Solved Anna University Problems

(NOV/DEC 2010 - NOV/DEC 2019)

EC8651 - Transmission Lines and RF Systems

UNIT 3 - Impedance Matching in HF Lines (Except smith chart)

UNIT 3 - IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

PROBLEMS

QUARTER WAVEGUIDE:

1. Design a quarter wave transformer to match a load of 200Ω to a source resistance of 500Ω. The operating frequency is 200 MHz.

[EC6503-MAY/JUNE 2016](10 Marks) [EC6503-APR/MAY 2018](7 Marks), [EC6503-APR/MAY 2017](6 Marks)

Solution:

Given: f=200MHz, Z_R=200 Ω, Z_{in}=500 Ω

For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_S = \frac{R_o^2}{Z_R}$$

$$500 = \frac{R_o^2}{200}$$

Ro = 316.22 t

The wavelength is given by,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5\,\text{m}$$

a **= 1.5 m**

∴The length of quarter wave line is given by,

$$s = l = \frac{\lambda}{4} = \frac{1.5}{4} = 0.375\,\text{m}$$

y **s (Or) l = 0.375 m**

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2. It is required to match a 200 ohms load to a 300 ohms transmission line to reduce the SWR along the line to 1. What must be the characteristic impedance of the quarter wave transformer used for this purpose if it is directly connected to the load? **[EC2305-NOV/DEC 2010](4 Marks)**

Solution:

Given: Z_R=200 Ω, Z_S=300 Ω

For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_S = \frac{R_o^2}{Z_R}$$

$$300 = \frac{R_o^2}{200}$$

Ro = 244.95 t

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3. An ideal lossless quarter wave transmission line of characteristic impedance 60 ohm is terminated in a load impedance Z_L. Give the value of the input impedance of the line when Z_L = 0, ∞ and 60 ohm. **[EC2305-MAY/JUNE 2012](6 Marks)**

Solution:

Given: R_O=60 Ω, Z_L=0 Ω, Z_L=∞ Ω, Z_L=60 Ω,

For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_S = \frac{R_o^2}{Z_R}$$

i) When Z_L =0 t

$$Z_S = \frac{60^2}{0} = \infty$$

Z_S = ã t

ii) When Z_L =ã t

$$Z_S = \frac{60^2}{\infty} = 0$$

Z_S = 0 t

i) When Z_L =60 t

$$Z_S = \frac{60^2}{60} = 60$$

Z_S = 60 t

SINGLE STUB MATCHING:

1. A single stub is to match a 400 Ω line to a load of 200-j100 Ω. The wavelength is 3m. Determine the position and length of the short circuited stub.

[EC6503-NOV/DEC 2019](8 Marks) [EC6503-APR/MAY 2019](8 Marks)

Solution:

Given: $Z_O = R_O = 400\ \Omega$, $Z_R = 200 - j100\ \Omega$, $\lambda = 3$ m

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{200 - j100 - 400}{200 - j100 + 400} = \frac{-200 - j100}{600 - j100}$$

$$= -0.2973 - j0.2162 = 0.3676\angle -143.973^\circ$$

Stub Location 1:

$$l_s = \frac{\lambda}{4\pi}\left[\phi + \pi - \cos^{-1}|K|\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[-143.973^\circ + 180^\circ - \cos^{-1} 0.3676\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[-143.973^\circ + 180^\circ - 68.432^\circ\right]$$

l_s = -0.045 λ

(Note: The distance or position or location must be 0 to 0.5 λ. i) If the value is more than 0.5 λ, then subtract 0.5 λ from it until the value comes <=0.5 λ and ii) if the value is <0 λ, then add 0.5 λ to the value until the value becomes positive)

Therefore, l_s = -0.045 a + 0.5 a = 0.455 a

Stub Location 2:

$$l'_s = \frac{\lambda}{4\pi}\left[\varphi + \pi + \cos^{-1}|K|\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[-143.973^\circ + 180^\circ + \cos^{-1} 0.3676\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[-143.973^\circ + 180^\circ + 68.432^\circ\right]$$

l_s = 0.1451 a

Since stub location 2 is lesser in value when compared to the stub location 1, we select the stub location 2.

Therefore the location of the stub is **0.1451** ^{a}

We selected stub location 2. so, its corresponding length have to be calculated. The stub 2 length can be found from the length of stub 1.

Stub Length 1:

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2\times 180^\circ}\tan^{-1}\left(\frac{\sqrt{1-0.3676^2}}{2\times 0.3676}\right)$$

$$l_t = \frac{\lambda}{360^\circ}\tan^{-1}(1.2649)$$

$$l_t = \frac{\lambda}{360^\circ}\times 51.672^\circ$$

$\mathbf{l_t = 0.1435}$ ^{a}

Stub Length 2:

$l_t' = 0.5\lambda - l_t$

$l_t' = 0.5\lambda - 0.1435\lambda$

$\mathbf{l_t' = 0.3565}$ ^{a}

Summary:

The stub is located at a distance of **0.1451** ^{a} **or 0.4353 m** from the load and its corresponding length is **0.3565** ^{a} **or 1.0695 m**

