PANIMALAR ENGINEERING COLLEGE

DEPARTMENT OF ECE

INTERNAL ASSESSMENT I

EC 8651 – TRANSMISSION LINES AND RF SYSTEMS - ANSWER KEY

Date: **Max. Marks:** 50
Class: III ECE A, B, C, D & E **Duration:** 2 Hours

PART A (5 X 2 =10)

1. Define characteristic impedance.

In a uniform transmission line it is the ratio of the voltage amplitude to the current amplitude of a single wave traveling down it. This is also called as "Surge impedance".

The impedance measured at any point in a transmission line is called as characteristic impedance. It is denoted as Z_o.

It is given by, $Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$ ohms/Km.

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2. When does reflection occur in a line?

When the load impedance (Z_R) is not equal to the characteristic impedance (Z_o) of transmission line (i.e., $Z_R \neq Z_o$), reflection takes place.

Reflection occurs because of the following cases:

1) when the load end is open circuited

2) when the load end is short-circuited

3) when the line is not terminated in its characteristic impedance

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3. Find the attenuation and phase constant of a wave propagating along the line whose propagation constant is $1.048 \times 10^{-4} \angle 88.8^\circ$

$\gamma = \alpha + j\beta \quad = 1.048 \text{x} 10^{-4} \angle 88.8^\circ$

$= 2.1947 \text{x} 10^{-6} + j1.04777 \text{x} 10^{-4}$

$\alpha = 2.1947 \text{x} 10^{-6}$ Np/m

$\beta = 1.04777 \text{x} 10^{-4}$ rad/m

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4. Define reflection coefficient.

Reflection Coefficient can be defined as the ratio of the reflected voltage or current to the incident voltage or current at the receiving end of the line.

It is denoted by 'K' or 'r' or 'ρ'

$$\text{Reflection Coefficient, } K = \frac{\text{Reflected Voltage or current at load}}{\text{Incident voltage or current at the load}}$$

$$K = V_r / V_i$$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

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5. Find SWR and reflection coefficient of a 50Ω transmission line when it is terminated by a load impedance of (60 + j40) ohm.

Solution:

$Z_R = (60+j40)\Omega$

$Z_O = 50\Omega$

The reflection coefficient is given by,

$$K = \frac{Z_R - Z_O}{Z_R + Z_O} = \frac{(60+j40)-50}{(60+j40)+50} = \frac{10+j40}{110+j40} = \frac{41.231\angle 75.96°}{117.0469\angle 19.98°} = 0.3522\angle 55.98°$$

$\mathbf{K = 0.3522\angle 55.98°}$

$$S = \frac{1+|K|}{1-|K|} = \frac{1+0.3522}{1-0.3522} = 2.087$$

$\mathbf{S = 2.087}$

PART B (2 X 13 = 26)

6.a) An open wire line which is 200 Km long is properly terminated. The generator at the sending end has V = 10V, f = 1 KHz and internal impedance of 500 Ω. At that frequency, the characteristic impedance of the line is (700+ j100) ohms and γ=0.007+j0.04 per Km. Determine the sending end voltage, current and power and the receiving end voltage, current and power.

(13 Marks)

Solution:

Given:

E_g = 10V, f= 1000 Hz, l=200 Km, $Z_R = Z_O$, Z_O = 700 +j 100 ohms, $Z_S = Z_O$, γ=0.007+j0.04

$\omega = 2\pi f$ = 2x3.14x1000 = 6280

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{10}{500 + 700 + j100^{\circ}} = 8.3045 \times 10^{-3} \angle -4.764^{\circ}\ A$$

$\mathbf{I_S = 8.3045x10^{-3} \angle -4.764^{\circ}}$

$E_S = I_S \times Z_S$

$E_S = 8.3045x10^{-3} \angle -4.764^{\circ} \times 700+j100$

$\mathbf{E_S = 5.872 \angle 3.366^{\circ}\ V}$

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)} e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, and put K=0, because $Z_R = Z_O$.

$$I_S = I_R\, e^{\sqrt{ZY}l}$$

$$I_R = I_S\, e^{-\sqrt{ZY}l}$$

$$e^{-\sqrt{ZY}l} = e^{-\gamma l} = e^{-(0.007+j0.04)\times 200} = e^{-1.4} \angle 98.36^{\circ}$$

$$e^{-\sqrt{ZY}l} = 0.247 \angle 98.36^{\circ}$$

$$\therefore I_R = 8.3045 \times 10^{-3} \angle -4.764^{\circ} \times 0.247 \angle 98.36^{\circ}$$

