

UNIT III
ANTENNA ARRAYS

UNIT - 3 ANTENNA ARRAYS

Antenna Array refers to group of elements used together to improve gain and directivity.

The radiation characteristics of an array depends on

1. The geometrical configuration of an array
2. The relative spacing between the elements
3. The Excitation amplitude of the individual elements
4. The Excitation phase of the individual elements
5. The relative Pattern of the individual elements

Classification of Antenna Array :

I Based on Geometrical Configuration

1. Linear array
2. Planar array (Rectangular or circular)
3. Conformal array

II Based on excitation of current in individual Elements

1. Broadside array
2. End fire array

III Based on Elements

1. Parasitic array
2. Non Parasitic array

IV Based on current amplitude distribution

1. uniform array
2. Binomial array
3. Edge array
4. Optimum array

V Based on signal input (or output) of each element

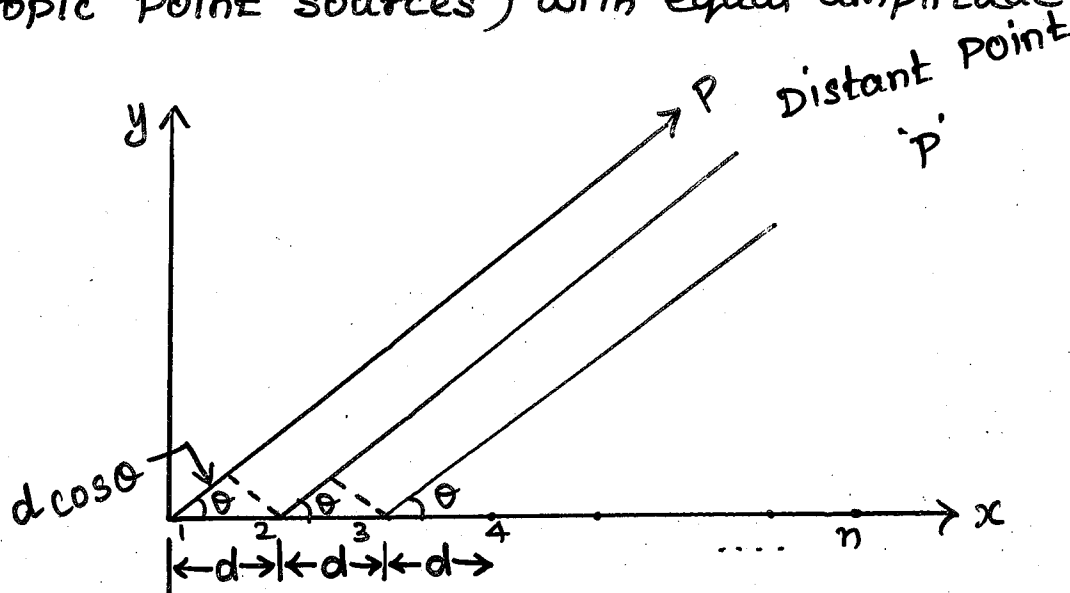
1. Phased array
2. self focussing array
3. Adaptive array

n Element Linear Array :-

Linear array :

An array is said to be linear if all its elements are spaced equally along a line.

Consider a linear array consisting of 'n' elements (isotropic point sources) with equal amplitude & spacing



as shown in Fig.

The total far field pattern at point P, can be found by adding vectorially the fields of individual sources as,

$$E_t = E_1 \cdot e^{j0\gamma} + E_2 \cdot e^{j\gamma} + E_3 \cdot e^{j2\gamma} + \dots + E_n \cdot e^{j(n-1)\gamma} \rightarrow \textcircled{1}$$

where, γ refers total phase difference

$$\gamma = \beta d \cos \theta + \alpha$$

sources

$\alpha \rightarrow$ progressive phase shift of adjacent $\wedge$

Let all elements has equal amplitude

$$\therefore E_1 = E_2 = E_3 = E_n = E_0$$

$\therefore$ equ $\textcircled{1}$ becomes,

$$E_t = E_0 [1 + e^{j\gamma} + e^{j2\gamma} + \dots + e^{j(n-1)\gamma}] \rightarrow \textcircled{2}$$

multiply equ $\textcircled{2}$ by $e^{j\gamma}$

$$E_t \cdot e^{j\gamma} = E_0 [e^{j\gamma} + e^{j2\gamma} + e^{j3\gamma} + \dots + e^{jn\gamma}] \rightarrow \textcircled{3}$$

subtract equ $\textcircled{3}$ from equ $\textcircled{2}$

$$E_t - E_t \cdot e^{j\gamma} = E_0 - E_0 \cdot e^{jn\gamma}$$

$$E_t (1 - e^{j\gamma}) = E_0 (1 - e^{jn\gamma})$$

$$E_t = \frac{E_0 (1 - e^{jn\gamma})}{1 - e^{j\gamma}}$$

$$= E_0 \left[\frac{e^{\frac{jn\gamma}{2}} \cdot e^{-\frac{jn\gamma}{2}} - e^{\frac{jn\gamma}{2}} \cdot e^{\frac{jn\gamma}{2}}}{e^{\frac{j\gamma}{2}} \cdot e^{-\frac{j\gamma}{2}} - e^{\frac{j\gamma}{2}} \cdot e^{\frac{j\gamma}{2}}} \right]$$

$$E_t = E_0 \cdot \frac{e^{jn\gamma/2} (e^{-jn\gamma/2} - e^{jn\gamma/2})}{e^{j\gamma/2} (e^{-j\gamma/2} - e^{j\gamma/2})}$$

$$E_t = E_0 \cdot e^{j(n-1)\gamma/2} \left[\frac{\sin(n\gamma/2)}{\sin(\gamma/2)} \right] \rightarrow \textcircled{4}$$

If the reference point is shifted to the center of the array, then

$$E_t = E_0 \cdot \left[\frac{\sin(n\gamma/2)}{\sin(\gamma/2)} \right] \rightarrow \textcircled{5}$$

$\frac{\sin(n\gamma/2)}{\sin(\gamma/2)}$ is called Array Factor

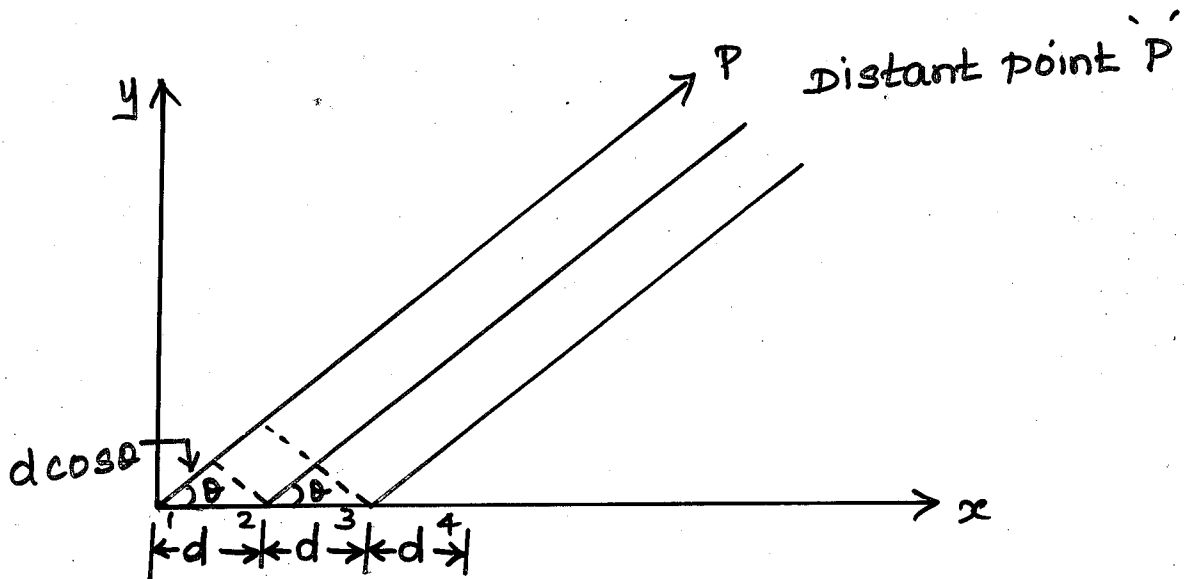
n element Broadside array :

* If individual elements in an array are equally spaced along a line and fed with current of equal magnitude and same phase, then the array is said to be Broadside array.

* In this, Direction radiation (major lobe) is perpendicular to array axis

* consider a linear array consisting of n elements each separated by 'd'. Let the array is broadside such that $\alpha = 0$.

$$\therefore \gamma = \beta d \cos \theta$$



Major Lobe maxima :

In broadside array, same Phase excitation is used in all elements, $\gamma = 0$

$$\gamma = \beta d \cos \theta \quad \rightarrow \textcircled{1}$$

$$0 = \beta d \cos \theta$$

$$\cos \theta = 0$$

$$\therefore \theta = 90^\circ \text{ or } 270^\circ$$

$\therefore$ In broadside array, major lobe maxima occurs at 90° or 270°

Directions of Pattern maxima :

$$\text{Consider Array Factor, } AF = \frac{\sin(n\gamma/2)}{\sin(\gamma/2)} \quad \rightarrow \textcircled{2}$$

maximum E_t can be obtained if $\sin(n\gamma/2)$ is maximum

$$\text{i.e., } \sin\left(\frac{n\gamma}{2}\right) = 1$$

$$\frac{n\gamma}{2} = \sin^{-1}(1)$$

$$\therefore \frac{n\gamma}{2} = \pm (2N+1) \frac{\pi}{2} \rightarrow (3)$$

where, N is an integer and $N=0$ corresponds to major lobe maxima.

sub $\gamma = \beta d \cos \theta$ in equ (3)

$$\frac{n}{2} \cdot \beta d \cos \theta = \pm (2N+1) \frac{\pi}{2}$$

$$\therefore \theta = \cos^{-1} \left[\pm (2N+1) \frac{\pi}{2} \cdot \frac{2}{n\beta d} \right]$$

Let $n=4$, $d = \lambda/2$

$$\therefore \theta = \cos^{-1} \left[\pm (2N+1) \cdot \frac{1}{4} \right] \rightarrow (4)$$

$$\left[\begin{array}{l} n\beta d = 4 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{2} \\ \frac{1}{n\beta d} = \frac{1}{4\pi} \end{array} \right]$$

sub $N=1$ in equ (4)

$$\theta = \pm \cos^{-1} \left(\frac{3}{4}, -\frac{3}{4} \right)$$

$$(\theta_{\max})_{\min} = \pm 41.4^\circ, \pm 138.6^\circ$$

Thus 4 minor lobe maxima occurs at $\pm 41.4^\circ$, -41.4° , $+138.6^\circ$, -138.6°

