Solved Anna University Problems
(NOV/DEC 2010 - NOV/DEC 2019)

EC8651 - Transmission Lines and RF Systems
UNIT 2 - HIGH FREQUENCY TRANSMISSION LINES

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PROBLEMS

1. A transmission line has Z_O = 1.0, Z_L = 0.-j0.2Ω (i) What is Z at l=λ/4 = 0.25λ? (ii) What is the VSWR on the line? (iii) How far from the load is at the first minimum? **[EC6503-NOV/DEC 2018](13 Marks)**

Solution:

Given: Z_O = 1.0, Z_L = 0.0-j0.2Ω, s=l=0.25λ

$$\beta s = \frac{2\pi}{\lambda} \times 0.25\lambda = 90^\circ$$

The input impedance is given by,

$$Z_{in} = Z_s = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s} \right]$$

$$Z_{in} = Z_s = R_O \frac{\tan\beta s}{\tan\beta s} \left[\frac{\frac{Z_R}{\tan\beta s} + jR_O}{\frac{R_O}{\tan\beta s} + jZ_R} \right]$$

$$Z_{in} = Z_s = R_O \left[\frac{0 + jR_O}{0 + jZ_R} \right]$$

$$Z_{in} = Z_s = R_O \left[\frac{R_O}{Z_R} \right]$$

$$Z_{in} = Z_s = \frac{R_O^2}{Z_R}$$

$$Z_{in} = Z_s = \frac{1}{-j0.2}$$

Z_{in} =Z_s = 0.2∠-90°

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{-j0.2 - 1}{-j0.2 + 1}$$

K = 1∠-157.38°

SWR is given by,

$$S = VSWR = \frac{1+|K|}{1-|K|} = \frac{1+1}{1-1} = \infty$$

$$l_{(V\min)} = \frac{\varphi + \pi}{2\beta} = \frac{\left(-157.38^\circ + 180^\circ\right)\lambda}{4 \times 180^\circ} = 0.0314\lambda$$

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2. A 30m long lossless transmission line with Zo = 50Ω operating at 2 MHz is terminated with a load Z_L =60+j40 Ω. If u=0.6c (c is velocity of light, u is phase velocity) on the line, find (i) Reflection Coefficient , (ii) Standing wave ratio (iii) Input impedance

[EC6503-NOV/DEC 2017](7 Marks), [EC2305-NOV/DEC 2012](16 Marks)[EC2305-APR/MAY 2011](6 Marks)

Solution:

Given: Z_0=50 Ω, $Z_R = 60 + j40\Omega$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{60 + j40 - 50}{60 + j40 + 50} = \frac{10 + j40}{110 + j40}$$

$$= 0.1971 + j0.2920 = 0.3523\angle 55.98^\circ$$

K = 0.3523∠55.98°

SWR is given by,

$$S = VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.3523}{1 - 0.3523} = \frac{1.3523}{0.6477} = 2.088$$

The velocity on the line is given by

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \frac{\omega}{v_p}$$

But the electrical length of the line is βs.

$$\therefore \beta s = \frac{\omega s}{v_p} = \frac{2\pi fs}{v_p} = \frac{2\pi \times 2 \times 10^6 \times 30}{0.6 \times 3 \times 10^8} = 0.6666\pi = 120^\circ$$

The input impedance is given by,

$$Z_{in} = Z_s = R_O\left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

$$Z_{in} = Z_s = 50\left[\frac{(60 + j40) + j50\tan(120)}{50 + j(60 + j40)\tan(120)}\right]$$

$$Z_{in} = Z_s = 50\left[\frac{-26.6025 + j40}{119.2820 - j103.923}\right]$$

$\mathbf{Z_{in} = Z_s = 15.183\angle 164.69^\circ}$

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3. Calculate SWR and reflection coefficient on a line having the characteristic impedance Z_0=300 Ω and terminating impedance $Z_R = 300 + j400\Omega$.

[EC6503-NOV/DEC 2016](2 Marks)

Solution:

Given: Z_0=300 Ω, $Z_R = 300 + j400\Omega$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(300 + j400) - 300}{(300 + j400) + 300} = \frac{j400}{600 + j400}$$

$$K = 0.3077 + j0.4615 = 0.5547\angle 56.31^\circ$$

$\mathbf{K = 0.5547\angle 56.31^\circ}$

SWR is given by,

$$S = VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.5547}{1 - 0.5547} = \frac{1.5547}{0.4453} = 3.4914$$

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4. Find the sending end impedance of a HF line having characteristic impedance of 50Ω. The line is of length 1.185 λ and is terminated in a load of 110+j80 Ω.