$\mathbf{I_R = 2.0512x10^{-3} \angle 93.60^{\circ}}$

$E_R = I_R \times Z_R = 2.0512x10^{-3} \angle 93.60^{\circ} \times (700+j100)$

$\mathbf{E_R = 1.45 \angle 101.73^{\circ}\ V}$

$P_S = |E_S| . |I_S| .\cos(E_S \wedge I_S)$

$P_S = 5.872 \times 8.3045x\ 10^{-3} \times \cos(8.13)$

$\mathbf{P_S = 0.048\ W}$

and $P_R = |E_R| . |I_R| .\cos(E_R \wedge I_R)$

$P_R = 1.45 \times 2.0512x10^{-3} \cos(8.13)$

$\mathbf{P_R = 2.944x10^{-3}\ W}$

Transmission Efficiency, $\eta = \frac{P_R}{P_S} \times 100$

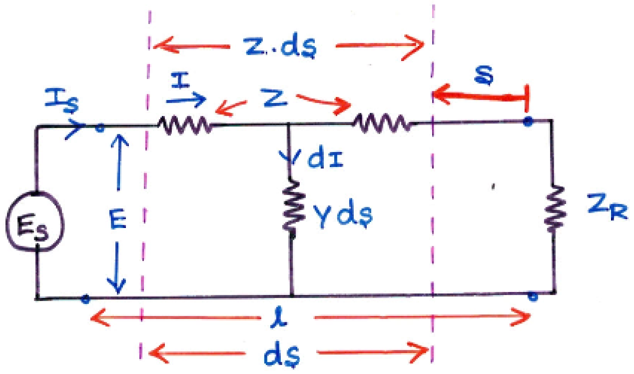
$$\eta = \frac{2.944x10^{-3}}{0.048} \times 100$$

$$\eta = 6.134\%$$

(Or)

b) Derive the expression for voltage and current at any point on a transmission line in terms of receiving end voltage and current. Also derive it for a line terminated by Z_0. **(13 Marks)**

When the voltage or current is transmitted a transmission line, it will not be constant through out the line. There will be drop in the voltage or current. To find the voltage and current at any point in a transmission line, let us derive general solution of transmission line.



Let R,L,G, and C be the primary constants of transmission line

l - Total length of the line

s - Distance from the load to the point of observation

ds - Small section of transmission line

Es - Source voltage

Is - Source current

E - Voltage at any point on a line

I - Current at ant point on a line

Consider a small section 'ds' having the series impedance 'Zds'. Let the current flowing through this section be 'I' then the voltage drop across this section will be

$$dE = I.Zds$$

$$\frac{dE}{ds} = I.Z \qquad (1)$$

Similarly, the current drop cross this section will be,

$$dI = E.Yds$$

$$\frac{dI}{ds} = E.Y \qquad (2)$$

Differentiate equations (1) and (2) w.r.to 's'

$$\frac{d^2E}{ds^2}=\frac{dI}{ds}.Z$$

$$\frac{d^2E}{ds^2}=EY.Z \qquad (3)$$

similarly, $\frac{d^2I}{ds^2}=IZ.Y \qquad (4)$

Equations (3) and (4) are called as the differential equations of transmission line. To find the solution for the differential equation,

put $\frac{d}{ds}=m$ in equations (3) and (4)

$m^2E = EYZ$ $\qquad$ $m^2I = IYZ$

$m^2 = YZ$ $\qquad$ $m^2 = YZ$

$m=\pm\sqrt{YZ}$ (5a) $\qquad$ $m=\pm\sqrt{YZ}$ (5b)

$\therefore$ the solution of the differential equations (3) and (4) are,

$$E=Ae^{\sqrt{ZY}s}+Be^{-\sqrt{ZY}s} \qquad (6)$$

$$I=Ce^{\sqrt{ZY}s}+De^{-\sqrt{ZY}s} \qquad (7)$$

To find the voltage and current at the receiver end,

put $I = I_R$, $E = E_R$, and s=0 in equs. (6) and (7)

$E_R = A + B$ (8)

$I_R = C + D$ (9)

Now, Differentiate equs. (6) and (7) w.r.to 's'

$$\frac{dE}{ds}=A.\sqrt{ZY}.e^{\sqrt{ZY}.s}-B.\sqrt{ZY}.e^{-\sqrt{ZY}.s}$$

$$IZ=A.\sqrt{ZY}.e^{\sqrt{ZY}.s}-B.\sqrt{ZY}.e^{-\sqrt{ZY}.s}$$

$$I=A.\sqrt{\frac{Y}{Z}}.e^{\sqrt{ZY}.s}-B.\sqrt{\frac{Y}{Z}}.e^{-\sqrt{ZY}.s} \qquad (10)$$

Similarly,

$$E=C.\sqrt{\frac{Z}{Y}}.e^{\sqrt{ZY}.s}-D.\sqrt{\frac{Z}{Y}}.e^{-\sqrt{ZY}.s} \qquad (11)$$