Direction of Pattern minima :

The condition for obtaining pattern minima is

$$\sin\left(\frac{n\gamma}{2}\right) = 0 \quad [\because E_z = 0]$$

$$\left(\frac{n\gamma}{2}\right) = \sin^{-1}(0)$$

$$\frac{n\gamma}{2} = \pm N\pi$$

$$\gamma = \pm \frac{2N\pi}{n}$$

$$\psi = \beta d \cos \theta + \alpha ; \text{ for Broadside array, } \alpha = 0$$

$$\text{Let } d = \frac{\lambda}{2} \text{ and } \beta = \frac{2\pi}{\lambda}$$

$$\therefore \beta d \cos \theta_{\min} = \pm \frac{2N\pi}{n}$$

$$\theta_{\min} = \cos^{-1} \left[\frac{\pm 2N\pi}{n\beta d} \right]$$

$$\theta_{\min} = \cos^{-1} \left[\pm \frac{N\lambda}{nd} \right]$$

$$\frac{2N\pi}{n\beta d} = \frac{2N\pi}{n \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}} = \frac{N\lambda}{nd}$$

For $n=4$;

$$(\theta_{\min})_{\text{minor}} = \cos^{-1} \left[\pm \frac{N}{2} \right]$$

$$\frac{N\lambda}{nd} = \frac{N \cdot \lambda}{2 \cdot \frac{\lambda}{2}} = \frac{N}{2}$$

For $N=1$

$$\text{For } N=1 ; (\theta_{\min})_{\text{minor}} = \pm 60^\circ, \pm 120^\circ$$

$$\text{For } N=2 ; (\theta_{\min})_{\text{minor}} = 0^\circ, 180^\circ$$

Thus $0^\circ, 60^\circ, 120^\circ, 180^\circ, -60^\circ, -120^\circ$ are six minor lobe minima of the array of 4 isotropic point sources.

Beam Width of Major Lobe :

$$\text{BWFN} = 2 \times \gamma$$

where, $\gamma \rightarrow$ Angle between first null & maximum of major lobe

$$\text{W.K.T } \theta_{\min} = \cos^{-1} \left[\pm \frac{N\lambda}{nd} \right]$$

$$\theta + \gamma = 90^\circ$$

$$90^\circ - \gamma = \cos^{-1} \left[\pm \frac{N\lambda}{nd} \right]$$

$$\therefore \theta = 90^\circ - \gamma$$

$$\cos(90^\circ - \gamma) = \pm \frac{N\lambda}{nd}$$

$$\sin \gamma = \pm \frac{N\lambda}{nd}$$

As γ is small, $\sin \gamma = \gamma$

$$\gamma = \pm \frac{N\lambda}{nd}$$

First null occurs when $N=1$

$$\therefore \gamma = \pm \frac{\lambda}{nd}$$

$$BWFN = 2\gamma = \frac{2\lambda}{nd} = \frac{2\lambda}{L}$$

$$\because L = nd$$

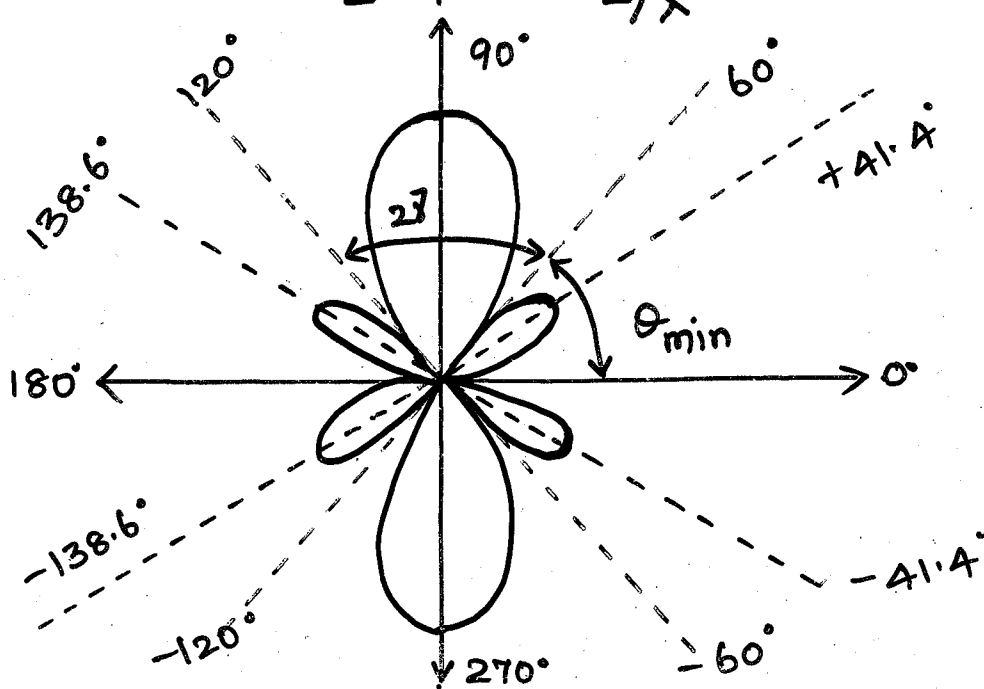
$\rightarrow$ Array Length

$$\Rightarrow \frac{2}{L/\lambda} \text{ radian}$$

$$\Rightarrow \frac{2 \times 57.3}{L/\lambda} \text{ degree}$$

$$\therefore BWFN = \frac{114.6}{L/\lambda}$$

$$HPBW = \frac{BWFN}{2} = \frac{57.3}{L/\lambda} \text{ degree}$$

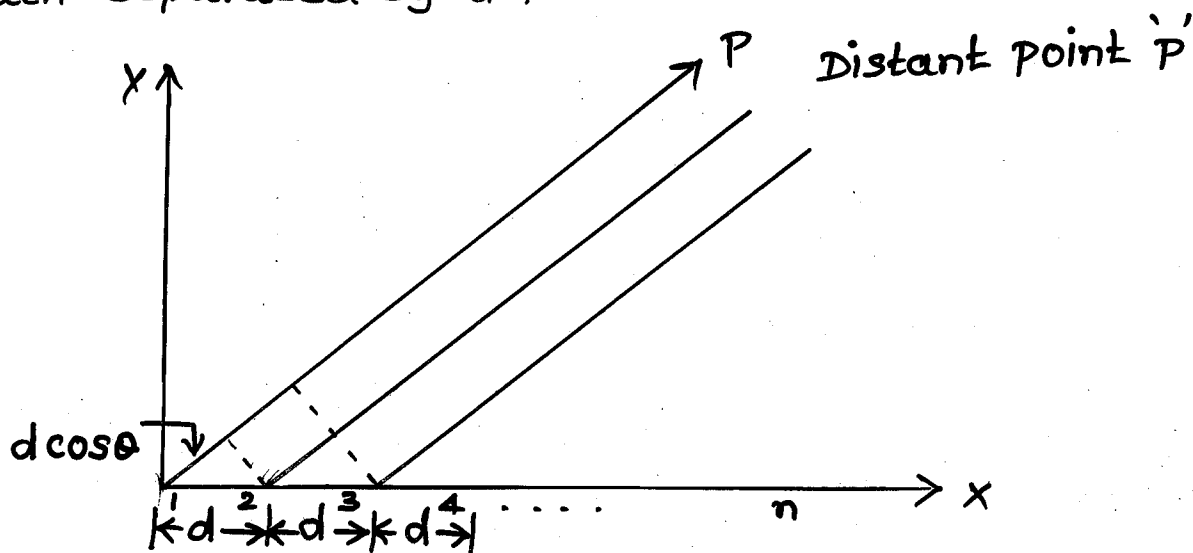


n Element End Fire Array :

* If individual elements in an array are equally spaced along a line and fed with current of equal magnitude and 180° phase shift then the array is said to be end Fire array.

* In this, Direction radiation (major lobe) is along the array axis.

* consider a linear array consisting of 'n' elements each separated by 'd'.



Major Lobe Maxima :

* For end Fire array, maximum radiation is along the array axis. i.e., at $\theta = 0^\circ$ and $\theta = 180^\circ$

$$\gamma = \beta d \cos \theta + \alpha \rightarrow (1)$$

sub $\gamma = 0$ & $\theta = 0^\circ$ in (1)

$$\therefore 0 = \beta d \cos(0) + \alpha$$

$$\therefore \alpha = -\beta d \rightarrow (2)$$

Directions of Pattern maxima :

consider Array Factor, $AF = \frac{\sin(n\gamma/2)}{\sin(\gamma/2)} \rightarrow \textcircled{3}$

maximum E_z can be obtained if $\sin(n\gamma/2)$ is maximum, i.e., $\sin\left(\frac{n\gamma}{2}\right) = 1 \rightarrow \textcircled{4}$

$$\frac{n\gamma}{2} = \sin^{-1}(1)$$

$$= \pm (2N+1) \frac{\pi}{2}$$

$$\therefore \gamma = \pm (2N+1) \frac{\pi}{2} \cdot \frac{2}{n}$$

$$\therefore \gamma = \pm (2N+1) \frac{\pi}{n} \rightarrow \textcircled{5}$$

For end fire array, $\alpha = -\beta d$

$$\beta d \cos \theta + \alpha = \pm (2N+1) \frac{\pi}{n}$$

$$\beta d \cos \theta - \beta d = \pm (2N+1) \frac{\pi}{n}$$

$$\beta d (\cos \theta - 1) = \pm (2N+1) \frac{\pi}{n}$$

$$\cos \theta - 1 = \pm (2N+1) \frac{\pi}{n\beta d}$$

$$\cos \theta = \pm (2N+1) \frac{\pi}{n\beta d} + 1$$

$$\theta = \cos^{-1} \left[\pm \frac{(2N+1)\pi}{n\beta d} + 1 \right]$$

Let $n=4$, $d = \lambda/2$

$$\therefore \theta = \pm \cos^{-1} \left[\pm \left(\frac{2N+1}{4} \right) + 1 \right] \quad \left(\pm \frac{3}{4} + 1 \right)$$

For $N=1$, $(\theta_{\max})_{\min} = \pm \cos^{-1} \left(\frac{7}{4}, \frac{1}{4} \right)$

$$(\theta_{\max})_{\min} = \pm 75.52^\circ$$

$$\text{For } N=2, (\theta_{\max})_{\min} = \pm \cos^{-1}\left(\frac{9}{4}, -\frac{1}{4}\right)$$

$$= \pm 104.48^\circ$$

$$\pm \frac{5}{4} + 1$$

$$\text{For } N=3, (\theta_{\max})_{\min} = \pm \cos^{-1}\left(\frac{11}{4}, -\frac{3}{4}\right)$$

$$= \pm 138.59^\circ$$

$$\pm \frac{7}{4} + 1$$

$$\text{For } N=4, (\theta_{\max})_{\min} = \pm \cos^{-1}\left(\frac{13}{4}, -\frac{5}{4}\right)$$

Thus $+75.5^\circ$, $+104.5^\circ$, $+138.6^\circ$, -75.5° , -104.5° and -138.6° are 6 minor lobe maxima of the array of 4 isotropic point sources.