[**EC6503-NOV/DEC 2016](8 Marks), [EC2305-NOV/DEC 2018](8 Marks)**

Solution:

Given: $R_O = Z_O$ = 50Ω, line length s=1.185 λ, Z_R=110+j80 Ω

$$\beta s = \frac{2\pi}{\lambda} \times 1.185\lambda = 66.6^\circ$$

The input impedance is given by,

$$Z_{in} = Z_s = R_O\left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

$$Z_{in} = Z_s = 50\left[\frac{(110 + j80) + j50\tan(66.6)}{50 + j(110 + j80)\tan(66.6)}\right]$$

$$Z_{in} = Z_s = 50\left[\frac{110 + j195.54}{-134.87 + j254.195}\right]$$

$\mathbf{Z_{in} = Z_s = 5530.036\angle -0.165^\circ\ \Omega}$

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5. A lossless transmission line has a shunt capacitance of 100pF/m and a series inductance of 4μH /m. Determine the characteristic impedance. **[EC6503-NOV/DEC 2015](2 Marks), [EC2305-NOV/DEC 2015](2 Marks), [EC2305-NOV/DEC 2013](2 Marks), [EC2305-APR/MAY 2016](2 Marks)**

Solution:

Given: C=100 pF/m, L=4μH/m

The characteristic impedance is given by,

$$R_O = \sqrt{\frac{L}{C}} = \sqrt{\frac{4 \times 10^{-6}}{100 \times 10^{-12}}} = 200\,\Omega$$

R_O = 200 Ω

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6. A lossless line in air having a characteristic impedance of 300Ω is terminated in unknown impedance. The first voltage minimum is located at 15cm from the load. The distance between successive voltage minima is 15cm. The standing wave ratio is 3.3. Calculate the wavelength and terminated impedance.

[EC6503-NOV/DEC 2015](6 Marks), [EC2305-NOV/DEC 2015](6 Marks), [EC2305-NOV/DEC 2013](6 Marks)

Solution:

Given: $Z_O = R_O = 300\ \Omega$, $l_{(V\min)} = 15\text{cm}$, S=3.3

The distance between two successive minima, 0.5λ =15cm

∴λ = 30cm = 0.3m

$$l_{(V\min)} = \frac{\varphi + \pi}{2\beta} = \frac{(\varphi + \pi)\lambda}{4\pi} = 0.15$$

$$\varphi = \frac{0.15 \times 4\pi}{\lambda} - \pi$$

$$\varphi = \frac{0.15 \times 4 \times 180^\circ}{0.3} - 180^\circ$$

ϕ=180°

$$|K| = \frac{S-1}{S+1} = \frac{3.3-1}{3.3+1} = \frac{2.3}{4.3} = 0.5349$$

Hence the reflection coefficient is given by,

$K = |K|\angle\phi = 0.5349\angle 180^\circ$

Hence the terminating impedance is given by,

$$Z_R = Ro\left[\frac{1+K}{1-K}\right]$$

$$Z_R = 300\left[\frac{1+0.5349\angle 180^\circ}{1-0.5349\angle 180^\circ}\right]$$

Z_R = 90.905 Ω

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7. A line with zero dissipation has R=0.006 ohm per m, C=4.45 pF per m, L=2.5μH per m. If the line is operated at 10MHz, find R_O, α, β, λ, v.

[EC6503-MAY/JUN 2016](8 Marks), [EC6503-APR/MAY 2015](8 Marks) [EC2305-NOV/DEC 2018](8 Marks)

Solution:

Given: R=0.006 ohm/m, C=4.45 pF/m, L=2.5μH/m, f=10MHz

The characteristic impedance is given by,

$$R_O = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.5\times10^{-6}}{4.45\times10^{-12}}} = 749.53\,\Omega$$

R_O = 749.53 Ω

The propagation constant is given by

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\gamma = \alpha + j\beta = j(2\pi\times10\times10^{6})\sqrt{2.5\times10^{-6}\times4.45\times10^{-12}}$$

$$\gamma = \alpha + j\beta = 0 + j0.2095$$

The attenuation constant, α=0 NP/m

The phase constant, β= 0.2095 rad/m

the velocity of propagation is given by,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi\times10\times10^{6}}{0.2095} = 2.998\times10^{8}\ m/sec$$

v_p = 2.998x10^8 m/sec

The wavelength is given by,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2095} = 29.98\,m$$

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8. A lossless line has a standing wave ratio of 4. The R_O is 150 ohms and the maximum voltage measured in the line is 135V. Find the power delivered to the load.