To find the voltage and current at receiver end,

put $I = I_R$, $E = E_R$, and s=0 in equs. (10) and (11)

$$I_R = A.\sqrt{\frac{Y}{Z}} - B.\sqrt{\frac{Y}{Z}} \qquad (12)$$

$$E_R = C.\sqrt{\frac{Z}{Y}} - D.\sqrt{\frac{Z}{Y}} \qquad (13)$$

To find the arbitrary constants A, B, C, and D solve equs. (8), (9), (12), and (13)

$$E_R = A + B$$

$$E_R = A + B$$

$$\sqrt{\frac{Z}{Y}} I_R = A - B$$

$$E_R + \sqrt{\frac{Z}{Y}} I_R = 2A$$

$$A = \frac{E_R}{2} + \sqrt{\frac{Z}{Y}} \frac{I_R}{2}$$

$$A = \frac{E_R}{2} + Z_o \frac{E_R}{2Z_R} \qquad \because \sqrt{\frac{Z}{Y}} = Z_o, I_R = \frac{E_R}{Z_R}$$

$$A = \frac{E_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) \qquad (14)$$

similarly,

$$B = \frac{E_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) \qquad (15)$$

$$C = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right) \qquad (16)$$

$$D = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right) \qquad (17)$$

sub equs. (14) and (15) in equs. (6) and (7)

$$E = A e^{\sqrt{ZY}s} + B e^{-\sqrt{ZY}s}$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}s} + \frac{E_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}s}$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_o}{Z_R}\right)\left[e^{\sqrt{ZY}s} + \frac{\left(1 - \frac{Z_o}{Z_R}\right)}{\left(1 + \frac{Z_o}{Z_R}\right)} e^{-\sqrt{ZY}s} \right]$$

$$E=\frac{E_R}{2}\left(1+\frac{Z_o}{Z_R}\right)\left[e^{\sqrt{ZY}s}+\frac{\left(\frac{Z_R-Z_o}{Z_R}\right)}{\left(\frac{Z_R+Z_o}{Z_R}\right)}e^{-\sqrt{ZY}s}\right]$$

$$E=\frac{E_R}{2}\left(\frac{Z_R+Z_o}{Z_R}\right)\left[e^{\sqrt{ZY}s}+\frac{(Z_R-Z_o)}{(Z_R+Z_o)}e^{-\sqrt{ZY}s}\right]$$

$$E=\frac{E_R}{2}\left(\frac{Z_R+Z_o}{Z_R}\right)\left[e^{\sqrt{ZY}s}+K\,e^{-\sqrt{ZY}s}\right] \qquad (18)$$

Similarly, $I=\frac{I_R}{2}\left(\frac{Z_R+Z_o}{Z_o}\right)\left[e^{\sqrt{ZY}s}-K\,e^{-\sqrt{ZY}s}\right]$ (19)

where, $K=\frac{Z_R-Z_O}{Z_R+Z_O}$

Equations (18) and (19) are the useful equations of transmission line.

The differential Equations (6) and (7) can also be solved as,

$$E=\frac{E_R}{2}\left(1+\frac{Z_o}{Z_R}\right)e^{\sqrt{ZY}s}+\frac{E_R}{2}\left(1-\frac{Z_o}{Z_R}\right)e^{-\sqrt{ZY}s}$$

$$E=\frac{E_R}{2}e^{\sqrt{ZY}s}+\frac{E_R}{2}\frac{Z_o}{Z_R}e^{\sqrt{ZY}s}+\frac{E_R}{2}e^{-\sqrt{ZY}s}-\frac{E_R}{2}\frac{Z_o}{Z_R}e^{-\sqrt{ZY}s}$$

$$E=\frac{E_R}{2}e^{\sqrt{ZY}s}+\frac{I_R Z_o}{2}e^{\sqrt{ZY}s}+\frac{E_R}{2}e^{-\sqrt{ZY}s}-\frac{I_R Z_o}{2}e^{-\sqrt{ZY}s} \qquad \because \frac{E_R}{Z_R}=I_R$$

$$E=\frac{E_R}{2}e^{\sqrt{ZY}s}+\frac{E_R}{2}e^{-\sqrt{ZY}s}+\frac{I_R Z_o}{2}e^{\sqrt{ZY}s}-\frac{I_R Z_o}{2}e^{-\sqrt{ZY}s}$$

$$E=\frac{E_R}{2}\left(e^{\sqrt{ZY}s}+e^{-\sqrt{ZY}s}\right)+\frac{I_R Z_o}{2}\left(e^{\sqrt{ZY}s}-e^{-\sqrt{ZY}s}\right)$$

$$E=E_R\cosh\sqrt{ZY}s+I_R Z_o\sinh\sqrt{ZY}s \qquad (20)$$

Similarly, $I=I_R\cosh\sqrt{ZY}s+\frac{E_R}{Z_o}\sinh\sqrt{ZY}s$ (21)

When line is terminated by Z_o, the general solution becomes,

$Z_R = Z_o$, K=0

$$E=\frac{E_R}{2}\left(\frac{Z_o+Z_o}{Z_o}\right)\left[e^{\sqrt{ZY}s}+0.e^{-\sqrt{ZY}s}\right]$$

$$E=E_R\,e^{\sqrt{ZY}s} \qquad (22)$$

$I = I_R\, e^{\sqrt{ZY}s}$ (23)

7.a) i) Derive the input impedance of open and short circuited lines and also derive the expression of transfer impedance. **(7 Marks)**

Input Impedance of Transmission Line:

The input impedance is defined as the ratio of voltage to current measured at the input end of the transmission line. It is given as,

$$Z_s = \frac{E_s}{I_s} \qquad ---(1)$$

From general solution of transmission line, the voltage and current measured at any point on the line can be given as,

$$E = E_R \cosh\sqrt{ZY}\,s + I_R Z_O \sinh\sqrt{ZY}\,s \qquad ---(2)$$

$$I = I_R \cosh\sqrt{ZY}\,s + \frac{E_R}{Z_O}\sinh\sqrt{ZY}\,s \qquad ---(3)$$

To find the voltage and current at the source end, put E=E$_S$, I=I$_S$ and s=l in equation (2) and (3)

$$E_S = E_R \cosh\sqrt{ZY}\,l + I_R Z_O \sinh\sqrt{ZY}\,l \qquad ---(4)$$

$$I_S = I_R \cosh\sqrt{ZY}\,l + \frac{E_R}{Z_O}\sinh\sqrt{ZY}\,l \qquad ---(5)$$

Substitute equ. (4) and (5) in equ. (1)

$$Z_s = \frac{E_R \cosh\sqrt{ZY}\,l + I_R Z_O \sinh\sqrt{ZY}\,l}{I_R \cosh\sqrt{ZY}\,l + \frac{E_R}{Z_O}\sinh\sqrt{ZY}\,l}$$

$$Z_s = \frac{I_R Z_R \cosh\sqrt{ZY}\,l + I_R Z_O \sinh\sqrt{ZY}\,l}{I_R \cosh\sqrt{ZY}\,l + \frac{I_R Z_R}{Z_O}\sinh\sqrt{ZY}\,l}$$

$$Z_s = \frac{I_R Z_R \cosh\sqrt{ZY}\,l + I_R Z_O \sinh\sqrt{ZY}\,l}{I_R \cosh\sqrt{ZY}\,l + \frac{I_R Z_R}{Z_O}\sinh\sqrt{ZY}\,l}$$

$$Z_s = \frac{I_R}{I_R}\left[\frac{Z_R \cosh\sqrt{ZY}\,l + Z_O \sinh\sqrt{ZY}\,l}{\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_O}\sinh\sqrt{ZY}\,l}\right]$$

$$Z_s = Z_O\left[\frac{Z_R \cosh\sqrt{ZY}\,l + Z_O \sinh\sqrt{ZY}\,l}{Z_O \cosh\sqrt{ZY}\,l + Z_R \sinh\sqrt{ZY}\,l}\right] \qquad ----(6)$$

Case 1: Short Circuit

For short circuit put Z_R=0 in equ. (6)

$$Z_{SC} = Z_O\left[\frac{0 + Z_O \sinh\sqrt{ZY}\,l}{Z_O \cosh\sqrt{ZY}\,l + 0}\right]$$

$$Z_{SC} = Z_O\left[\frac{Z_O \sinh\sqrt{ZY}\,l}{Z_O \cosh\sqrt{ZY}\,l}\right]$$

$$Z_{SC} = Z_O \tanh\sqrt{ZY}\,l \qquad ---(7)$$

Case 2: Open Circuit

For open circuit put $Z_R = \infty$ in equ. (6)

$$Z_{OC} = Z_O \cdot \frac{Z_R}{Z_R}\left[\frac{\cosh\sqrt{ZY}\,l + \frac{Z_O}{Z_R}\sinh\sqrt{ZY}\,l}{\frac{Z_O}{Z_R}\cosh\sqrt{ZY}\,l + \sinh\sqrt{ZY}\,l}\right]$$

$$Z_{OC} = Z_O \cdot \frac{Z_R}{Z_R}\left[\frac{\cosh\sqrt{ZY}\,l + 0}{0 + \sinh\sqrt{ZY}\,l}\right]$$

$$Z_{OC} = Z_O\left[\frac{\cosh\sqrt{ZY}\,l}{\sinh\sqrt{ZY}\,l}\right]$$

$$Z_{OC} = Z_O \coth\sqrt{ZY}\,l \qquad ---(8)$$

Transfer Impedance:

Transfer impedance is defined as the ratio of source voltage to receiver current. It is denoted by 'Z_T'

$$Z_T = \frac{E_s}{I_R} \qquad ---(1)$$

W.K.T the voltage at any point on a line is given as,

$$E = E_R \cosh\sqrt{ZY}s + I_R Z_o \sinh\sqrt{ZY}s \qquad (2)$$

To find the voltage at source end,

Put $E=E_s$ and s=l in equ. (2)

$$E_s = E_R \cosh\sqrt{ZY}l + I_R Z_o \sinh\sqrt{ZY}l$$

$$E_s = I_R Z_R \cosh\sqrt{ZY}l + I_R Z_o \sinh\sqrt{ZY}l$$

$$E_s = I_R\left(Z_R \cosh\sqrt{ZY}l + Z_o \sinh\sqrt{ZY}l\right)$$

$$\frac{E_s}{I_R} = Z_R \cosh\sqrt{ZY}l + Z_o \sinh\sqrt{ZY}l$$

$$Z_T = \frac{E_s}{I_R} = Z_R \cosh\sqrt{ZY}l + Z_o \sinh\sqrt{ZY}l \qquad (3)$$

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ii) A line has the following primary constants R = 100Ω / km, L = 0.001 H/Km, G = 1.5 μmho / Km, C = 0.062 μF /Km. Find the characteristic impedance and propagation constant at frequency 1000 Hz. **(6 Marks)**