Direction of Pattern minima:

The condition for obtaining Pattern minima is

$$\sin\left(\frac{n\gamma}{2}\right) = 0$$

$$\left(\frac{n\gamma}{2}\right) = \sin^{-1}(0)$$

$$\left(\frac{n\gamma}{2}\right) = \pm N\pi$$

$$\gamma = \pm \frac{2N\pi}{n}$$

$$\beta d \cos\theta + \alpha = \pm \frac{2N\pi}{n}$$

$$\beta d \cos\theta - \beta d = \pm \frac{2N\pi}{n}$$

$$\beta d (\cos\theta - 1) = \pm \frac{2N\pi}{n}$$

$$\cos\theta - 1 = \pm \frac{2N\pi}{n\beta d}$$

$$\cos\theta = \pm \frac{2N\pi}{n\beta d} + 1$$

$$\therefore \theta = \pm \cos^{-1} \left[\pm \frac{2N\pi}{n\beta d} + 1 \right]$$

For $n=4$, $d = \lambda/2$

$$(\theta_{\min})_{\min} = \pm \cos^{-1} \left[\pm \frac{N}{2} + 1 \right] \quad \pm \frac{1}{2} + 1$$

$$\begin{aligned} \text{For } N=1, (\theta_{\min})_{\min} &= \pm \cos^{-1} \left(\frac{3}{2}, \frac{1}{2} \right) \\ &= \pm 60^\circ \quad \pm 1 + 1 \end{aligned}$$

$$\begin{aligned} \text{For } N=2, (\theta_{\min})_{\min} &= \pm \cos^{-1} (2, 0) \\ &= \pm 90^\circ \quad \pm \frac{3}{2} + 1 \end{aligned}$$

$$\begin{aligned} \text{For } N=3, (\theta_{\min})_{\min} &= \pm \cos^{-1} \left(\frac{5}{2}, -\frac{1}{2} \right) \\ &= \pm 120^\circ \end{aligned}$$

$$\begin{aligned} \text{For } N=4, (\theta_{\min})_{\min} &= \pm \cos^{-1} \left(\frac{7}{2}, -1 \right) \quad \pm \frac{4}{2} + 1 \\ &= \pm 180^\circ \end{aligned}$$

Thus $+60^\circ, +90^\circ, +120^\circ, +180^\circ, -60^\circ, -90^\circ, -120^\circ, -180^\circ$ are 8 minor lobe minima of the array of 4 isotropic sources.

Beam Width of Major Lobe :

$$BWFN = 2\theta$$

$$(\theta_{\min})_{\min} = \pm \cos^{-1} \left[\pm \frac{2N\pi}{n\beta d} + 1 \right]$$

$$\cos \theta - 1 = \pm \frac{N\lambda}{nd}$$

$$1 - 2\sin^2\left(\frac{\theta}{2}\right) - 1 = \pm \frac{N\lambda}{nd}$$

$$2\sin^2\left(\frac{\theta}{2}\right) = \pm \frac{N\lambda}{nd}$$

$$\therefore \cos \theta = 1 - 2\sin^2(\theta/2)$$

$$\sin^2\left(\frac{\theta}{2}\right) = \pm \frac{N\lambda}{2nd}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{N\lambda}{2nd}}$$

$$\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{N\lambda}{2nd}}$$

$$\theta = \pm \sqrt{\frac{2N\lambda}{nd}} \Rightarrow \pm \sqrt{\frac{2N\lambda}{L}}$$

$$\sin \frac{\theta}{2} = \frac{\theta}{2}$$

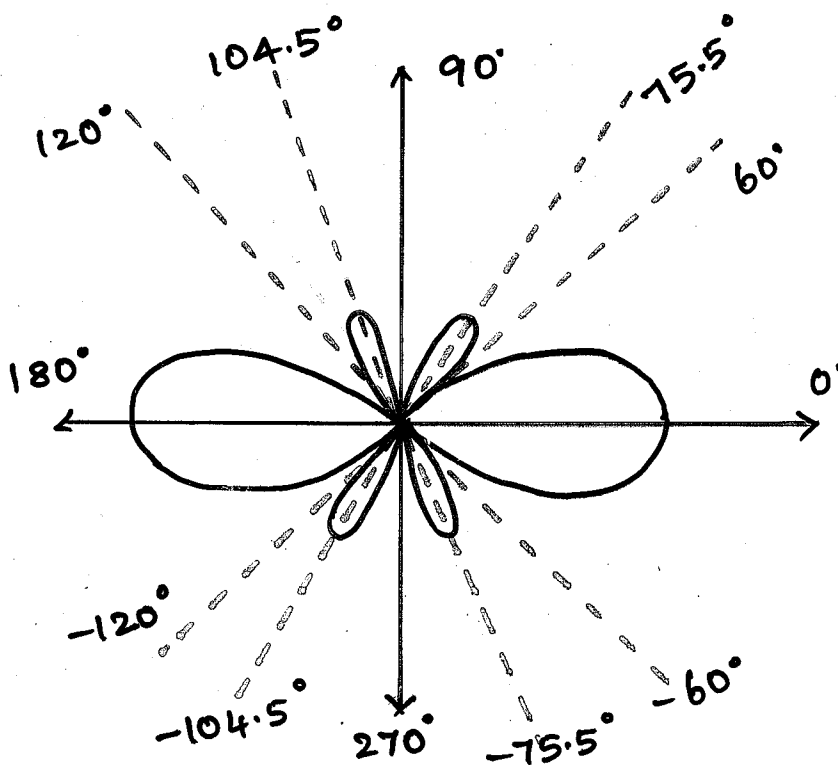
$\therefore \frac{\theta}{2}$ is very small!

$$\therefore \text{BWFN} = 2\theta = 2 \sqrt{\frac{2N\lambda}{L}}$$

$$\text{For } N=1, \text{ BWFN} = 2 \sqrt{\frac{2\lambda}{L}} \text{ radian (or)} 2 \sqrt{\frac{2\lambda}{L}} \times 57.3 \text{ degree.}$$

$$\text{HPBW} = \frac{\text{BWFN}}{2} = 57.3 \sqrt{\frac{2\lambda}{L}} \text{ degree}$$

The field pattern of end fire array of 4 elements is shown in figure below.



Pattern Multiplication:

The field pattern of an array of non isotropic but similar point sources is the product of the Pattern of individual source and the Pattern of an array of isotropic point sources having the same locations, relative amplitudes and phase as the non isotropic point sources.

$$\text{i.e., } E = \underbrace{f(\theta, \phi) F(\theta, \phi)}_{\text{Field pattern}} \underbrace{|f_p(\theta, \phi) + F_p(\theta, \phi)|}_{\text{phase pattern}}$$

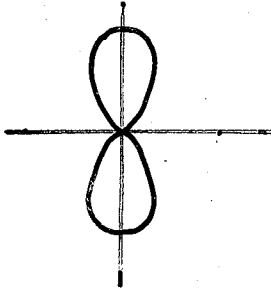
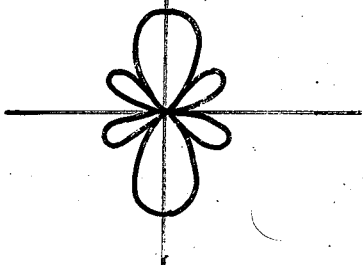
where,

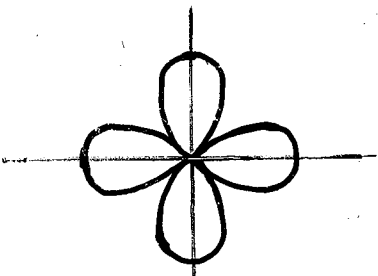
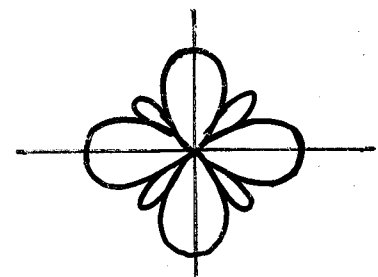
$f(\theta, \phi) \rightarrow$ Field Pattern of individual source.

$f_p(\theta, \phi) \rightarrow$ phase Pattern of individual source.

$F(\theta, \phi) \rightarrow$ Field Pattern of array of isotropic sources.

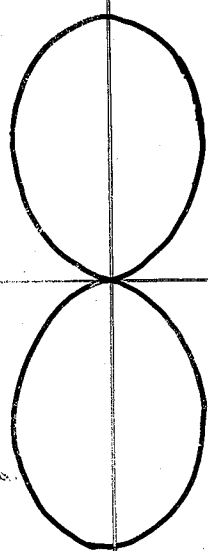
$F_p(\theta, \phi) \rightarrow$ Phase Pattern of array of isotropic source.

No. of Point sources	d	Radiation Pattern
2	$\lambda/2$	
4	$\lambda/2$	

2	γ	
2	2γ	

Example 1 :

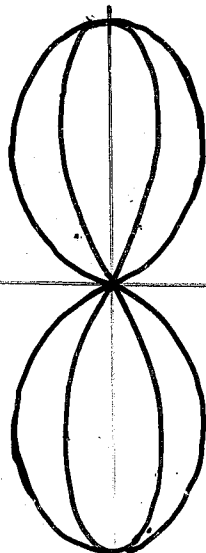
Let the individual Source Pattern be as in Fig (a) and the Pattern of an array of 2 isotropic point sources as in Fig(b) and the total array pattern as in Fig(c)



