{\cancel{\mu} r^2 \sin\theta}\cdot\frac{\partial}{\partial\theta}\left[\cancel{r}\sin\theta\cdot\frac{jk\cancel{\mu} a^2 I_0\sin\theta}{4\cancel{r}}\cdot e^{-jkr}\left(1+\frac{1}{jkr}\right)\right]$$

$$H_r = \frac{1}{r^2\sin\theta}\,\frac{jka^2I_0}{4}\,e^{-jkr}\left(1+\frac{1}{jkr}\right)\cdot\frac{\partial}{\partial\theta}\sin^2\theta$$

$$H_r = \frac{1}{r^2\cancel{\sin}\theta}\cdot\frac{jka^2I_0}{\cancel{4}\,2}\cdot e^{-jkr}\left(1+\frac{1}{jkr}\right)\cdot\cancel{2}\cancel{\sin}\theta\cos\theta$$

$$\therefore H_r = \frac{jka^2I_0\cos\theta}{2r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr} \longrightarrow (25)$$

$$H_\theta = \frac{-1}{\mu r^{\cancel{2}}\sin\theta}\cdot\cancel{r}\left[\frac{\partial}{\partial r}\cdot r\sin\theta A_\phi - \frac{\partial}{\partial\phi}\cdot\underset{\substack{\uparrow\\0}}{A_r}\right]$$

$$= \frac{-1}{\mu r\sin\theta}\,\frac{\partial}{\partial r}\,r\sin\theta A_\phi$$

$$= \frac{-1}{\mu r\cancel{\sin}\theta}\cdot\cancel{\sin}\theta\,\frac{\partial}{\partial r}\,\cancel{r}\cdot\frac{jk\mu a^2I_0\sin\theta}{4\cancel{r}}\left[1+\frac{1}{jkr}\right]\cdot e^{-jkr}$$

$$= \frac{-\sin\theta}{\cancel{\mu} r}\cdot\frac{jk\cancel{\mu}a^2I_0}{4}\cdot\frac{\partial}{\partial r}\left[1+\frac{1}{jkr}\right]\cdot e^{-jkr}$$

$$H_\theta = \frac{-jka^2I_0\sin\theta}{4r}\left[\frac{\partial}{\partial r}e^{-jkr} + \frac{\partial}{\partial r}\cdot\frac{e^{-jkr}}{jkr}\right]$$

$$H_\theta = \frac{-jka^2I_0\sin\theta}{4r}\left[e^{-jkr}(-jk) + \frac{jkr\cdot e^{-jkr}\cdot(-jk) - e^{-jkr}\cdot jk}{(jk)^2}\right]$$

$$H_\theta = \frac{-jka^2 I_0 \sin\theta}{4r}\left[-jke^{-jkr} - \frac{jke^{-jkr}}{jkr} - \frac{jke^{-jkr}}{(jkr)^2}\right]$$

$$H_\theta = \frac{-k^2a^2 I_0 \sin\theta}{4r} \cdot e^{-jkr}\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right] \rightarrow (26)$$

similarly,

$$H_\phi = \frac{1}{\mu r^2 \sin\theta} \cdot r\sin\theta\left[\frac{\partial}{\partial r} \underset{\underset{0}{\uparrow}}{rA_\theta} - \frac{\partial}{\partial\theta} \underset{\underset{0}{\uparrow}}{A_r}\right]$$

$$\therefore H_\phi = 0 \qquad \rightarrow (27)$$

To Find Electric Field :-

$$E = \frac{1}{j\omega\varepsilon} \cdot \nabla \times H$$

$$= \frac{1}{j\omega\varepsilon} \cdot \frac{1}{r^2\sin\theta}\begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{bmatrix}$$

$$E_r = \frac{1}{j\omega\varepsilon} \cdot \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta} r\sin\theta \underset{\underset{0}{\uparrow}}{H_\phi} - \frac{\partial}{\partial\phi} rH_\theta\right]$$

$$E_r = \frac{1}{j\omega\varepsilon} \cdot \frac{1}{r^2\sin\theta}\left[-\frac{\partial}{\partial\phi} \cdot rH_\theta\right]$$

$$E_r = \frac{1}{j\omega\varepsilon} \frac{1}{r^2\sin\theta}\left[\frac{-\partial}{\partial\phi} \cdot r \cdot \frac{-(ka)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jkr} - \frac{1}{(jkr)^2}\right] \cdot e^{-jkr}\right]$$

$$E_r = \frac{1}{j\omega\varepsilon r^2 \sin\theta} \cdot \frac{r(ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]\cdot e^{-jkr} \cdot \underbrace{\frac{\partial}{\partial\phi}(1)}_{0}$$

$$E_r = 0 \longrightarrow (28)$$

$$E_\theta = \frac{1}{j\omega\varepsilon r^{\cancel{2}} \sin\theta} \cdot \cancel{r} \cdot \left[\frac{\partial}{\partial r} r\sin\theta \underbrace{H_\phi}_{0} - \frac{\partial}{\partial\phi}\cdot H_r\right]$$

$$= \frac{1}{j\omega\varepsilon r \sin\theta}\left[-\frac{\partial}{\partial\phi} H_r\right]$$

$$E_\theta = \frac{-1}{j\omega\varepsilon r\sin\theta} \cdot \frac{\partial}{\partial\phi} \frac{jka^2 I_0 \cos\theta}{2r^2}\left(1+\frac{1}{jkr}\right)\cdot e^{-jkr}$$

$$E_\theta = \frac{-1}{j\omega\varepsilon r\sin\theta} \cdot \frac{jka^2 I_0 \cos\theta}{2r^2}\left(1+\frac{1}{jkr}\right) e^{-jkr} \cdot \underbrace{\frac{\partial}{\partial\phi}(1)}_{0}$$