[EC6503-MAY/JUN 2016](6 Marks), [EC6503-APR/MAY 2015](6 Marks)

Solution:

Given: S=4, V_{max} = 135V, R_O = 150Ω

The power delivered to the load is given by,

$$P = \frac{E_{max}^2}{R_{max}} = \frac{E_{max}^2}{R_O S} = \frac{135^2}{150 \times 4} = 30.375W$$

P=30.375W

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9. A lossless line has a characteristic impedance of 400 Ω. Determine the standing wave ratio if the receiving end impedance is 800+j0 Ω.

[EC6503-APR/MAY 2017](2 Marks), [EC2305-NOV/DEC 2010](2 Marks)

Solution:

Given: Z_O = 400 Ω, Z_R = 800+j0 Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{800 - 400}{800 + 400} = \frac{400}{1200}$$

K = 0.3333∠0°

SWR is given by,

$$S = VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.3333}{1 - 0.3333} = \frac{1.3333}{0.6667} = 2.0$$

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10. A radio frequency line with Z_O = 70 Ω is terminated by Z_L = 115-j80 Ω at λ=2.5m. Find the VSWR and the maximum and minimum line impedances. **[EC6503-APR/MAY 2017](6 Marks)**

Solution:

Given: Z_O = 70 Ω, Z_R = 115-j80 Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(115 - j80) - 70}{(115 - j80) + 70} = \frac{45 - j80}{185 - j80}$$

K = 0.4554∠-37.26°

SWR is given by,

$$S = VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.4554}{1 - 0.4554} = \frac{1.4554}{0.54467} = 2.672$$

S=2.672

maximum line impedance = Z_O.S = 70 x 2.672 = **187.04 Ω**

minimum line impedances = Z_O/S = 70 / 2.672 = **26.198 Ω**

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11. A line having characteristic impedance of 50 Ω is terminated in load impedance (75+j75) Ω. Determine the reflection coefficient and voltage standing wave ratio.
[EC2305-NOV/DEC 2016](4 Marks), [EC2305-NOV/DEC 2014](4 Marks)

Solution:

Given: Z_O = 50 Ω, Z_R = (75+j75) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(75 + j75) - 50}{(75 + j75) + 50} = \frac{25 + j75}{125 + j75}$$

$$K = 0.4118 + j0.3529 = 0.5423\angle 40.60^\circ$$

K = 0.5423∠40.60°

SWR is given by,

$$S = VSWR = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.5423}{1 - 0.5423} = \frac{1.5423}{0.4577} = 3.3697$$

S=3.3697

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12. A line having characteristic impedance of 50 Ω is terminated in load impedance (75+j75) Ω. Determine the reflection coefficient.
[EC2305-NOV/DEC 2014](2 Marks)

Solution:

Given: Z_O = 50 Ω, Z_R = (75+j75) Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(75 + j75) - 50}{(75 + j75) + 50} = \frac{25 + j75}{125 + j75}$$

$$K = 0.4118 + j0.3529 = 0.5423\angle 40.60^\circ$$

K = 0.5423∠40.60°

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13. characteristic impedance of a transmission line at 8 MHz is (40-2j) ohm and the propagation constant is (0.01+j0.18) per meter. Find the primary constants. **[EC2305-NOV/DEC 2012](8 Marks)**

Solution:

Given: Z_o=40-j2 Ω , f=8 MHz, P=γ=0.01+j0.18

$\omega = 2\pi f = 2 \times 3.14 \times 8\times10^6 = 50.24\times10^6$

$R+j\omega L = Z_o P = (40-j2) \times (0.01+j0.18)$

$= 0.76+j7.18$

Comparing real and imaginary terms,

R = 0.76 ohms/m

$\omega L = 7.18$

$$L = \frac{7.18}{50.24 \times 10^6} = 0.1429 \mu H/m$$

L=0.1429 μH/m

Similarly,

$$G + j\omega C = \frac{\gamma}{Z_o} = \frac{0.01 + j0.18}{40 - j2}$$

$$G + j\omega C = 2.494 \times 10^{-5} + j4.501 \times 10^{-3}$$

Comparing real and imaginary terms,

G = 24.94x10^{-6} mho/m

$\omega C = 4.501\times10^{-3}$

$$C = \frac{4.501 \times 10^{-3}}{50.24 \times 10^6} = 0.08959\, nF/m$$

C = 0.08959nF/m

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14. A low loss transmission line of 100 ohms characteristic impedance is connected to a load of 200 ohm. Calculate the voltage reflection coefficient and the standing wave ratio. **[EC2305-MAY/JUN 2012](6 Marks)**

Solution:

Given: Z_O = 100 Ω, Z_R = 200 Ω

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{200-100}{200+100} = \frac{100}{300}$$

K = 0.3333∠0°

SWR is given by,

$$S = VSWR = \frac{1+|K|}{1-|K|} = \frac{1+0.3333}{1-0.3333} = \frac{1.3333}{0.6667} = 2.0$$

S=2.0