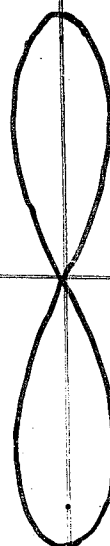
(a)
unit pattern



(b)
Group Pattern

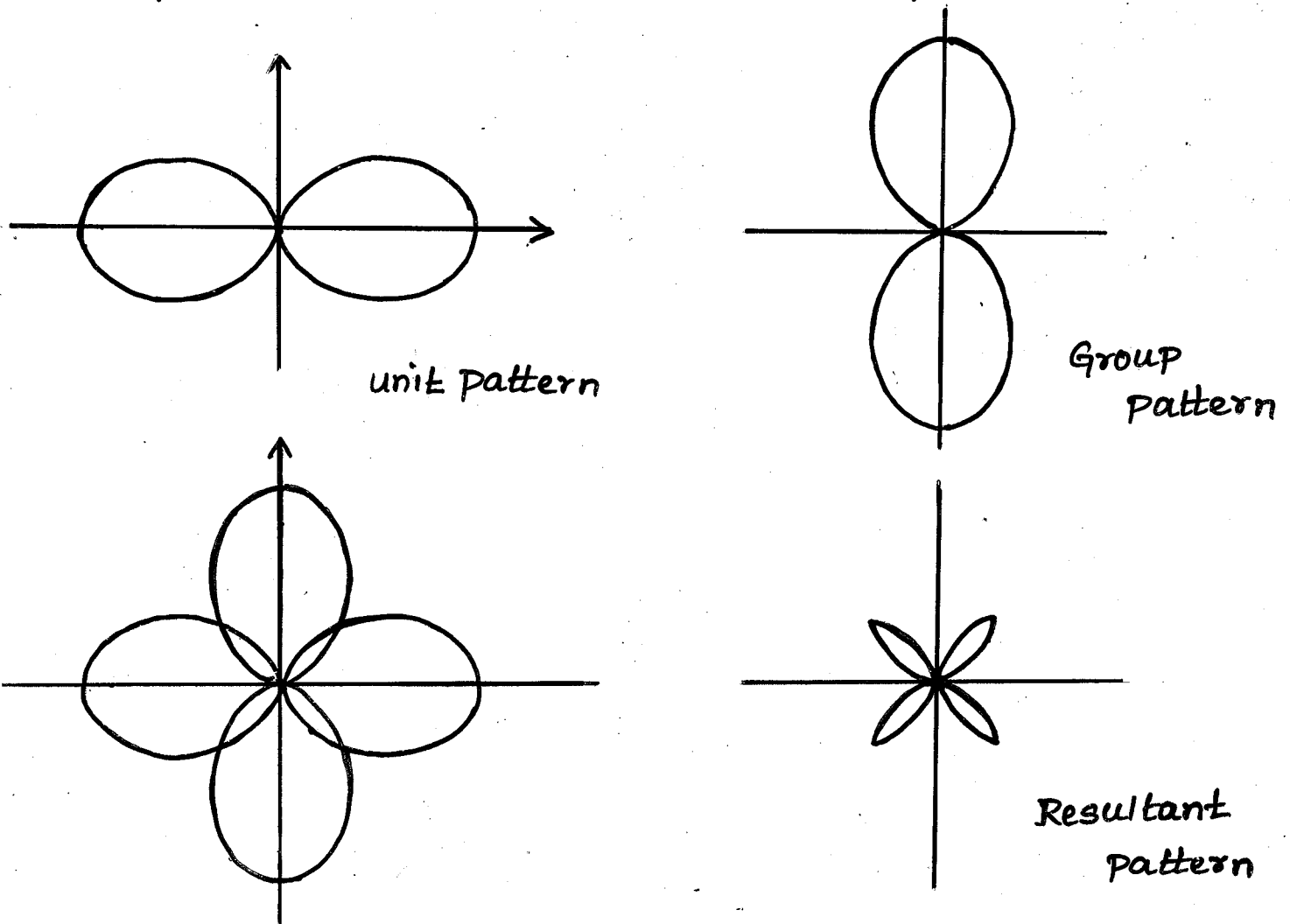


Overlapping of
Group & unit pattern



(c)
Resultant
Pattern

Example 2: Let the individual source pattern be

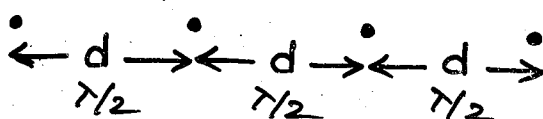


Radiation Pattern of 4 isotropic elements spaced $\frac{\lambda}{2}$ apart:

Consider a linear array consisting of 4 isotropic elements. The spacing between the elements is $\frac{\lambda}{2}$ and the currents are in phase, i.e., $\alpha = 0$

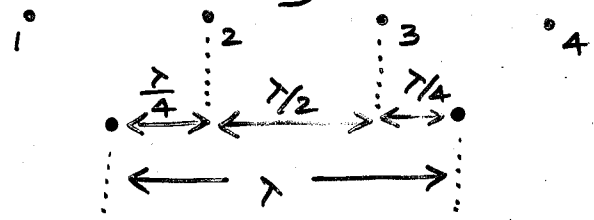
Let the elements 1 and 2 be unit one and this new unit is placed between 1 and 2 (exactly at the midpoint). Similarly, the elements 3 and 4 are considered as unit 2 and these are placed between the elements 3 and 4.

Original Array



$\Rightarrow$ 4 elements each separated by $\frac{\lambda}{2}$

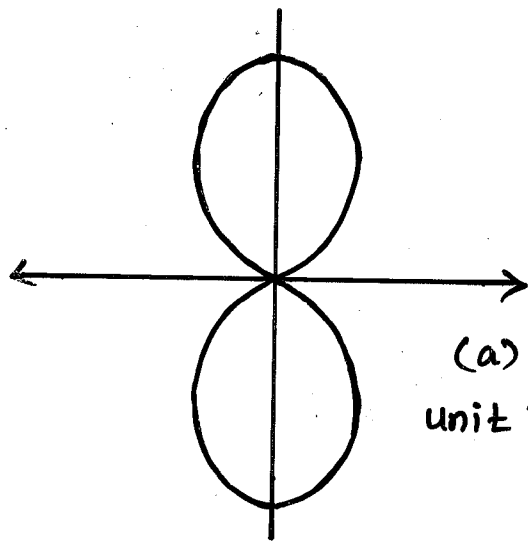
Replaced Array



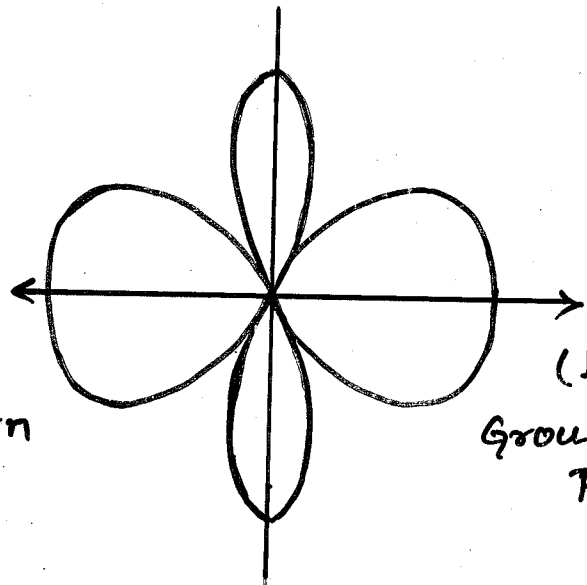
Two elements separated by λ

Individual pattern :

Pattern of 2 point sources separated by $\lambda/2$ is shown in Fig (a) and the pattern of 2 elements separated by λ is shown in Fig (b).



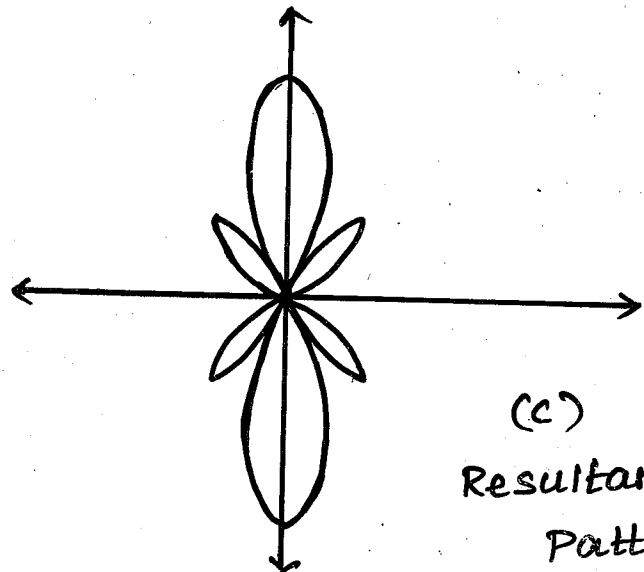
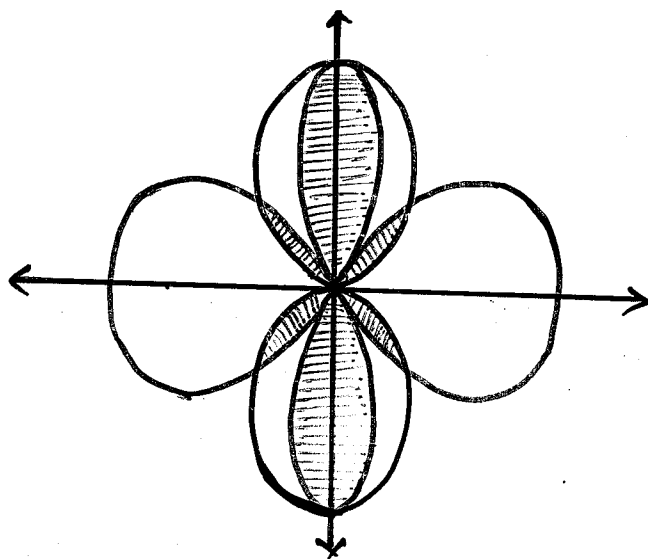
(a)
unit Pattern



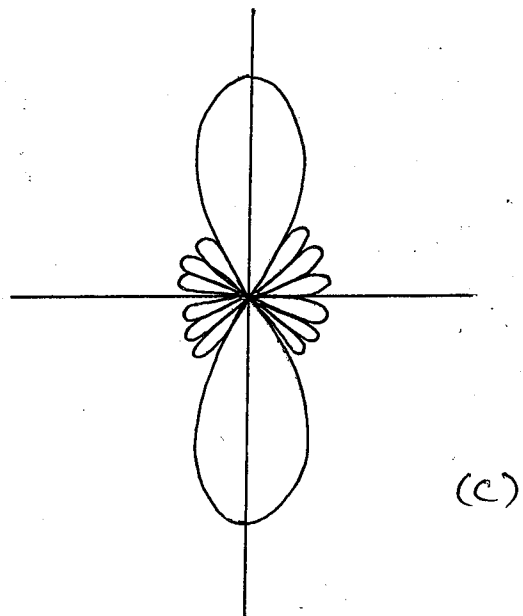
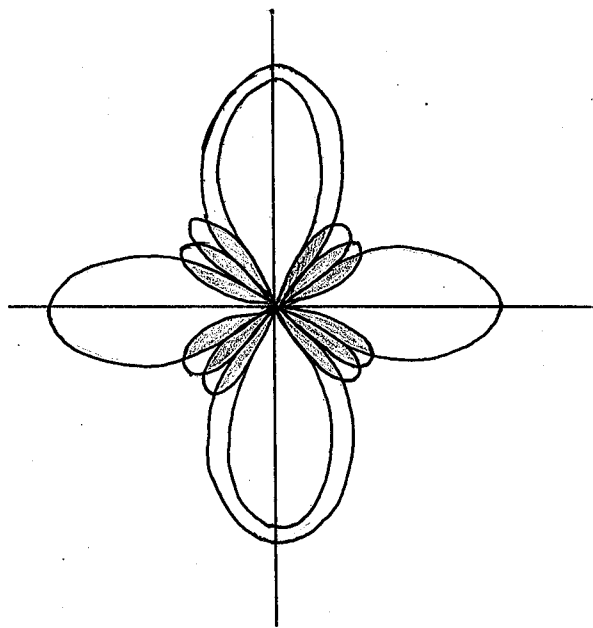
(b)
Group Pattern

Resultant Pattern:-

The resultant Pattern is shown in fig (c)



(c)
Resultant Pattern



Thus the radiation pattern of 8 isotropic element is obtained by multiplying the unit pattern of 4 individual elements and group pattern of 2 isotropic element spaced 2λ . Hence the resultant pattern is as shown in above fig.(c).