Solution:

Given:

f= 1000 Hz, R = 100 ohms/Km, L = 0.001 H/Km, G = 1.5 μmho/ Km, C = 0.062 μF/Km

ω = 2πf = 2x3.14x1000 = 6280

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{100 + j(6280)(0.001)}{1.5\times10^{-6} + j(6280)(0.062\times10^{-6})}}$$

$$Z_o = \sqrt{\frac{100 + j6.28}{1.5\times10^{-6} + j3.8936\times10^{-4}}} = \sqrt{\frac{100.196\angle 3.593^\circ}{3.8936\times10^{-4}\angle 89.78^\circ}}$$

$$Z_o = \frac{10.01}{0.0197}\angle(3.593^\circ - 89.78^\circ)/2$$

$\mathbf{Z_o = 508.12 \angle{-43.094}^\circ}$

The propagation constant is given by,

$$P = \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{(100 + j(6280)(0.001)) \times \left(1.5 \times 10^{-6} + j(6280)(0.062 \times 10^{-6})\right)}$$

$$P = \gamma = \sqrt{(100 + j6.28) \times \left(1.5 \times 10^{-6} + j3.8936 \times 10^{-4})\right)}$$

$$P = \gamma = \sqrt{100.196\angle 3.593^{\circ} \times 3.8936 \times 10^{-4} \angle 89.78^{\circ}}$$

$$P = \gamma = \sqrt{100.196 \times 3.8936 \times 10^{-4} \angle 89.78^{\circ} + 3.593^{\circ}}$$

$P=\gamma = 0.1975 \angle 46.687°$

(Or)

b) i) The characteristic impedance of a uniform transmission line is 2039.5 Ω at a frequency of 800 Hz. At this frequency the propagation constant was found to be $0.054\angle 87.9^{\circ}$Determine the values of primary constants. **(6 Marks)**

Solution:

Given:

Z_o=2309.6 Ω , f=800Hz, $P=\gamma=0.054\angle 87.9^{\circ}$

$\omega = 2\pi f$ = 2 x 3.14 x 800 = 5024

$R=j\omega L = Z_oP$ = 2309.6 x $0.054\angle 87.9^{\circ}$

= 4.57+j124.63

Comparing real and imaginary terms,

R = 4.57 ohms/Km

ωL = 124.63

$$L = \frac{124.63}{5024} = 0.0248\,H/Km\,(Or)\,24.8mH/Km$$

L=24.8 mH/Km

Similarly,

$$G + j\omega C = \frac{\gamma}{Z_o} = \frac{0.054\angle 87.9^{\circ}}{2309.6}$$

$$G + j\omega C = 8.5675 \times 10^{-7} + j2.3365 \times 10^{-5}$$

Comparing real and imaginary terms,

$G = 8.5675\text{x}10^{-7}$ mho/Km

$\omega C = 2.3365\text{x}10^{-5}$

$$C=\frac{2.3365\times10^{-5}}{5024}=4.6507\,nF/Km$$

C = 4.6507nF/Km

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ii) Explain in detail about waveform distortion and also derive the condition for minimum attenuation in a distortionless line. **(7 marks)**

The signal transmitted through the transmission line will be in complex from and has many frequency components. In ideal transmission line, the signal received at the receiver end must be same as the transmitted signal. this condition is achieved only if all the frequency components are attenuated equally and transmitted with same delay. There are 2 types of waveform distortion.

1. Frequency distortion

2. Delay or Phase distortion

1. Frequency distortion:

It is a type of distortion in which all the frequency components are not attenuated at same level. This distortion can be avoided if 'α' is independent of 'ω'. In transmission line equalizers are used at the ends to reduce the distortion.

$$\alpha=\sqrt{\frac{(RG-\omega^2LC)+\sqrt{(RG-\omega^2LC)^2+\omega^2(RC+LG)^2}}{2}}$$

2. Delay or Phase distortion

It is a type of distortion in which all the frequency components are transmitted at different time intervals. This distortion can be avoided if 'β' is a constant multiplied by 'ω' and 'v_p' is independent of 'ω'. Coaxial cables are used to reduce this distortion.

$$\beta=\sqrt{\frac{(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+\omega^2(RC+LG)^2}}{2}}$$

Condition for minimum attenuation in distortionless Line:

A transmission line which satisfies the condition RC = LG is called as distortionless line.