1. Location of Stub = **0.1451** ^{a} **or 0.4353 m**

2. Length of Stub = **0.3565** ^{a} **or 1.0695 m**

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2. Determine the length and location of a single short circuited stub to produce an impedance match on a transmission line with R_o of 600 Ω and terminated in 1800Ω. **[EC6503-NOV/DEC 2016](8 Marks)**

Solution:

Given: $Z_O=R_O=600\Omega$, $Z_R=1800\Omega$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{1800 - 600}{1800 + 600} = \frac{1200}{2400} = 0.5 = 0.5\angle 0^\circ$$

Stub Location 1:

$$l_s = \frac{\lambda}{4\pi}\left[\phi + \pi - \cos^{-1}|K|\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[0^\circ + 180^\circ - \cos^{-1} 0.5\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[0^\circ + 180^\circ - 60^\circ\right]$$

l_s = 0.1667 a

Stub Location 2:

$$l_s' = \frac{\lambda}{4\pi}\left[\varphi + \pi + \cos^{-1}|K|\right]$$

$$l_s' = \frac{\lambda}{4 \times 180^\circ}\left[0^\circ + 180^\circ + \cos^{-1} 0.5\right]$$

$$l_s' = \frac{\lambda}{4 \times 180^\circ}\left[0^\circ + 180^\circ + 60^\circ\right]$$

l_s = 0.3333 a

Stub location 1 is lesser in value when compared to stub location 2. so, we select the stub location 1.

Therefore the location of the stub is **0.1667** a

We selected stub location 1. so, its corresponding length have to be calculated

Stub Length 1:

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1 - |K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2 \times 180^\circ}\tan^{-1}\left(\frac{\sqrt{1 - 0.5^2}}{2 \times 0.5}\right)$$

$$l_t = \frac{\lambda}{360^\circ}\tan^{-1}(0.866)$$

$$l_t = \frac{\lambda}{360^\circ}\times 40.893^\circ$$

l_t = 0.11361 ^{a}

(If frequency is given, calculate λ value. In this problem frequency value is not given so, no need to find λ value)

Summary:

The stub is located at a distance of **0.1667** ^{a} from the load and its corresponding length is **0.11361** ^{a}

1. Location of Stub = **0.1667** ^{a}

2. Length of Stub = **0.11361** ^{a}

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3. A UHF transmission line working at 1 GHz is connected to an unmatched line producing a voltage reflection coefficient of 0.5(0.866+j0.5). Calculate the length and position of the stub to match the line. **[EC2305-NOV/DEC 2010](8 Marks)**

Solution:

Given: $Z_O=R_O=600\Omega$, $Z_R=1800\Omega$, f=1GHz

$$K = 0.5(0.866 + j0.5) = 0.433 + j0.25 = 0.5\angle 30^\circ$$

Stub Location 1:

$$l_s = \frac{\lambda}{4\pi}\left[\phi + \pi - \cos^{-1}|K|\right]$$

$$l_s = \frac{\lambda}{4\times 180^\circ}\left[30^\circ + 180^\circ - \cos^{-1} 0.5\right]$$

$$l_s = \frac{\lambda}{4\times 180^\circ}\left[30^\circ + 180^\circ - 60^\circ\right]$$

l_s = 0.2083 ^{a}

Stub Location 2:

$$l_s' = \frac{\lambda}{4\pi}\left[\varphi + \pi + \cos^{-1}|K|\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[30^\circ + 180^\circ + \cos^{-1} 0.5\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[30^\circ + 180^\circ + 60^\circ\right]$$

l_s = 0.375 ^{a}

Since stub location 1 is lesser in value when compared to stub location 2, so, we select the stub location 1.

Therefore the location of the stub is **0.2083** ^{a}

We selected stub location 1. so, its corresponding length have to be calculated

Stub Length 1:

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2 \times 180^\circ}\tan^{-1}\left(\frac{\sqrt{1-0.5^2}}{2 \times 0.5}\right)$$

$$l_t = \frac{\lambda}{360^\circ}\tan^{-1}(0.866)$$

$$l_t = \frac{\lambda}{360^\circ} \times 40.893^\circ$$

l_t = 0.11361 ^{a}

(If frequency is given, calculate λ value. In this problem frequency value is 1 GHZ)

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{1 \times 10^9} = 0.3\,\text{m}$$

Summary:

The stub is located at a distance of **0.2083** ^{a} **or 0.0625 m** from the load and its corresponding length is **0.11361** ^{a} **or 0.0341 m**

1. Location of Stub = **0.2083** ^{a} **or 0.0625 m**

2. Length of Stub = **0.11361** ^{a} **or 0.0341 m**

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4. A 100 ohm, 200m long lossless transmission line operates at 10 MHz and is terminated into an impedance of 50-j200 ohm. The transit time of the line is 1 μs. Determine the length and location of a short circuited stub line.