The important features of resultant pattern are,

1. The width of the Principal lobe (between nulls) is the same as the width of the corresponding lobe of the group pattern.
2. The sum of the nulls in the unit pattern and group pattern gives the number of nulls in the resultant pattern assuming none of the nulls are coincident
3. The Number of secondary lobes in the resultant pattern can be determined from the number of nulls in resultant pattern.

PHASED ARRAY: (SCANNING ARRAY)

* In Broadside and End-fire array, the maximum radiation can be obtained by adjusting the phase excitation between elements in the direction normal and along the axis of array.

* The array in which the phase and the amplitude of the elements is variable provided that the direction of maximum radiation (Beam radiation) and pattern shape along with the side lobes are controlled is called as phased array.

* Suppose the array gives maximum radiation in the radiation $\theta = \theta_0$ where $0 \leq \theta_0 \leq 180^\circ$ then the phase shift that must be controlled can be obtained as,

$$\psi = \beta d \cos \theta + \alpha$$

$$0 = \beta d \cos \theta_0 + \alpha$$

$$\therefore \alpha = -\beta d \cos \theta_0 \rightarrow \textcircled{1}$$

So, by controlling the progressive phase difference between the elements, the maximum radiation can be oriented in any desired direction to form a scanning array.

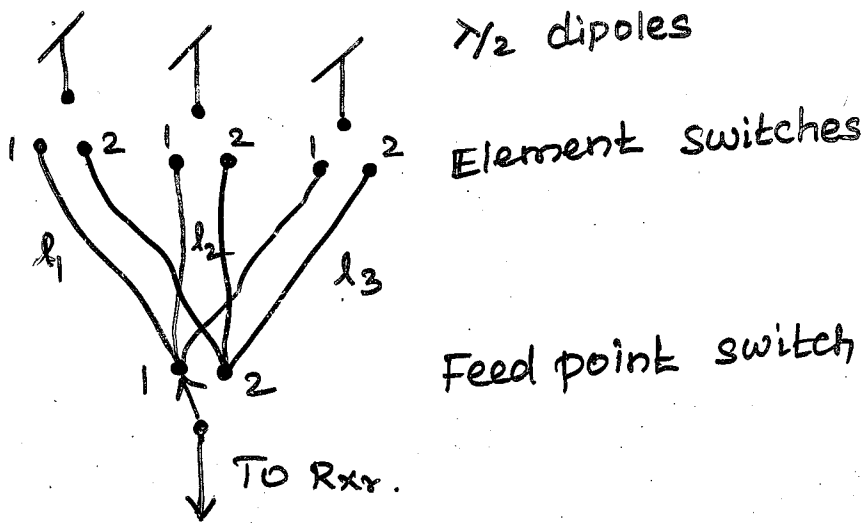
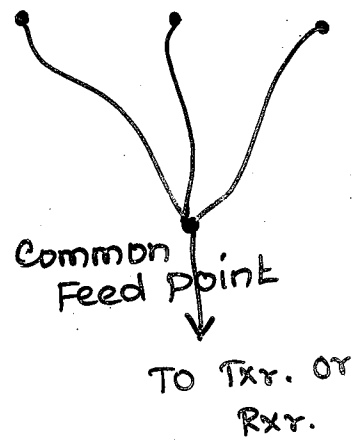
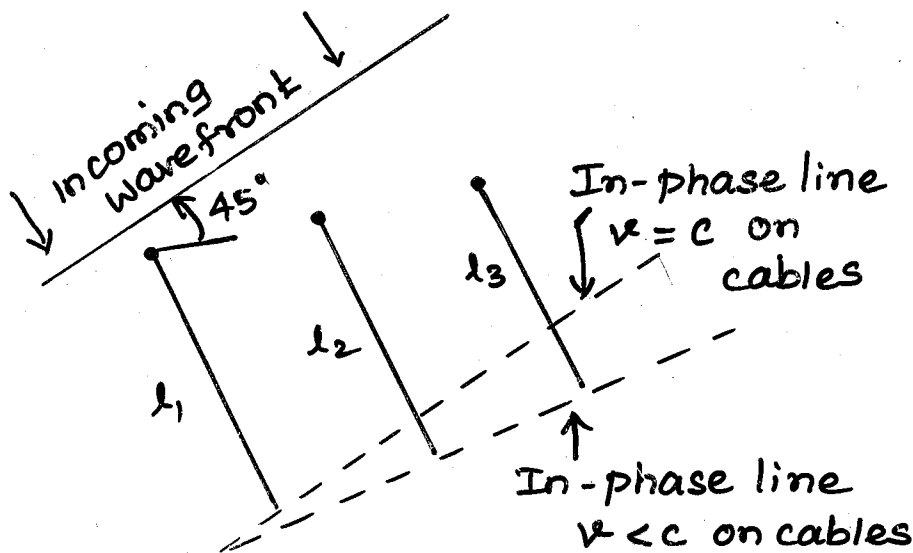
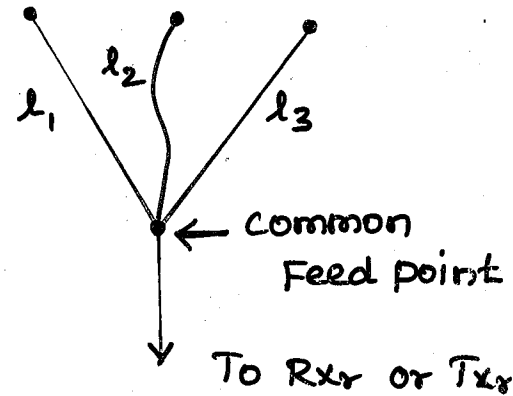
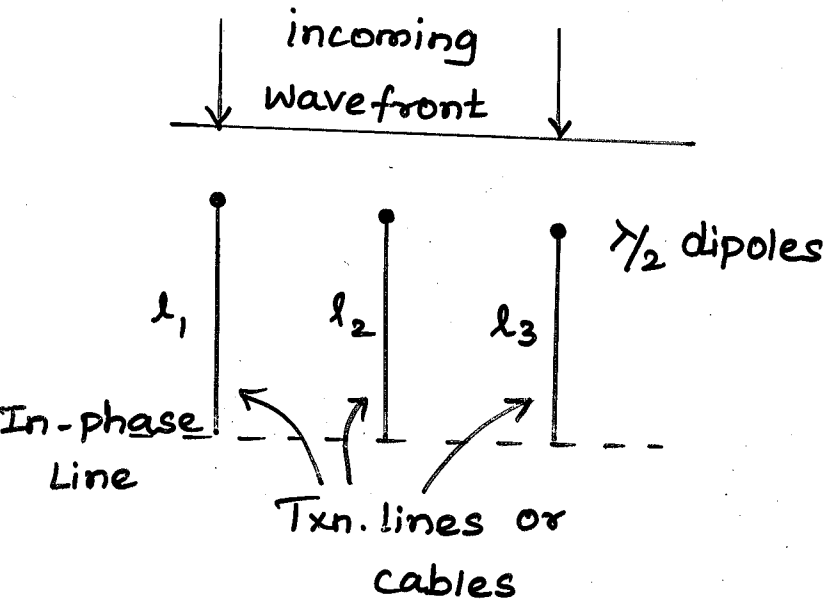
In phased array, beam steering can be done using

1. mechanical switches
2. Electronic switches

3. phase shifters

4. Insertion of cables

1. Beam steering Using Mechanical Switches :-



* Consider a 3 element array. Let each element be a $\frac{\lambda}{2}$ dipole connected to a common feed point through a cable of equal length.

* An incoming wave arriving broadside will induce voltages in the transmission line (cables) in the same phase.

* By bringing all the cables to a common point, the 3 element array will operate as a broadside array.

* For impedance matching, the cable connected to the transmitter or receiver should be one third impedance of the 3 cables or a 3 to 1 impedance transformer can be inserted at the common point.

* Now consider a wave arriving at an angle of 45° (fig(b)). If $v = c$, in-phase line is parallel to incoming wavefront. If $v < c$, lengths l_2 & l_3 must be increased in order for all phases to be the same. Thus if these cables are joined, the 3 element array will have its beam 45° from broadside.

* Beam can be shifted from broadside to 45° by operating the ganged switch. (switches are installed at each antenna element and one at the common feed point. These switches are ganged mechanically).

2. Beam Steering Using Electronic switches (PIN diodes) :

To make operation reliable and simple, the ganged mechanical switches are replaced by PIN diodes. which acts as electronic switch.

Advantages :

1. Low cost
2. Fast switching
3. occupies less space
4. Available in IC Form.

3. Beam steering Using Phase shifters :

* In many applications, phase shifters are used instead of controlling phase by switching cables.

* Phase shifting can be achieved by using ferrite device.

* The conducting wires are wrapped around the Phase shifter. The current flowing through these wires controls the magnetic field within ferrite and then the magnetic field in the ferrite controls the phase shift.

4. Using insertion of cables :-

Phase shifter effect may be produced by the

insertion of sections of cable (delay line) by electronic switching. Insertion of cables of length $\frac{\lambda}{4}$, $\frac{\lambda}{2}$, $\frac{3\lambda}{4}$ provides phase increments of 90° .