RC = LG (1)

W.K.T $\gamma=P=\alpha+j\beta=\sqrt{ZY}$

$$\gamma=P=\alpha+j\beta=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = P = \alpha + j\beta = \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

$$\gamma = P = \alpha + j\beta = \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{R}{L} + j\omega\right)}$$

$$\gamma = P = \alpha + j\beta = \sqrt{LC}\left(\frac{R}{L} + j\omega\right) \text{ (Or) } \sqrt{LC}\left(\frac{G}{C} + j\omega\right)$$

$$\alpha + j\beta = \sqrt{LC}\,\frac{R}{L} + j\omega\sqrt{LC} \text{ (Or) } \sqrt{LC}\,\frac{G}{C} + j\omega\sqrt{LC}$$

$$\alpha + j\beta = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} \text{ (Or) } G\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}$$

$$\therefore\ \alpha = R\sqrt{\frac{C}{L}} \text{ (Or) } G\sqrt{\frac{L}{C}} \qquad (2)$$

$$\beta = \omega\sqrt{LC} \qquad (3)$$

From equs. (2) and (3) we come to know that 'α' is independent of frequency and 'β' is a constant multiplied by 'ω'.

PART C (1 X 14 = 14)

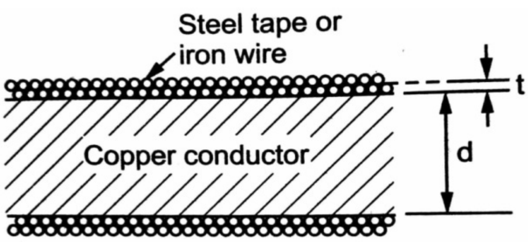
8.a) What is loading? Explain the different types of loading and also prove that distortionless line can be achieved by means of loading.

The process of increasing the inductance 'L' of a line is called as loading of a line. Loading is introduced in telephone cables. There are 3 types of loading.

a. Continuous loading
b. Lumped loading
c. patch loading

a) Continuous loading:

In this method, the inductance of the line is increased uniformly along the length of the line. In this type, iron or high permeability magnetic material in the form of a wire or tape is wound around the copper conductor as shown in figure.



The increase in the inductance for a continuously loaded line is,

$$L \approx \frac{\mu}{\frac{d}{nt}+1} \text{ mH}$$

where, μ - Permeability of surrounding material

d - Diameter of copper conductor

n - number of layers

t - Thickness gir layer of tape or iron wire

Advantages:

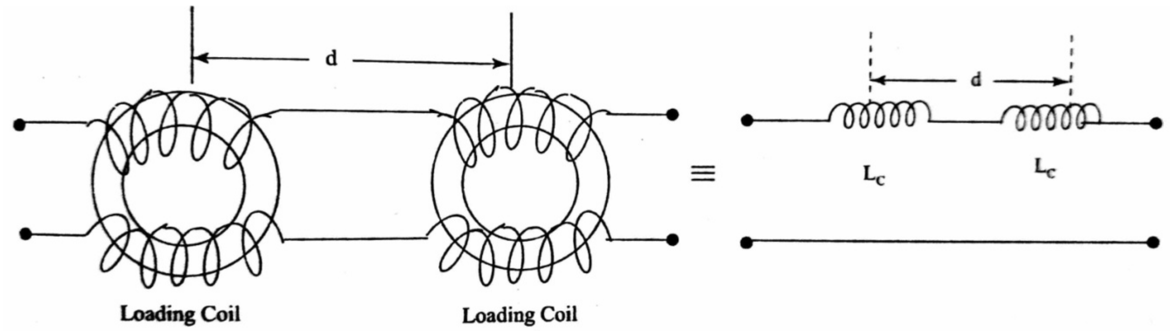
1. Attenuation is constant over a wide range of frequency.
2. Continuous loading is used on submarine cables.

Disadvantages:

1. Very expensive due to high cost of manufacture
2. Only low inductance value is possible
3. Since loading is done with iron wire, eddy current and hysteresis losses increases with frequency

b) Lumped Loading:

In this type of loading, the inductors are introduced in lumps at uniform distances in the line. The inductors are introduced in both the limbs to keep the line as balanced circuit. the lumped loading is preferred for open wire lines.



where, d - Spacing between two successive loading coils

Lc - Total lumped inductance in a length d of the line (both conductors)

Advantages:

1. Due to high toroidal cores, large values of inductance is possible
2. Eddy current and hysteresis losses are less.
3. cost is less.

Disadvantages:

1. Lumped loading is useful only for voice band circuits upto 3KHz.
2. Inductance value is not uniform throughout the line

c) Patch Loading:

This type of loading employs sections of continuously loaded cable separated by sections of unloaded cable. The typical length for the patch loading is normally 0.25Km.