$$E_\theta = 0 \longrightarrow (29)$$

$$E_\phi = \frac{1}{j\omega\varepsilon r^{\cancel{2}} \cancel{\sin\theta}} \cdot \cancel{r}\cancel{\sin\theta}\left[\frac{\partial}{\partial r} rH_\theta - \frac{\partial H_r}{\partial\theta}\right]$$

$$E_\phi = \frac{1}{j\omega\varepsilon r} \cdot \left[\frac{\partial}{\partial r} rH_\theta - \frac{\partial H_r}{\partial\theta}\right] \rightarrow (30)$$

$$\frac{\partial}{\partial r} rH_\theta = \frac{\partial}{\partial r} \cdot \cancel{r} \cdot \frac{-(ka)^2 I_0 \sin\theta}{4\cancel{r}}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right] e^{-jkr}$$

$$\frac{\partial}{\partial r} rH_\theta = \frac{-(ka)^2 I_0 \sin\theta}{4}\left[\frac{\partial}{\partial r} e^{-jkr} + \frac{\partial}{\partial r}\frac{e^{-jkr}}{jkr} + \frac{\partial}{\partial r}\frac{e^{-jkr}}{(jkr)^2}\right]$$

$$\frac{\partial}{\partial r} rH_\theta = \frac{-(ka)^2 I_0 \sin\theta}{4}\left[-jk\cdot e^{-jkr} + \frac{jkr\, e^{-jkr}(-jk) - e^{-jkr}\cdot jk}{(jkr)^2} + \frac{(jkr)^2\cdot e^{-jkr}(-jk) - e^{-jkr}\cdot 2(jkr)\cdot jk}{(jkr)^4}\right]$$

$$\frac{\partial}{\partial r} rH_\theta = -\frac{(ka)^2}{4} I_0 \sin\theta \cdot e^{-jkr}\left[-jk - \frac{\cancel{jkr}\cdot jk}{(jkr)^{\cancel{2}}} \frac{-jk}{(jkr)^2} + \frac{\cancel{(jkr)^2}(-jk)}{(jkr)^{\cancel{4}\,2}} - \frac{2\cancel{(jkr)}\cdot jk}{(jkr)^{\cancel{4}\,3}}\right]$$

$$\frac{\partial}{\partial r} rH_\theta = -\frac{(ka)^2}{4} I_0 \sin\theta\cdot e^{-jkr}\cdot(-jk)\left[1 + \frac{1}{jkr} + \frac{2}{(jkr)^2} + \frac{2}{(jkr)^3}\right] \rightarrow (31)$$

$$\frac{\partial}{\partial\theta} H_r = \frac{\partial}{\partial\theta}\, \frac{jka^2 I_0 \cos\theta}{2r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

$$= \frac{jka^2 I_0}{2r^2}\left[1 + \frac{1}{jkr}\right]\cdot e^{-jkr}\cdot\frac{\partial}{\partial\theta}\cos\theta$$

$$\frac{\partial}{\partial\theta} H_r = \frac{-jka^2 I_0}{2r^2}\left[1 + \frac{1}{jkr}\right]\cdot e^{-jkr}\cdot \sin\theta \rightarrow (32)$$

substitute equ 31 and 32 in 30

$$E_\phi = \frac{1}{j\omega\varepsilon r}\left[\frac{j(ka)^2}{4} \cdot I_0 \sin\theta \cdot e^{-jkr} \cdot k \cdot \left(1+\frac{1}{jkr}+\frac{2}{(jkr)^2}+\frac{2}{(jkr)^3}\right) + \frac{jka^2 I_0}{2r^2}\left(1+\frac{1}{jkr}\right)e^{-jkr} \cdot \sin\theta\right]$$

$$E_\phi = \frac{\cancel{j}k}{\cancel{j}\omega\varepsilon r} a^2 I_0 \sin\theta\, e^{-jkr}\left[\frac{k^2}{4}\left(1+\frac{1}{jkr}+\frac{2}{(jkr)^2}+\frac{2}{(jkr)^3}\right) + \frac{1}{2r^2}\left(1+\frac{1}{jkr}\right)\right]$$

$$E_\phi = \overset{\eta}{\frac{k}{\omega\varepsilon}}\frac{a^2}{r} I_0 \sin\theta\, e^{-jkr} \cdot \frac{k^2}{4}\left[1+\frac{1}{jkr}-\frac{2}{(kr)^2}+\frac{j2}{(kr)^3} + \frac{\cancel{4}^{2}}{k^2} \cdot \frac{1}{\cancel{2}r^2}\left(1+\frac{1}{jkr}\right)\right]$$

$$E_\phi = \eta a^2 I_0 \sin\theta\, e^{-jkr} \cdot \frac{k^2}{4r}\left[1+\frac{1}{jkr}-\cancel{\frac{2}{(kr)^2}}+\cancel{\frac{j2}{(kr)^3}} + \cancel{\frac{2}{(kr)^2}} - \cancel{\frac{j2}{(kr)^3}}\right]$$

$$\therefore E_\phi = \eta a^2 I_0 \sin\theta\, e^{-jkr} \cdot \frac{k^2}{4r}\left[1+\frac{1}{jkr}\right] \rightarrow (33)$$

Summary

$$H_r = \frac{jka^2 I_0 \cos\theta}{2r^2}\left[1+\frac{1}{jkr}\right]\cdot e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]\cdot e^{-jkr}$$

$$H_\phi = 0$$

$$E_r = E_\theta = 0$$

$$E_\phi = \eta\frac{(ka)^2}{4r} I_0 \sin\theta\left[1+\frac{1}{jkr}\right]\cdot e^{-jkr}$$