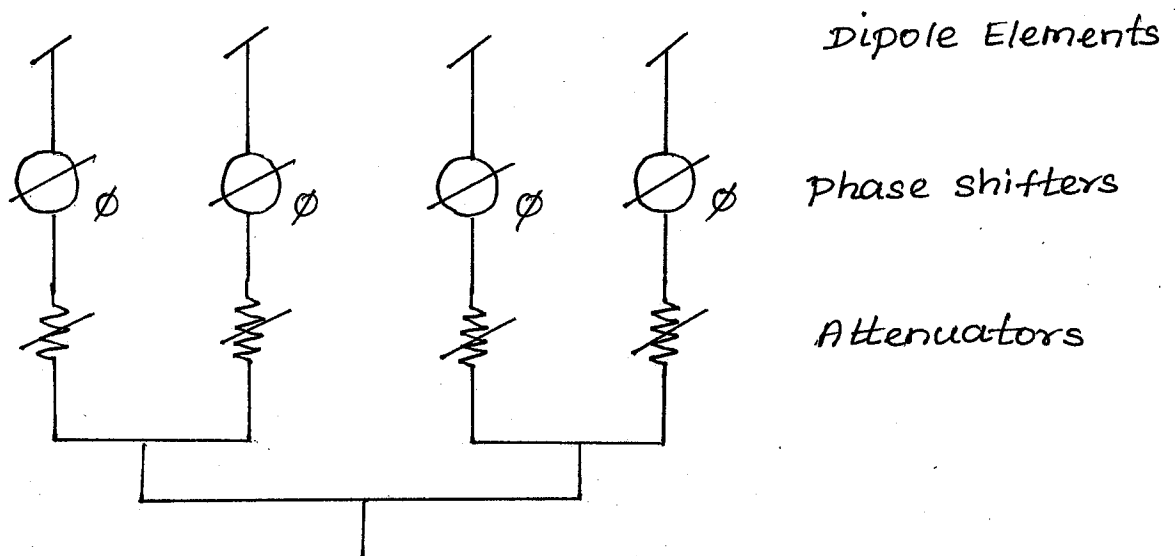
For more precise phasing, cables with smaller incremental differences are used.

Feeding Mechanisms of phased array :-

1. corporate Feed
2. Line Feed
3. coupler Feed

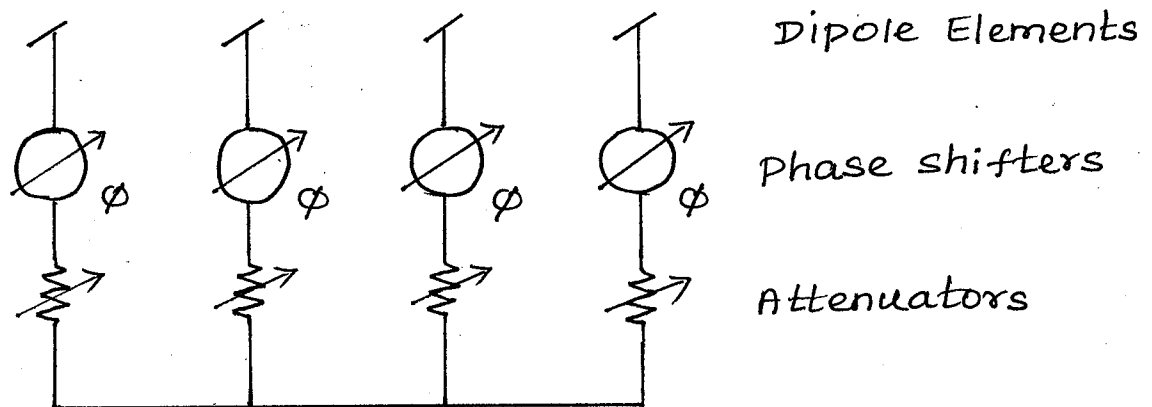
1. corporate Feed :-

In corporate Feed, the Feed cables are all of equal length. A Phase shifter and attenuator is connected at each element.



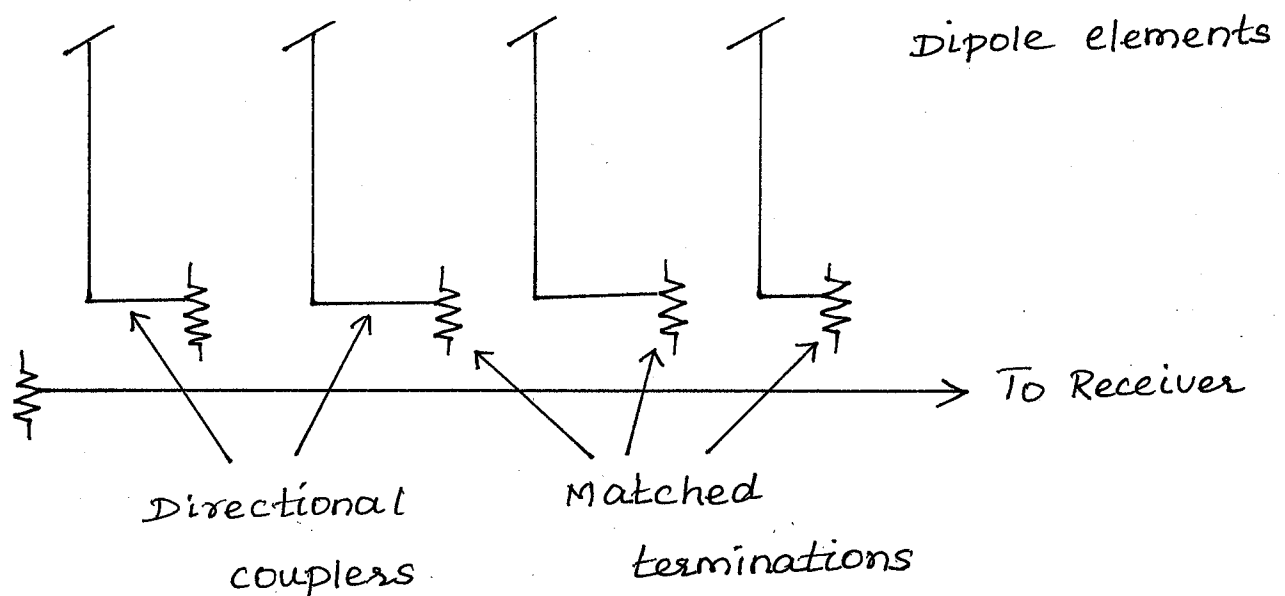
2. Line Feed :-

A progressive phase shift is introduced between elements using phase shifters and the array is of end fire array.



3. coupler Feed :-

In this Phasing is accomplished by physically sliding the directional coupler along the line.



The Phased array for specialized functional utility are recognized by different names such as,

1. Retroarray
2. Frequency scanning array
3. Adaptive array

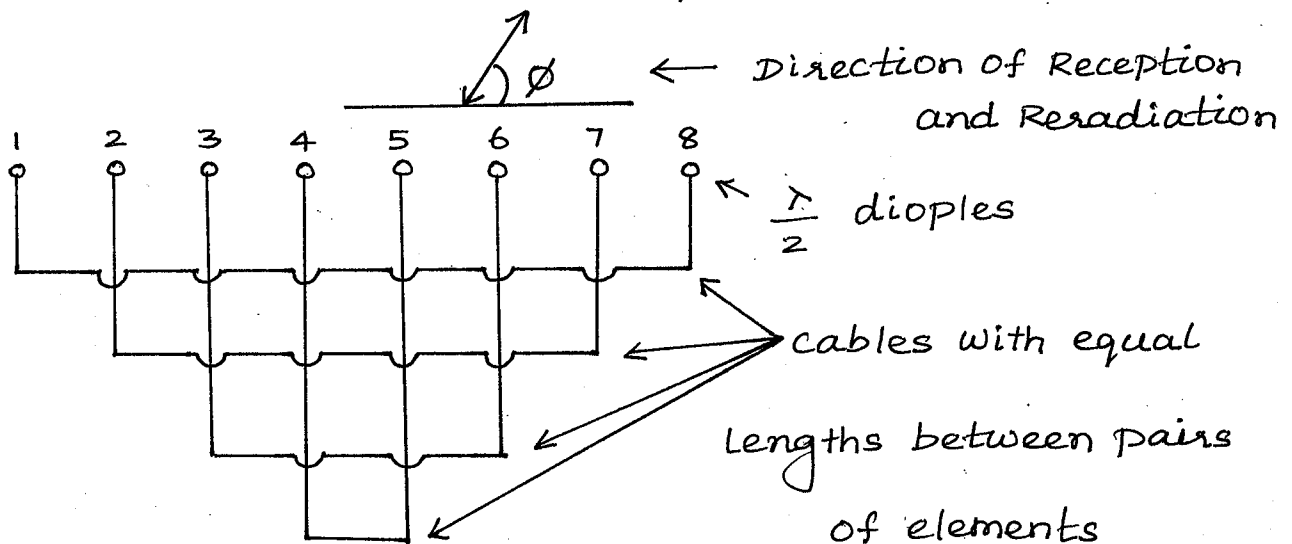
1. Retroarray :

* The array which automatically reflects an incoming signal back to the source is called retroarray.

* It acts as a retroreflector similar to the passive square corner reflector. That means the wave incident on the array is received and transmitted back in the same direction.

* Each element of the retroarray reradiates signal which is actually the conjugate of the received one.

* Simplest form of the retroarray is the van Atta array as shown in fig below, in which 8 identical $\lambda/2$ dipole elements are used with pairs formed between



elements 1 and 8, 2 and 7, 3 and 6, 4 and 5 using cables of equal length.