To understand the significance of loading, consider a loaded telephone cable. the primary constants of a loaded cable are R, L, and C. (G=0)

$$Z=R + j\omega L \qquad (1)$$

$$Y = j\omega C \qquad (2)$$

$$\gamma=\sqrt{ZY}=\sqrt{|Z|\angle Z\times|Y|\angle Y} \qquad (3)$$

$$\gamma=\sqrt{\sqrt{R^2+\omega^2L^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right)\times\omega C\angle\frac{\pi}{2}}$$

$$\gamma=\sqrt{\sqrt{\omega^2L^2\left(1+\frac{R^2}{\omega^2L^2}\right)}\angle\frac{\pi}{2}-\tan^{-1}\left(\frac{R}{\omega L}\right)\times\omega C\angle\frac{\pi}{2}}$$

$$\gamma=\sqrt{\sqrt{\omega L\left(1+\frac{R^2}{\omega^2L^2}\right)}\omega C\angle\pi-\tan^{-1}\left(\frac{R}{\omega L}\right)}$$

$$\gamma=\omega\sqrt{LC}\sqrt{\sqrt{1+\frac{R^2}{\omega^2L^2}}}\angle\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)$$

Since $R<<\omega L$, $\frac{R^2}{\omega^2L^2}\approx 0$

$$\gamma=\omega\sqrt{LC}\angle\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)$$

Let $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\approx\theta$

$$\gamma=\omega\sqrt{LC}\angle\theta$$

$$\gamma=\omega\sqrt{LC}\,e^{j\theta}$$

$$\gamma=\omega\sqrt{LC}(\cos\theta+j\sin\theta) \qquad (4)$$

To find cos θ:

$$\cos\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)=\cos\left(\frac{\pi}{2}-\frac{1}{2}\left(\frac{R}{\omega L}\right)\right)$$

$$=\sin\left(\frac{1}{2}\left(\frac{R}{\omega L}\right)\right)=\frac{R}{2\omega L}$$

To find Sinθ:

$$\sin\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)=\sin\left(\frac{\pi}{2}-\frac{1}{2}\left(\frac{R}{\omega L}\right)\right)$$

$$=\cos\left(\frac{1}{2}\left(\frac{R}{\omega L}\right)\right)\approx 1$$

Therefore equ. (4) can be written as,

$$\gamma=\omega\sqrt{LC}\left(\frac{R}{2\omega L}+j1\right)$$

$$\alpha+j\beta=\frac{R}{2}\sqrt{\frac{C}{L}}+j\omega\sqrt{LC}$$

By equating the real and imaginary terms, we get

$$\alpha=\frac{R}{2}\sqrt{\frac{C}{L}},\ \beta=\omega\sqrt{LC}\ \text{and}\ v_p=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}$$

From the above expressions, it is obvious that by increasing the inductance value, distortionless line can be achieved.

(Or)

b) A generator of 1V, 1000 cycles, supplies power to a 100 mile open wire line terminated in 200 ohms resistance. The line parameters are R = 10.4 ohms per mile, L = 0.00367 Henry per mile, G = 0.8×10^{-6} mho per mile, C = 0.00835 µF per mile. Determine the following parameters; **Reflection coefficient**, Sending end impedance, Sending end current, Receiving end current, Receiving end voltage, Input power, Power delivered to the load and Efficiency of transmission line.

Solution:

Given:

E_g = 1V, f=1000 cycles = 1000 Hz, l=100 mile, Z_R = 200 ohms, R = 10.4 ohms/mile, L = 0.00367 H/mile, G = 0.8 x10^{-6} mho/ mile, C = 0.00835 µF/mile

$\omega = 2\pi f$ = 2x3.14x1000 = 6280

The characteristic impedance is given by,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{10.4 + j(6280)(0.00367)}{0.8 \times 10^{-6} + j(6280)(0.00835 \times 10^{-6})}}$$

$$Z_o = \sqrt{\frac{10.4 + j23.0476}{0.8 \times 10^{-6} + j5.2438 \times 10^{-5}}} = \sqrt{\frac{25.285\angle 65.713^\circ}{5.244 \times 10^{-5}\angle 89.126^\circ}}$$

$$Z_o = \frac{5.0284}{7.2415 \times 10^{-3}} \angle (65.713^\circ - 89.126^\circ)/2$$

Z_o = 694.38 $\angle$-11.71°

The propagation constant is given by,

$$P = \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$P = \gamma = \sqrt{(10.4 + j(6280)(0.00367)) \times (0.8 \times 10^{-6} + j(6280)(0.00835 \times 10^{-6}))}$$

$$P = \gamma = \sqrt{(10.4 + j23.0476) \times (0.8 \times 10^{-6} + j5.2438 \times 10^{-5})}$$

$$P = \gamma = \sqrt{25.285\angle 65.713 \times 5.244 \times 10^{-5}\angle 89.126^\circ}$$

$$P = \gamma = \sqrt{1.326 \times 10^{-3}\angle 65.713^\circ + 89.126^\circ}$$

$P=\gamma$ = 0.0364 $\angle$77.42° (Or) 7.928x10^{-3}+j0.0355

α = 7.928x10^{-3}

β = 0.0355

1. Reflection Coefficient, K

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{200 - 694.38\angle -11.71^\circ}{200 + 694.38\angle -11.71^\circ} = -0.5568 + j0.071 = 0.5613\angle 172.73^\circ$$