[EC2305-MAY/JUNE 2012](8 Marks)

Solution:

Given: $Z_O=R_O=100\ \Omega$, $Z_R=50-j200\ \Omega$, f=10MHz

$$K=\frac{Z_R-Z_O}{Z_R+Z_O}$$

$$K=\frac{50-j200-100}{50-j200+100}=\frac{-50-j200}{150-j200}$$

$$=0.52-j0.64=0.8246\angle-50.91^\circ$$

Stub Location 1:

$$l_s=\frac{\lambda}{4\pi}\left[\phi+\pi-\cos^{-1}|K|\right]$$

$$l_s=\frac{\lambda}{4\times180^\circ}\left[-50.91^\circ+180^\circ-\cos^{-1}0.8246\right]$$

$$l_s=\frac{\lambda}{4\times180^\circ}\left[-50.91^\circ+180^\circ-34.452^\circ\right]$$

$\mathbf{l_s = 0.1314\ \lambda}$

Stub Location 2:

$$l'_s=\frac{\lambda}{4\pi}\left[\varphi+\pi+\cos^{-1}|K|\right]$$

$$l'_s=\frac{\lambda}{4\times180^\circ}\left[-50.91^\circ+180^\circ+\cos^{-1}0.8246\right]$$

$$l'_s=\frac{\lambda}{4\times180^\circ}\left[-50.91^\circ+180^\circ+34.452^\circ\right]$$

$\mathbf{l_s = 0.2271\ \lambda}$

Stub location 1 is lesser in value when compared to the stub location 2, we select stub location 1. Therefore the location of the stub is **0.1314 λ**

We selected stub location 1. so, its corresponding length have to be calculated.

Stub Length 1:

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2\times 180^\circ}\tan^{-1}\left(\frac{\sqrt{1-0.8246^2}}{2\times 0.8246}\right)$$

$$l_t = \frac{\lambda}{360^\circ}\tan^{-1}(0.343)$$

$$l_t = \frac{\lambda}{360^\circ}\times 18.93^\circ$$

l_t = 0.0526 ª

$$\lambda = \frac{C}{f} = \frac{3\times 10^8}{1\times 10^6} = 300\,\text{m}$$

Summary:

The stub is located at a distance of **0.1314** ª **or 39.42 m** from the load and its corresponding length is **0.0526** ª **or 15.78 m**

1. Location of Stub = **0.1314** ª **or 39.42 m**

2. Length of Stub = **0.0526** ª **or 15.78 m**

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5. A single stub is to match a 300 Ω line to a load of (180+j120) Ω. The wavelength is 2 meters. Determine the shortest distance from the load to the stub location and proper length of a short circuited stub using relevant formula. **[EC2503-APR/MAY 2014](8 Marks)**

Solution:

Given: $Z_O=R_O=300\ \Omega$, $Z_R=180+j120\ \Omega$, $\lambda=2$ m

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$K = \frac{180 + j120 - 300}{180 + j120 + 300} = \frac{-120 + j120}{480 + j120}$$

$$= -0.1765 + j0.2941 = 0.343\angle 120.964^\circ$$

Stub Location 1:

$$l_s = \frac{\lambda}{4\pi}\left[\phi + \pi - \cos^{-1}|K|\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[120.964^\circ + 180^\circ - \cos^{-1} 0.343\right]$$

$$l_s = \frac{\lambda}{4 \times 180^\circ}\left[120.964^\circ + 180^\circ - 69.94^\circ\right]$$

l_s = 0.3209 ª

Stub Location 2:

$$l'_s = \frac{\lambda}{4\pi}\left[\varphi + \pi + \cos^{-1}|K|\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[120.964^\circ + 180^\circ + \cos^{-1} 0.343\right]$$

$$l'_s = \frac{\lambda}{4 \times 180^\circ}\left[120.964^\circ + 180^\circ + 69.94^\circ\right]$$

l'_s = 0.5151 λ

(Note: The distance or position or location must be 0 to 0.5 λ. i) If the value is more than 0.5 λ, then subtract 0.5 λ from it until the value comes <=0.5 λ and ii) if the value is <0 λ, then add 0.5 λ to the value until the value becomes positive)

Therefore, l'_s = 0.5151 ª - 0.5 ª = 0.0151 ª

Stub location 2 is lesser in value when compared to the stub location 1, we select stub location 1. Therefore the location of the stub is **0.0151 ª**

We selected stub location 2. so, its corresponding length have to be calculated.

The stub 2 length can be found from the length of stub 1.

Stub Length 1:

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2 \times 180^\circ}\tan^{-1}\left(\frac{\sqrt{1-0.343^2}}{2 \times 0.343}\right)$$

$$l_t = \frac{\lambda}{360^\circ} \tan^{-1}(1.369)$$

$$l_t = \frac{\lambda}{360^\circ} \times 53.859^\circ$$

l_t = 0.1496 ^{a}

Stub Length 2:

l_t' = 0.5λ - l_t

l_t' = 0.5λ - 0.1496λ

l_t' = 0.3504 ^{a}

Summary:

The stub is located at a distance of **0.0151** ^{a} **or 0.0302 m** from the load and its corresponding length is **0.3504** ^{a} **or 0.7008 m**

1. Location of Stub = **0.0151** ^{a} **or 0.0302 m**

2. Length of Stub = **0.3504** ^{a} **or 0.7008 m**