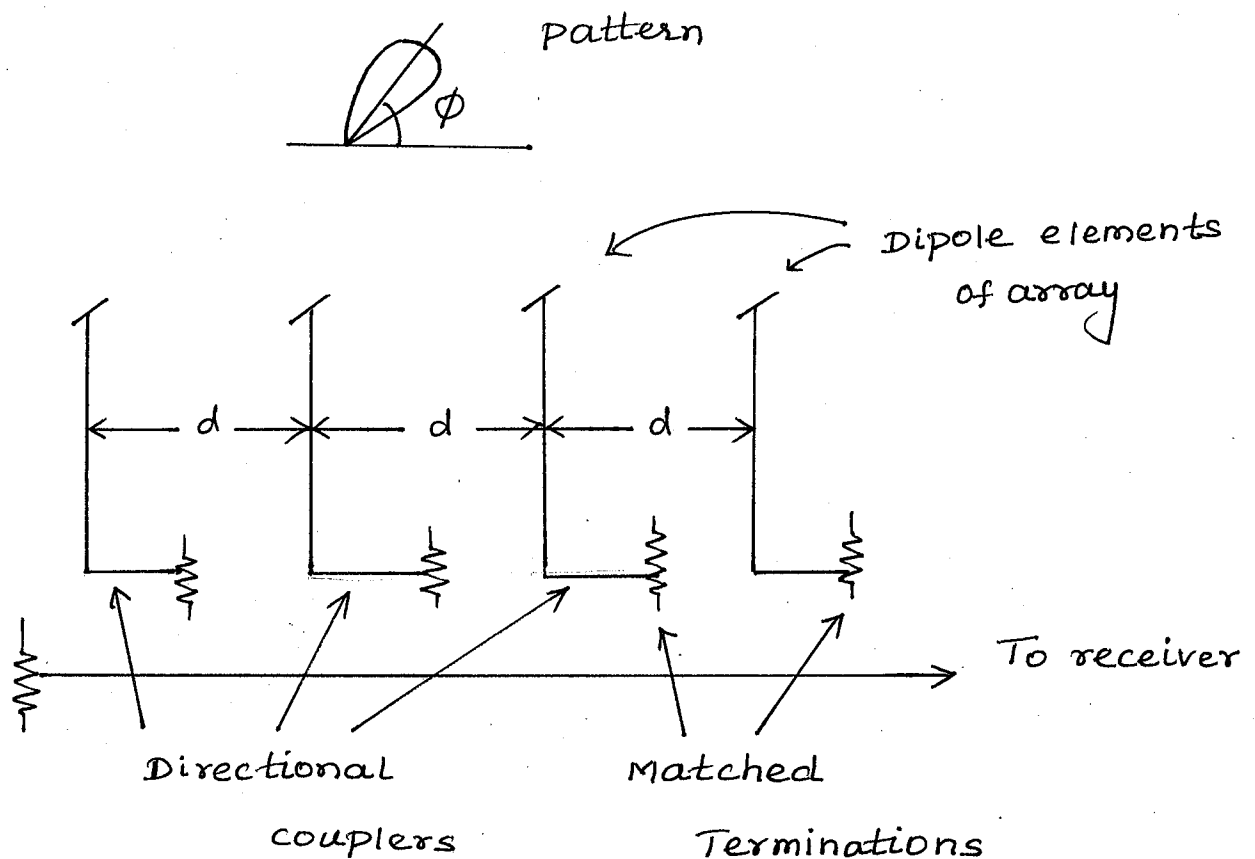
If the wave arrives at angle ϕ say then it gets transmitted in the same direction.

2. Frequency Scanning Array :-

The array in which the phase change is controlled by varying the frequency is called frequency scanning array.

It is the simplest phased array as at each element separate phase control is not necessary.

A simple transmission line fed frequency scanning array is shown in fig. below.



- * Each element of the scanning array is fed by a transmission line via directional coupler. The directional couplers are fixed in position, while the beam scanning is done with a frequency change.

- * To avoid reflections and to obtain pure form of the travelling wave, the transmission line is properly terminated.

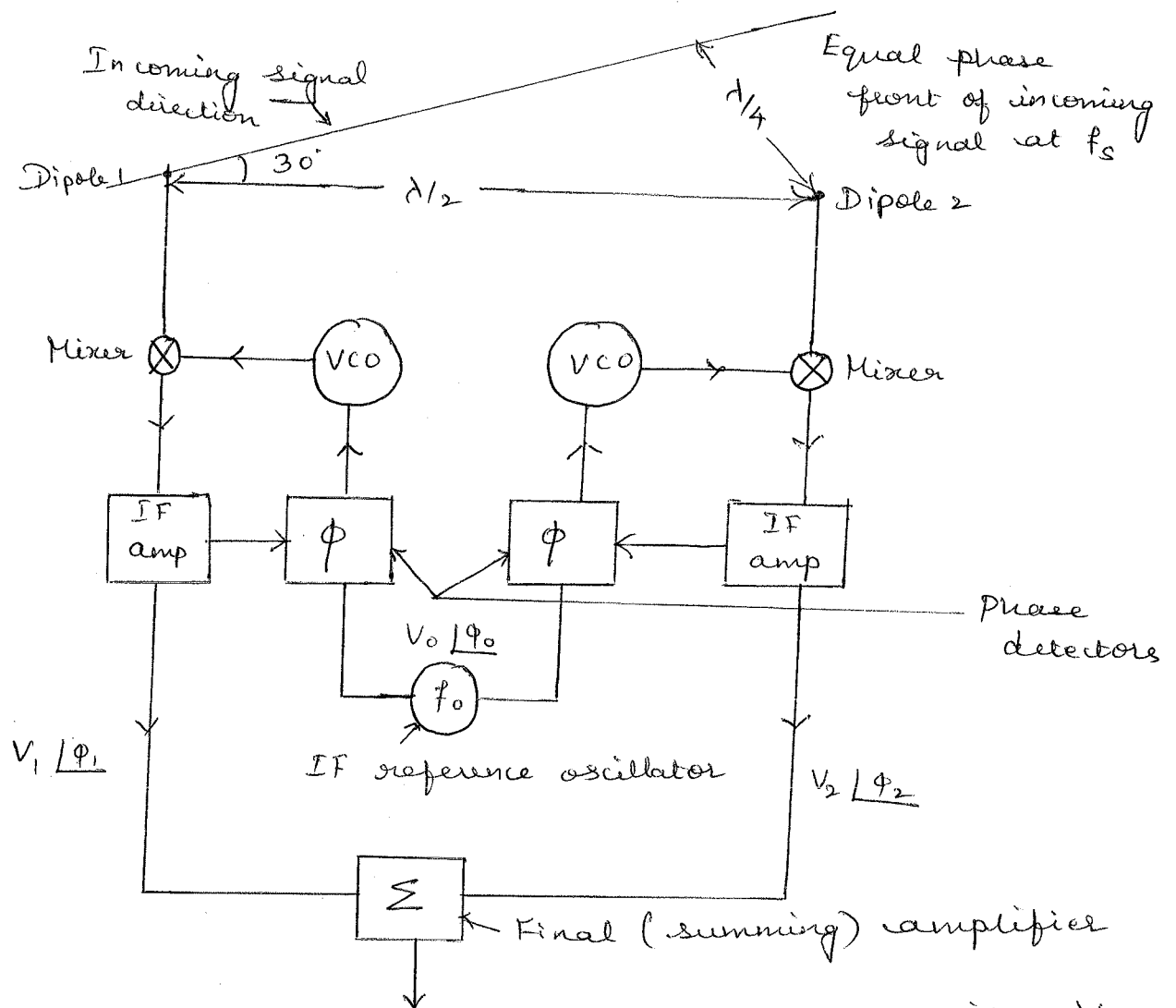
- * The main advantage of the frequency scanning array is that there are no moving parts and no switches.

3. Adaptive Array :-

- * An array which automatically turn the maximum beam in the desired direction while turn the null in the undesired direction is called adaptive array.

- * The adaptive array adjust itself in the desired direction with awareness of its environment.

- * In modern adaptive arrays, the output of each element in the array is sampled, digitized and then processed using computers. Such arrays are commonly called as smart antennas or Digital beam forming (DBF) antennas.



Consider an array of 2 element with $\lambda/2$ spacing between the elements at the signal frequency f_s . Let each element be a $\lambda/2$ dipole and the pattern of the elements are uniform.

Let a signal has 30° inclination from broadside. The wave arriving at element 2 travels $\lambda/4$ farther than to element 1, thus retarding the phase of the signal by 90° at element 2.

Each element is equipped with its own mixer, Voltage-controlled Oscillator (VCO), intermediate frequency amplifier and Phase detector.

A reference oscillator at frequency f_0 is connected to each phase detector as reference. The phase detector compares the phase of the downshifted signal with the phase of the reference oscillator and produces a voltage proportional to the phase difference. This voltage, advances or retards the phase of the VCO output so as to reduce the phase difference to zero (phase locking).

The voltage for the VCO of element 1 would be equal in magnitude and opposite to the voltage for the VCO of element 2, so that the downshifted signals from both elements are locked in phase, thus

$$\phi_1 = \phi_2 = \phi_0$$

Where,

ϕ_1 = Phase of downshifted signal from element 1

ϕ_2 = Phase of downshifted signal from element 2

ϕ_0 = Phase of reference oscillator

With equal gain from both IF amplifiers, the voltages V_1 and V_2 from both elements should be equal so that,

$$V_1 \angle \phi_1 = V_2 \angle \phi_2$$

Voltage from the summing amplifier is proportional to $2V_1 (= 2V_2)$, maximising the response of the array to the incoming signal by steering the beam onto the incoming signal. In this example, 45° phase corrections of opposite sign would be required by the VCOs (+ for element 1, - for element 2).

Advantages of Adaptive array:

1. Higher capacity
2. Higher coverage
3. Higher bit rate
4. Improved link quality
5. Spectral efficiency
6. Mobility

Disadvantages of adaptive array:

1. Complex
2. More Expensive
3. Larger size
4. Location

Applications:

1. Cellular & Wireless networks
2. Radar
3. Electronic warfare as a counter measure to electronic jamming
4. Satellite systems

Antenna Synthesis

The method of designing antenna to yield acceptable radiation pattern and other system constraints is called Antenna synthesis.

3 categories:

Antenna synthesis falls into 3 categories.

1. Requirement of antenna pattern to possess null in desired direction.
2. Requirement of pattern to exhibit a desired distribution in the entire visible region. This is referred to beam shaping.
3. Requirement of pattern with narrow beams and low side lobes.

Antenna synthesis methods:

1. Category 1 can be accomplished using Schelkunoff polynomial method.
2. Category 2 can be accomplished using Fourier transform and Woodward Lawson method.

3. category 3 can be achieved using

Binomial, Dolph - Tchebyshev method ~~and~~ and

Taylor line source.

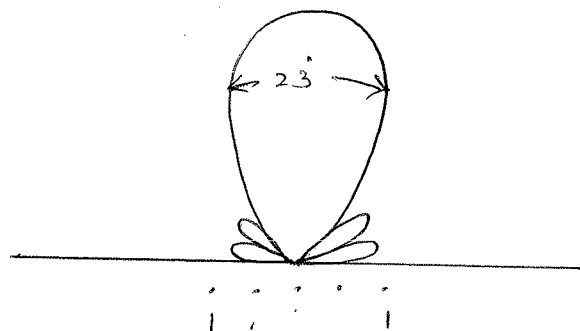
Array with non-uniform excitation amplitude:

Non-uniform amplitude distributions are classified into,

1. Binomial array
2. Edge array &
3. Optimum array.

Uniform distribution:

1. Equal amplitude of radiating sources, forms uniform array.
2. The minor lobes are relatively large.
3. Pattern has HPBW of 23° , but side lobes are relatively large.



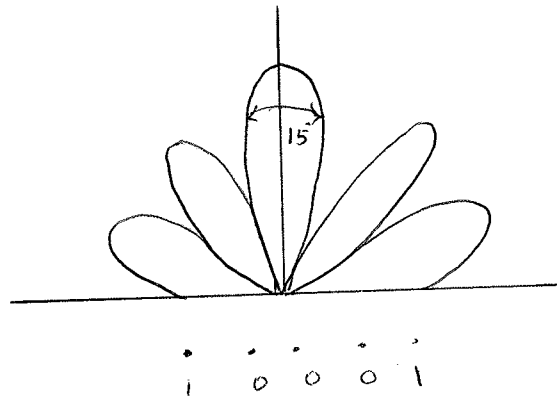
Edge array:

In edge distribution, only the end sources of the array are supplied with power, the remaining centre sources being either omitted or inactive.