K=0.5613$\angle$172.73°

$$e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l}\angle \beta l = e^{(7.928 \times 10^{-3})(100)}\angle (0.0355)(100)$$

$$e^{\gamma l} = e^{(7.928 \times 10^{-3})(100)}\angle (0.0355)(100)\,\text{rad}$$

$e^{\gamma l}$ = $e^{0.7928}$ $\angle$203.4°

$e^{\gamma l}$ = 2.20957 $\angle$203.4°

$$e^{-\gamma l} = e^{-(\alpha + j\beta)l} = e^{-\alpha l} \angle -\beta l = e^{-(7.928\times10^{-3})(100)} \angle -(0.0355)(100)$$

$$e^{-\gamma l} = e^{-(7.928\times10^{-3})(100)} \angle -(0.0355)(100)\,\text{rad}$$

$e^{-\gamma l} = e^{-0.7928} \angle -203.4°$

$e^{-\gamma l} = 0.4526 \angle -203.4°$

$$Z_s = Z_o \left\{ \frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}} \right\}$$

$$Z_s = 694.38 \angle -11.71^\circ \left\{ \frac{2.20957\angle 203.4^\circ + (0.5613\angle 172.73^\circ)(0.4526\angle -203.4^\circ)}{2.20957\angle 203.4^\circ - (0.5613\angle 172.73^\circ)(0.4526\angle -203.4^\circ)} \right\}$$

$$Z_s = 694.38 \angle -11.71^\circ \left\{ \frac{2.20957\angle 203.4^\circ + 0.254\angle -30.67^\circ}{2.20957\angle 203.4^\circ - 0.254\angle -30.67^\circ} \right\}$$

$$Z_s = 694.38 \angle -11.71^\circ \left\{ \frac{2.071\angle -150.90^\circ}{2.368\angle -161.58^\circ} \right\}$$

$Z_S = 694.38 \angle -11.71^\circ \times 0.8747\angle 10.684^\circ$

$Z_S = 607.4\angle -1.026^\circ$

$$I_S = \frac{E_g}{Z_g + Z_{in}} = \frac{E_g}{Z_g + Z_S}$$

$$I_S = \frac{1}{0 + 607.4\angle -1.026^\circ} = 1.646\times10^{-3}\angle 1.026^\circ\ \text{A}$$

$I_S = 1.646\times10^{-3} \angle 1.026^\circ$

$E_S = I_S \times Z_S$

$E_S = 1.646\times10^{-3} \angle 1.026^\circ \times 607.4 \angle -1.026^\circ$

$E_S = 1$ V

We know that,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}s} - \frac{(Z_R - Z_o)}{(Z_R + Z_o)}e^{-\sqrt{ZY}s}\right]$$

At sending end, s=l, as s is measured from the receiving end.

$$I_S = \frac{I_R}{2}\left(\frac{Z_R + Z_o}{Z_0}\right)\left[e^{\sqrt{ZY}l} - K\,e^{-\sqrt{ZY}l}\right]$$

$Z_R + Z_O = 891.14\angle -9.1^\circ$

$Ke^{\sqrt{ZY}l} = 0.5613\angle 172.73^\circ \times 2.20957 \angle 203.4^\circ$

$Ke^{\sqrt{ZY}l} = 1.24\angle 16.13^\circ$

$$1.646x10^{-3}\angle 1.026^\circ = \frac{I_R}{2}\left(\frac{891.14\angle -9.1^\circ}{694.38\angle -11.71^\circ}\right)\left[2.20957\angle 203.4^\circ - 1.24\angle 16.13^\circ\right]$$

$$1.646x10^{-3}\angle 1.026^\circ = \frac{I_R}{2}\left(1.283\angle 2.61^\circ\right)\left[3.443\angle -159.21^\circ\right]$$

$$1.646x10^{-3}\angle 1.026^\circ = I_R \times 2.2088\angle -156.6^\circ$$

$\mathbf{I_R = 7.452x10^{-4}\angle 157.63^\circ}$

$E_R = I_R \times Z_R = 7.452x10^{-4}\angle 157.63^\circ \times 200 = 0.149\angle 157.63^\circ$

$\mathbf{E_R = 0.149\angle 157.63^\circ\ V}$

$P_S = |E_S|.\ |I_S|.\cos(E_S \wedge I_S)$

$P_S = 1 \times 1.646 \times 10^{-3} \times \cos(1.026)$

$\mathbf{P_S = 1.646 \times 10^{-3}\ W}$

and $P_R = |E_R|.\ |I_R|.\cos(E_R \wedge I_R)$

$P_R = 0.149 \times 7.452x10^{-4}\cos(0)$

$P_R = 0.149 \times 7.452x10^{-4}\ W$

$\mathbf{P_R = 1.110348x10^{-4}\ W}$

Transmission Efficiency, $\eta = \frac{P_R}{P_S} \times 100$

$$\eta = \frac{1.1103x10^{-4}}{1.646 \times 10^{-3}} \times 100$$

$$\eta = 6.746\%$$