* The relative amplitudes of the 5 sources are $1, 0, 0, 0, 1$.

* HPBW = 15° .

* Side lobes have same amplitude as the main lobe.



Optimum array:

In optimum array, the amplitudes of the sources follow Dolph Chebyshev polynomial and hence called Chebyshev array.

* HPBW = 27°

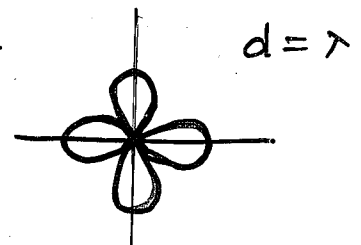
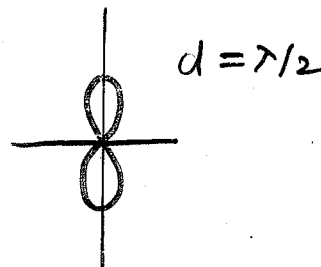
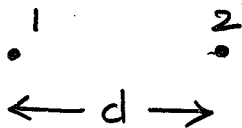
* Side lobes have lesser amplitude.

Binomial Array :

* In some practical applications, it is needed to have major lobe without minor lobes. so, in such cases Binomial array is preferred.

* Binomial array with spacing between 2 adjacent elements equal to less than $\lambda/2$ produces a pattern without side lobes.

* Basically Binomial array is a non uniform amplitude array.



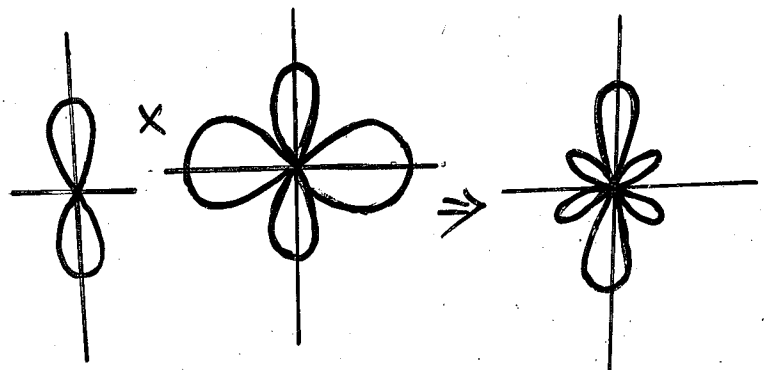
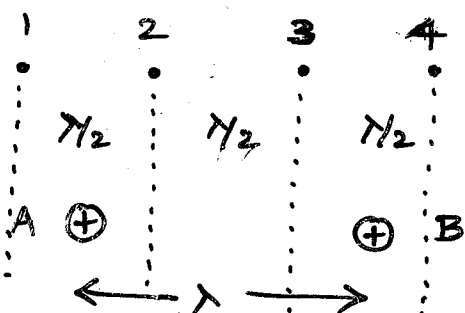
* For above array, the normalized electric field pattern is given by.

$$E_n = \cos^{n-1} \left[\frac{\pi d}{\lambda} \cdot \cos \theta \right] \rightarrow \textcircled{1}$$

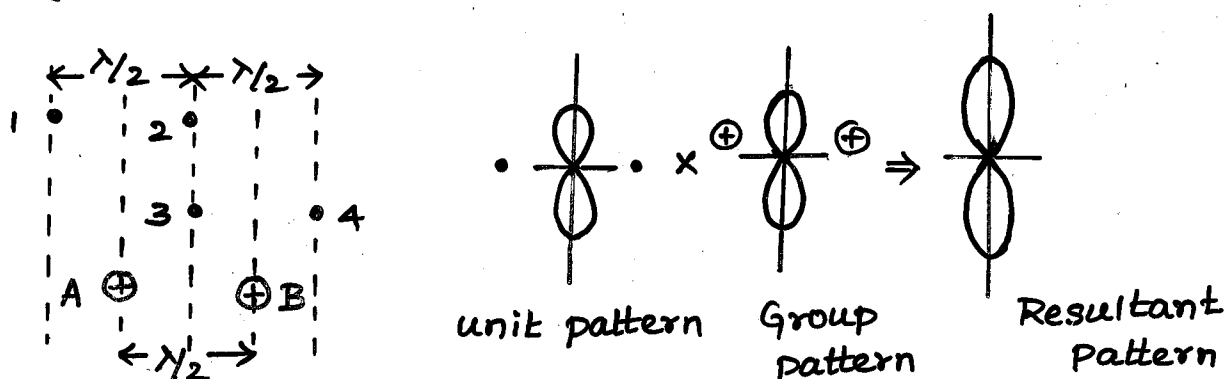
With $d = \lambda/2$, the pattern has no side lobes.

To increase directivity, array length is increased.

An array consisting of 4 elements spaced $\lambda/2$ apart.



So, By Pattern multiplication, the resultant pattern has side lobes. To reduce the side lobes, the elements are arranged as shown below.



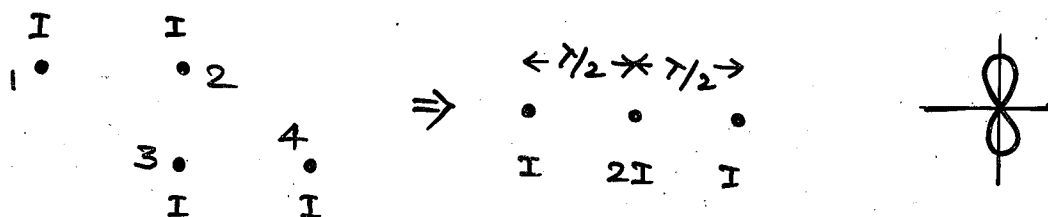
* To reduce or eliminate side lobes, the separation B/w group elements must be less than or equal to $\lambda/2$.

* To achieve figure of eight pattern, move group B towards group A till distance of separation is $\lambda/2$. When group B is at a distance $\lambda/2$ from A, the elements 2 and 3 are overlapped.

* The electric field of 2 elements with distance $\lambda/2$ is given by $E_n = \cos \left[\frac{\pi (\lambda/2)}{\lambda} \cos \theta \right] \Rightarrow \cos \left[\frac{\pi}{2} \cos \theta \right]$

Three Element Non-Uniform Array :

* To obtain 3 element Non-uniform Array, two 2-element uniform arrays are overlapped.



* Two elements 2 and 3 are coinciding with each other, so they can be replaced by a single element array carrying current $2I$.

* The resultant electric field is given by,

$$(E_R)_n = \cos \left[\frac{\pi}{2} \cos \theta \right] \times \cos \left[\frac{\pi}{2} \cos \theta \right] \Rightarrow \cos^2 \left[\frac{\pi}{2} \cos \theta \right]$$

Four Element Non-Uniform Array :

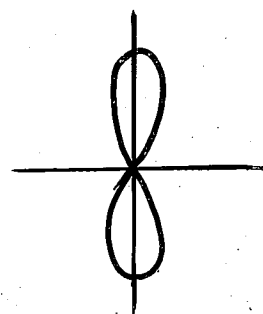
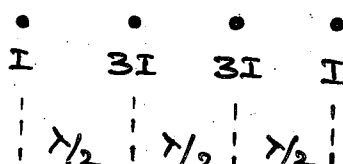
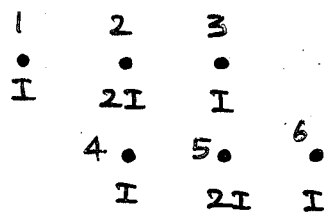
* To obtain 4-element non-uniform array, two 3-element non-uniform arrays are overlapped.

* In this case, element 2 of group A coincides with element 1 of group B and element 3 of group A coincides with element 2 of group B. So they can be replaced by a single elements carrying current $3I$.

* The resultant pattern is a product of a unit pattern (i.e., resultant of 3 element array which is figure eight squared) and a group pattern (which is figure eight pattern) so, the resultant pattern obtained is called figure eight cubed pattern.

* The resultant electric field is given by,

$$(E_R)_n = \cos^3 \left[\frac{\pi}{2} \cos \theta \right]$$



n-Element Binomial Array :

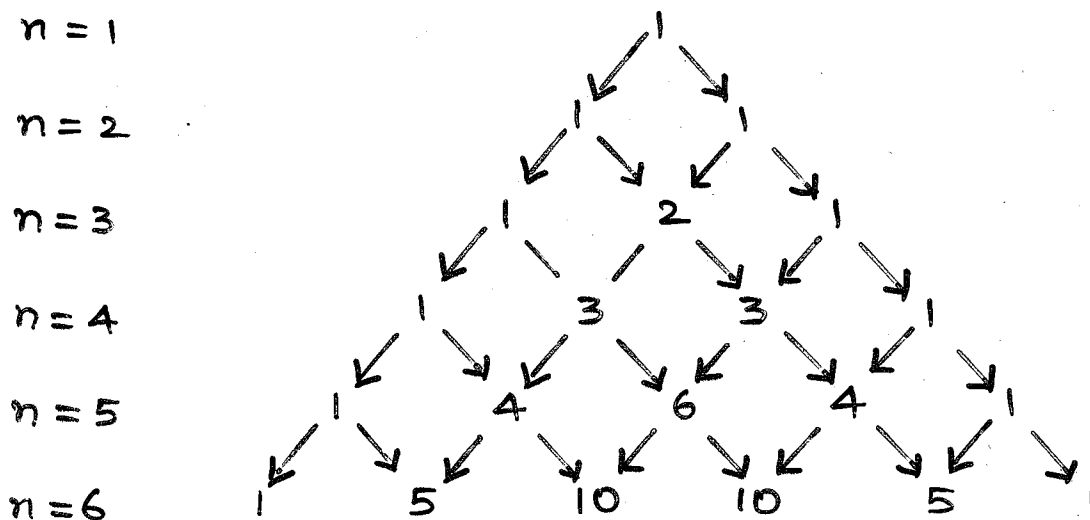
* The required directivity can be achieved using 'n' elements, where the current amplitudes of elements are arranged according to the coefficients of Binomial series.

* The coefficients of binomial series can easily be arranged using Pascal's triangle as given in table.

* In general, for Binomial array, the resultant pattern is given by,

$$E_n = \cos^{n-1} \left[\frac{\pi}{2} \cos \theta \right] ; \quad n \rightarrow \text{No. of elements in an array}$$

Pascal's Triangle (Coefficients of Binomial series)



* To design a binomial array to achieve required directivity, the required expressions are,

$$\text{HPBW} = \frac{1.06}{\sqrt{n-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}} \quad (d = \lambda/2)$$